Inequalities

- Avoid multiplying or dividing an inequality by an unknown (because we cannot tell whether we need to reverse the inequality sign)
- Make the RHS zero.
- Write LHS as a single fraction, factorized as much as possible
- Find the 'critical values' of x
- Use a table to determine the sign of LHS

Examples

① $$\frac{x-3}{2(x+2)} > \frac{1}{x-4}$$

$$\frac{x-3}{2(x+2)} - \frac{1}{x-4} > 0$$

$$\frac{(x-3)(x-4) - 2(x+2)}{2(x+2)(x-4)} > 0$$

$$\frac{x^2-7x+12-2x-4}{2(x+2)(x-4)} > 0$$

$$\frac{(x-8)(x-1)}{2(x+2)(x-4)} > 0$$

$-2 \quad 0 \quad 1 \quad 4 \quad 8$

$x-8$	$-$	$-$	$-$	$-$	$+$
$x-1$	$-$	$-$	$+$	$+$	$+$
$x+2$	$-$	$+$	$+$	$+$	$+$
$x-4$	$-$	$-$	$-$	$+$	$+$
LHS	$+$	$-$	$+$	$-$	$+$

Solution : $\underline{\underline{x < -2 \text{ or } 1 < x < 4 \text{ or } x > 8}}$

② $$\frac{x^2 - 2x - 4}{x^2 - 2x + 2} > 0$$

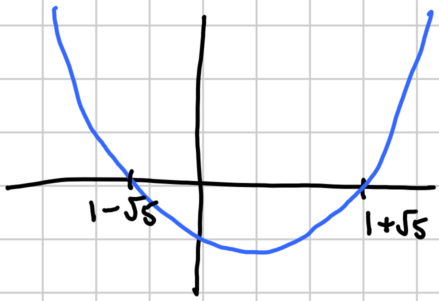
Neither top nor bottom will factorize

Complete the square:

$$\frac{(x-1)^2 - 5}{(x-1)^2 + 1} > 0$$

Bottom is $\underline{\text{always}}$ +ve.

Top:
$$(x-1)^2 - 5 = 0$$
$$\Rightarrow \quad (x-1)^2 = 5$$
$$x - 1 = \pm\sqrt{5}$$
$$x = 1 \pm \sqrt{5}$$



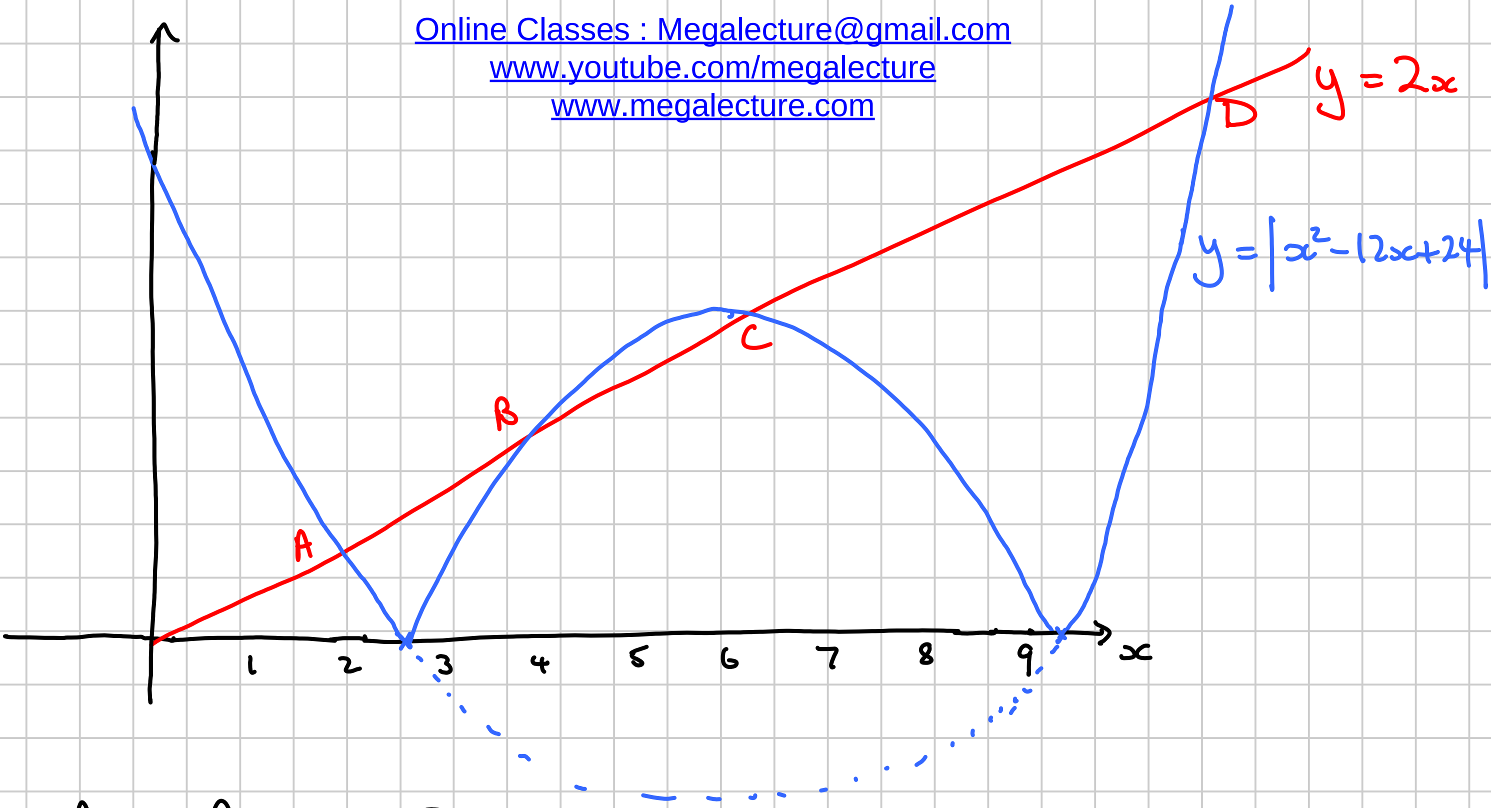
Top is +ve when $\underline{\underline{x < 1-\sqrt{5} \text{ or } x > 1+\sqrt{5}}}$

- For modulus inequalities always draw a graph

③ $$|x^2 - 12x + 24| < 2x$$

Solve $x^2 - 12x + 24 = 0$

$$x = \frac{12 \pm \sqrt{144 - 96}}{2}$$
$$= \frac{12 \pm 4\sqrt{3}}{2} = 6 \pm 2\sqrt{3}$$



At A and D,

$$x^2 - 12x + 24 = 2x$$
$$x^2 - 14x + 24 = 0$$
$$(x-2)(x-12) = 0$$
$$x = 2 \text{ (at A)} \quad \text{or} \quad x = 12 \text{ (at D)}$$

At B and C

$$-(x^2 - 12x + 24) = 2x$$
$$0 = x^2 - 10x + 24$$
$$0 = (x-4)(x-6)$$
$$x = 4 \text{ (at B)} \quad \text{or} \quad x = 6 \text{ (at C)}$$

So solution is $\underline{\underline{2 < x < 4 \quad \text{or} \quad 6 < x < 12}}$

p 6 Ex 1.2 Q 1g, 3, 4
p 9 Ex 1.3 Q 2e, 3b, 4f, 5f
p16 Ex 1.5 Q 1df, 2df, 5b, 6d