Physics (A-level)

Circular motion (chap.7):

- One **radian** (rad) is defined as the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle

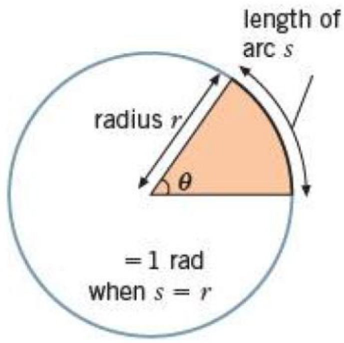


Figure 7.1 θ in radians = arc/radius

$$\theta = \frac{\text{length of arc}}{\text{radius of circle}} \left(= \frac{s}{r}\right)$$

- The **angular speed** is defined as the rate of change of angular displacement

$$\text{angular speed } \omega = \frac{\Delta\theta}{\Delta t}$$

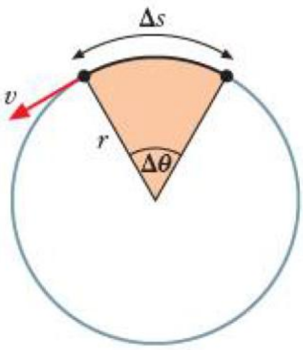


Figure 7.2 Angular velocity $\omega = v/r$

- Figure 7.2, v → constant, in Δt object moves along the arc Δs and sweeps out at ΔØ:
  - ➢ Δs = rΔØ and dividing both sides by Δt:
    - ▪ Δs/Δt = rΔØ/Δt
  - ➢ $v = r\omega$

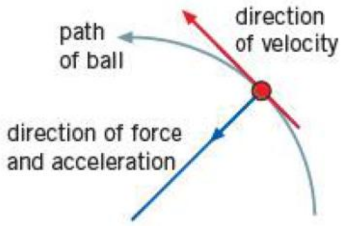


Figure 7.3 A ball swung in a circle on the end of a string

- Both the centripetal acceleration and force are towards the center (90° to that of the instantaneous velocity)
- Figure 7.4 & 7.5 shows the angle between two radii OA and OB & v_A and v_B (ΔØ)
  - ➢ Triangles OAB and CDE are similar
  - ➢ Consider angle ΔØ to be so small that arc AB approximated as a straight line
  - ➢ DE/CD = AB/OA
  - ➢ $\Delta v/v_A = \Delta s/r$
  - ➢ $\Delta v = \Delta s(v_A/r)$ and dividing both sides by Δt
  - ➢ $\Delta v/\Delta t = (\Delta s/\Delta t)(v_A/r)$
  - ➢ $A = v^2/r$

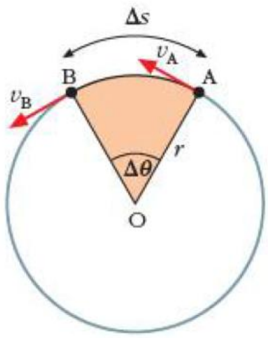
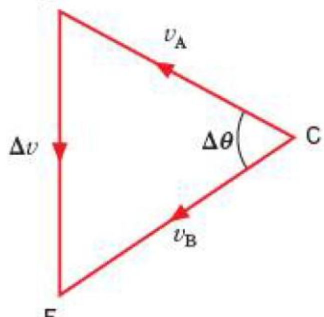


Figure 7.4 Diagram for proof of $a = v^2/r$



...5 Vector diagram ...f $a = v^2/r$

$$\text{centripetal acceleration} = \frac{v^2}{r} = r\omega^2$$

$$\text{centripetal force} = \frac{mv^2}{r} = mr\omega^2$$

Gravitational fields (chap.8):

- A **gravitational field** is a region of space where a mass experiences a force
- **Newton's law of gravitation** states that two point masses attract each other with a force that is proportional to the product of their masses and inversely proportional to the square of their separation:

$$F \propto m_1 m_2 / r^2 \qquad F = \frac{G m_1 m_2}{r^2}$$

- G (gravitational constant) = 6.67×10^{-11} N m^2 kg^{-2}
- Newton's law specifies that the two masses are **point masses**, however the law still holds where the diameter/size of the masses is small compared to their separation
- Differences between gravitational fields and electric fields:
  - The electric field acts on charges, whereas the gravitational field acts on masses
  - The electric field can be attractive or repulsive, whereas gravitational field always attractive
- The gravitational field outside a spherical uniform mass is radial (all the lines of gravitational force appear to radiate from the centre of the sphere)
- Circular motion:
  - $F_{grav} = F_{circ}$
  - $GMm/r^2 = mv^2/r$
  - The period T of the planet in its orbit is the time required for the planet to travel a distance $2\pi r$:
    - $V = 2\pi r/T$
  - $GMm/r^2 = m(4\pi^2 r^2/T^2)/r$
  - $T^2 = (4\pi^2/GM)r^3$

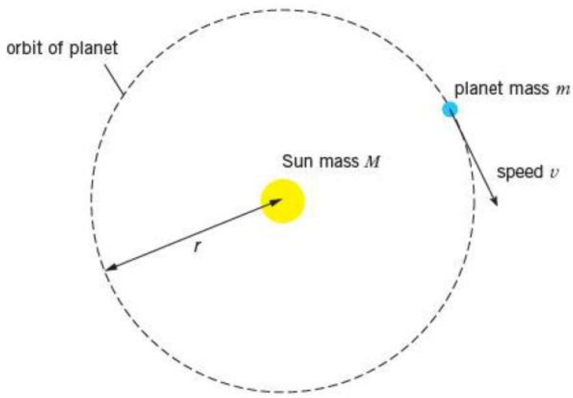


Figure 8.2 Circular orbit of a planet about the Sun

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

  - The right hand-side of the equation shows the constants (π and G), where M is the same (mass of the sun in the e.g.) when we are considering the relation between T and r
- **Kepler's third law of planetary motion** states that for planet or satellites describing circular orbits about the same central body, the square of the period is proportional to the cube of the radius of the orbit ($T^2 \propto r^3$)
- **Geostationary orbit** refers to communication satellites (called geostationary satellites) that are in equatorial orbits with exactly the same period of rotation as the Earth (24 hours), and move in the same direction as the Earth (west to east) so that they are always above the same point on the Equator
- The **gravitational field strength** at a point is defined as the force per unit mass acting on a small mass placed at that point
- Newton's second law: F = ma. Thus the gravitational field strength is given by g = F/m

$$g = \frac{F}{m} = \frac{GM}{r^2}$$

- For small distances above the Earth's surface, g is approximately constant and is called the acceleration of free fall
- **Gravitational potential** at a point in a gravitational field is defined as the work done per unit mass in bringing a unit mass from infinity to the point

$$\phi = -\frac{GM}{r}$$

  - The gravitational potential is negative due to the **always** attractive gravitational force, hence there is work done by the test mass, decreasing its potential
- **g.p.e:** the work done in bringing an object from infinity to the point
- For a body of mass m, then the gravitational potential energy of the body will be *m times* as large as for the body of the unit mass

$$\text{Gravitational potential energy} = m\phi = -\frac{GMm}{r}$$

Example questions:

A satellite is orbiting the Earth. For an astronaut in the satellite, his sensation of weight is caused by the contact force from his surroundings. The astronaut reports that he is 'weightless', despite being in the Earth's gravitational field. Suggest what is meant by the astronaut reporting that he is 'weightless'.

- gravitational force provides the centripetal force
- gravitational force is 'equal' to the centripetal force
  - (accept Gm1m2 / x2 = mxω2 or $F_C = F_G$)
- 'weight'/sensation of weight/contact force/reaction force is difference between FG and FC which is zero

Explain why the centripetal force acting on both stars has the same magnitude.

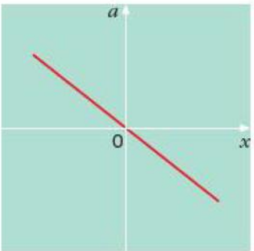
- gravitational force provides/is the centripetal force
- same gravitational force (by Newton III)

Oscillations (chap.13):

- The time taken for one complete oscillation or vibration is referred to as the **period T** of the oscillation
- The number of oscillations or vibrations per unit time is the **frequency f**
- Frequency f = 1/T
- The distance from the equilibrium position is known as the **displacement** (vector quantity)
- The **amplitude** (scalar quantity) is the maximum displacement
- **Simple harmonic motion** is defined as the motion of a particle about a fixed point such that its acceleration is proportional to its displacement from the fixed point, and is directed towards the point

$$a = -\omega^2 x$$

- A sinusoidal displacement-time graph is a characteristic of s.h.m.
- **Harmonic** oscillators move in s.h.m.



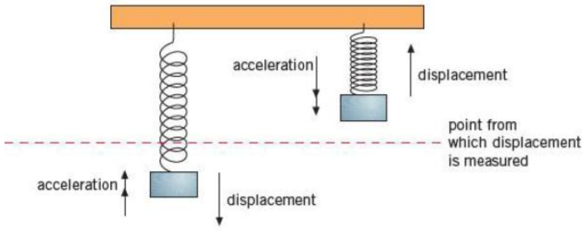


Figure 13.3 Directions of displacement and acceleration are always opposite.

- ω is known as the **angular frequency** of the oscillation
  - $\omega = 2\pi f$
- Newton's second law states that the force acting on the body is proportional to the acceleration of the body; hence the **restoring force** is proportional to the displacement and acting towards the fixed point
- Solution of equation for s.h.m.:

$$x = x_0 \sin \omega t$$
or
$$x = x_0 \cos \omega t$$

  - X_0 → amplitude of oscillation
  - V → the gradient of displacement-time graph

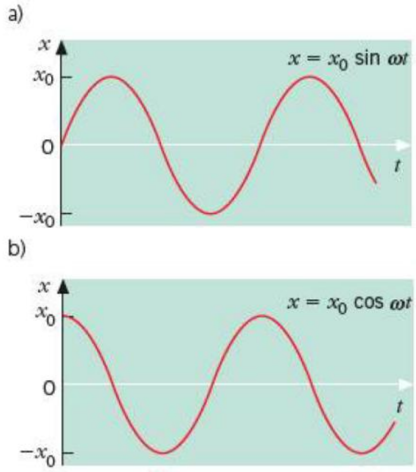


Figure 13.5 Displacement–time curves for the two solutions to the s.h.m. equation

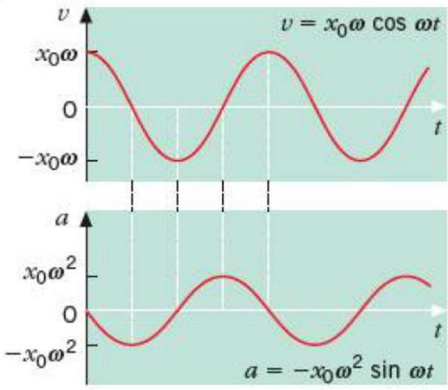
$$v = x_0\omega \cos \omega t \text{ when } x = x_0 \sin \omega t$$

  - For the case where x is zero at time t = 0, displacement and velocity are given by:
    - $x = x_0 \sin \omega t$
    - $v = x_0\omega \cos \omega t$
  - Applying $\sin^2\emptyset + \cos^2\emptyset = 1$:

$x^2/x_0^2 + v^2/x_0^2\omega^2 = 1$ leading to:

$v^2 = x_0^2\omega^2 - x^2\omega^2$ hence:

$$v = \pm\omega\sqrt{(x_0^2 - x^2)}$$



  - a → the gradient of velocity-time graph

$$a = -x_0\omega^2 \sin \omega t \text{ when } x = x_0 \sin \omega t$$

$$a = -\omega^2 x$$

- The K.E. of the particle oscillating with s.h.m. is ½mv^2:

$$E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)$$

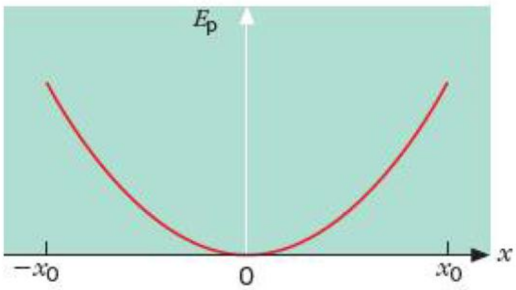


Figure 13.11 Variation of potential energy in s.h.m.

- The restoring force is F = ma:

$$F_{res} = -m\omega^2 x$$

- The potential energy:

$$E_p = \frac{1}{2} m\omega^2 x^2$$

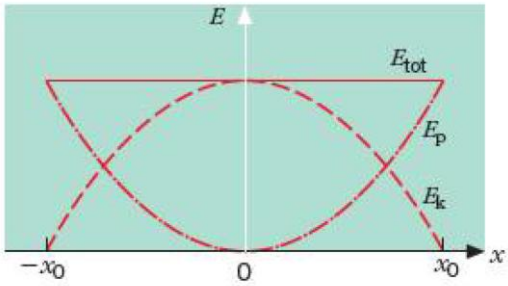


Figure 13.12 Energy variations in s.h.m.

- The total energy E_{tot} of the oscillating particle:

$$E_{tot} = E_k + E_p$$
$$= \frac{1}{2} m\omega^2 (x_0^2 - x^2) + \frac{1}{2} m\omega^2 x^2$$

$$E_{tot} = \frac{1}{2} m\omega^2 x_0^2$$

- A particle is said to be undergoing **free oscillations** when the only external force acting on it is the restoring force (vibrating at its natural frequency):
  - No force to dissipate energy, hence constant amplitude and total energy remains constant, so s.h.m. are free oscillations
- In real situations, however, resistive forces cause the oscillator's energy to be dissipated, eventually converted into thermal energy. The oscillations are said to be **damped**
  - Light damping: the amplitude decreases gradually with time (T of the oscillation is slightly greater than the corresponding free oscillation)
  - Heavy damping: the oscillations will die away more quickly
  - Critically damped: the displacement decreases to zero in the shortest time, without any oscillation
  - Overdamping: the displacement decreases to zero in a longer time than for critical damping

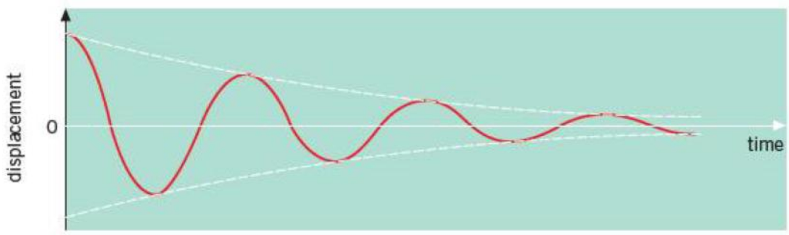


Figure 13.13 Lightly damped oscillations

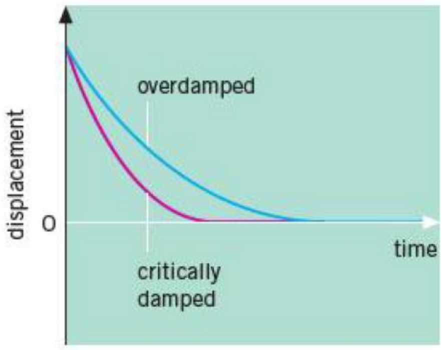


Figure 13.14 Critical damping and overdamping

- When a vibrating body undergoes free (undamped) oscillations, it vibrates at its **natural frequency**
- Periodic forces will make the object vibrate at the frequency of the applied force (forced vibrations)
- During forced oscillations, at first the amplitude is small, but increases with increasing frequency, reaches a maximum amplitude, then decreases (shown in a **resonance curve**)
  - **Resonance** occurs when a natural frequency of vibration of an object is equal to the driving frequency, giving a maximum amplitude of vibration
  - The frequency at which resonance occurs is called the **resonant frequency**

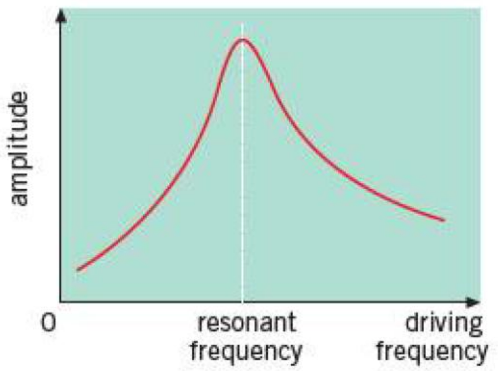


Figure 13.17 Resonance curve

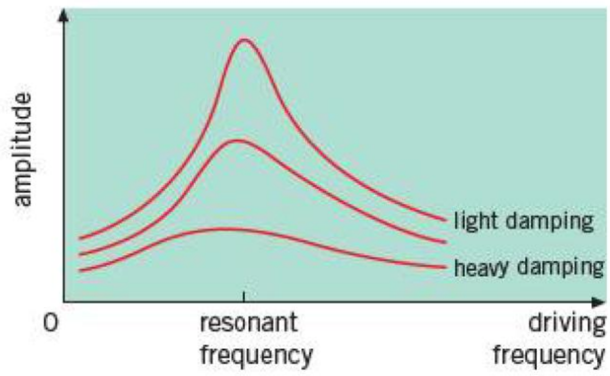


Figure 13.18 Effect of damping on the resonance curve

- As the degree of damping increases:
  - The amplitude of oscillation at all frequencies reduced
  - The frequency at maximum amplitude gradually shifts towards lower frequencies
  - The peak becomes flatter

Temperature (Chapter 11):

- Thermal energy is transferred from a region of higher temperature to a region of lower temperature
- **Thermal equilibrium** is a condition when two or more objects in contact have the same temperature so that there is no net flow of energy between them
- °C has two fixed point – the melting point of pure ice and the boiling point of pure water – divides the range between them into 100 equal intervals (changes when pressure changes or has impurities)
- Thermodynamic scale (Kelvin scale) is said to be an absolute scale, as it is not defined in terms of a property of any particular substance, based on the idea that the K.E. increases with increases in temperature; it has two fixed points:
  - Absolute zero (0 K) – (minimum internal energy of all substances, 0 K.E. and minimum electrical potential energy)
  - The triple point of water, the temperature at which ice, water and water vapour can co-exist, which is defined as 273.16 K (0.01 °C)

$\theta/°C = T/K - 273.15$ or $T/K = \theta/°C + 273.15$

Feature	Resistance thermometer (thermistor)	Thermocouple thermometer
Robustness	Very robust	Robust
Range	Narrow range	Can be very wide
Size	Larger, has greater thermal capacity hence slower acting	Smaller, has smaller thermal capacity hence quicker acting and can measure temp. at a point
Sensitivity	High sensitivity over a narrow range	Can be sensitive according to the metals chosen
linearity	Fairly linear over a narrow range	Non-linear so requires calibration
Remote operation	Long conducting wires allow the operator to be at a distance from the thermometer	Long conducting wires allow the operator to be at a distance from the thermometer

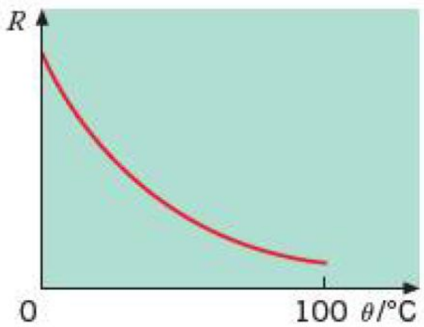


Figure 11.8 Resistance *R* of a thermistor over a small range of temperatures

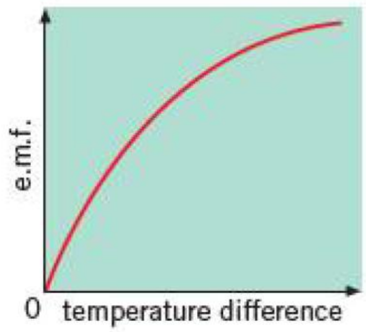


Figure 11.10 Graph of thermocouple e.m.f. against temperature

Thermal properties of materials (Chapter 12):

- A solid has fixed volume and shape (particles are close together, tightly bonded to their neighbors, and vibrating about fixed points)
  - During transition between solid and liquid, the energy supplied does not increase the K.E, hence the temperature of the solid, instead it is used to overcome the intermolecular forces between the atoms or molecules – increasing the electrical potential energy of the molecules, this increase is the latent heat of fusion of solid
- A liquid has fixed volume, no fixed shape and similar density as to solid
  - During transition between liquid and gas, the intermolecular forces in the liquid must be overcome, the latent heat of vaporization
- The graph shows that:
  - The electrical potential energy of two atoms very close together is large and negative
  - As the separation increases, their potential energy also increases
  - When atoms are completely separated, their potential energy is maximum and has a value of zero

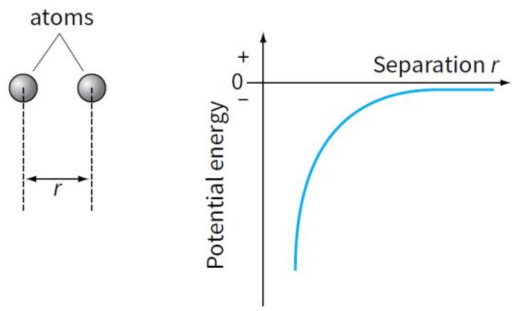


Figure 21.5 The electrical potential energy of atoms is negative and increases as they get further apart.

- A gas has no fixed shape or volume (widely separated and free to move around within their container)
- Latent heat of vaporization > latent heat of fusion, due to the greater energy required to completely separate the molecules than to break the rigid bonds in the solid (melting involves breaking of fewer bonds per molecule); energy is required to push back the atmosphere as liquid turns to vapour, vol. of vapour > vol. of liquid
- During evaporation, the most energetic molecules are most likely escape the surfaces of the liquid and hence reducing the average K.E. thus its temperature (cooling effect)
- **The internal energy** of a system is the sum of the random distribution of kinetic and potential energies of its atoms or molecules
  - Can be increased by heating and/or compression
- **First law of thermodynamics**: The increase in internal energy of a body is equal to the thermal energy transferred to it by heating plus the mechanical work done to it
  - $\Delta U = q + w$

- **Specific heat capacity:** The energy required per unit mass of a substance to raise its temperature by 1 K (or 1 °C). Unit: J kg^{-1} K^{-1}

$\Delta Q = mc\Delta\theta$

  - Assuming no energy losses to the surroundings:

$$M \times c \times (T_f - T_i) = I \times V \times t$$

  - Assuming energy losses (e.g. in 300s):
    - First reading:

thermal energy supplied = *thermal energy gained by liquid* + *energy losses to surroundings*

$$V_1 \times I_1 \times 300 = M_1 \times c \times (T_o - T_i) + h$$

    - Second reading:

$$V_2 \times I_2 \times 300 = M_2 \times c \times (T_o - T_i) + h$$

    - Subtracting:

$$(V_2 \times I_2 \times 300) - (V_1 \times I_1 \times 300) = (M_2 - M_1) \times c \times (T_o - T_i)$$

Hence, thermal energy losses have been eliminated and c can be det

- **Specific latent heat of fusion:** The energy required per unit mass of a substance to change it from solid to liquid without a change in temperature. Unit: J kg−1

$\Delta Q = mL_f$

- **Specific latent heat of vaporization:** The energy required per unit mass of a substance to change it from liquid to gas without a change in temperature. Unit: J kg−1

$\Delta Q = mL_v$

  - Assuming no energy losses to the surroundings:

$$(M - m) \times L = V \times I \times 300$$

  - Assuming energy losses (e.g. in 300s):
    - First reading:

thermal energy supplied by heater = *energy used to vaporise water* + *energy losses to surroundings*

$$V_1 \times I_1 \times 300 = M_1 \times L + h$$

    - Second reading:

$$V_2 \times I_2 \times 300 = M_2 \times L + h$$

    - Subtracting:

$$(V_2 I_2) \times 300 - (V_1 I_1) \times 300 = (M_2 - M_1) \times L$$

- Exchanges of heat energy examples:

1 A mass of 0.30 kg of water at 95 °C is mixed with 0.50 kg of water at 20 °C. Calculate the final temperature of the water, given that the specific heat capacity of water is 4200 J kg^{-1} K^{-1}.
Hint: always start by writing out a word equation containing all the gains and losses of heat energy.
heat energy lost by hot water = heat energy gained by cold water

$$(m \times c \times \Delta\theta_1) = (M \times c \times \Delta\theta_2)$$

$$0.30 \times 4200 \times (95 - \theta) = 0.50 \times 4200 \times (\theta - 20)$$

where θ is the final temperature of the water.

$$1260 \times (95 - \theta) = 2100 \times (\theta - 20)$$

$$119700 - 1260\theta = 2100\theta - 42000$$

$$161700 = 3360\theta$$

$$\theta = \mathbf{48\,°C}$$

2 A mass of 12 g of ice at 0 °C is placed in a drink of mass 210 g at 25 °C. Calculate the final temperature of the drink, given that the specific latent heat of fusion of ice is 334 kJ kg^{-1} and that the specific heat capacity of water and the drink is 4.2 kJ kg^{-1} K^{-1}.
energy lost by drink = energy gained by melting ice + energy gained by ice water

$$(m \times c \times \Delta\theta_1) = (M \times L_f) + (M \times c \times \Delta\theta_2)$$

$$M \times c \times \Delta\theta_2 = \frac{12}{1000} \times 4.2 \times 1000 \times (\theta - 0)$$

$$= 50.4\theta$$

$$\frac{210}{1000} \times 4.2 \times 1000 \times (25 - \theta) = \frac{12}{1000} \times 334 \times 1000 + 50.4\theta$$

where θ is the final temperature of the drink. Simplifying,

$$22050 - 882\theta = 4008 + 50.4\theta$$

$$18\,042 = 932.4\theta$$

$$\theta = \mathbf{19\,°C}$$

Electric fields (Chapter 17):

- For any point outside a spherical conductor, the charge on the sphere may be considered to act as a point charge at the centre of the sphere
- **Coulomb's law**: Any two point charges exert an electrical force on each other that is proportional to the product of their charges and inversely proportional to the square of the distance between them

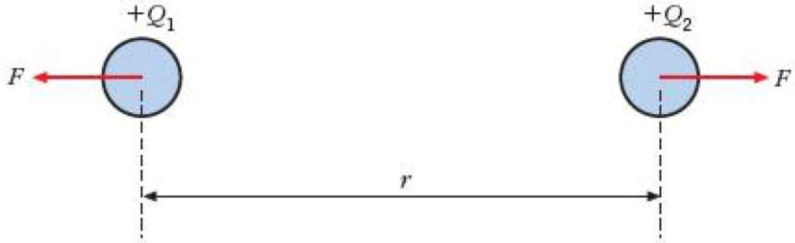


Figure 17.2 Force between charged spheres

- Figure 17.2 shows for point charges Q1 and Q2 a distance r apart. Coulomb's law gives the force F as:

$F \propto Q_1Q_2/r^2$

or

$$F = \frac{kQ_1Q_2}{r^2}$$

where k is a constant of proportionality, the value of which depends on the insulating medium around the charges and the system of units employed. In SI units, F is measured in newtons, Q in coulombs and r in metres. Then the constant k is given as

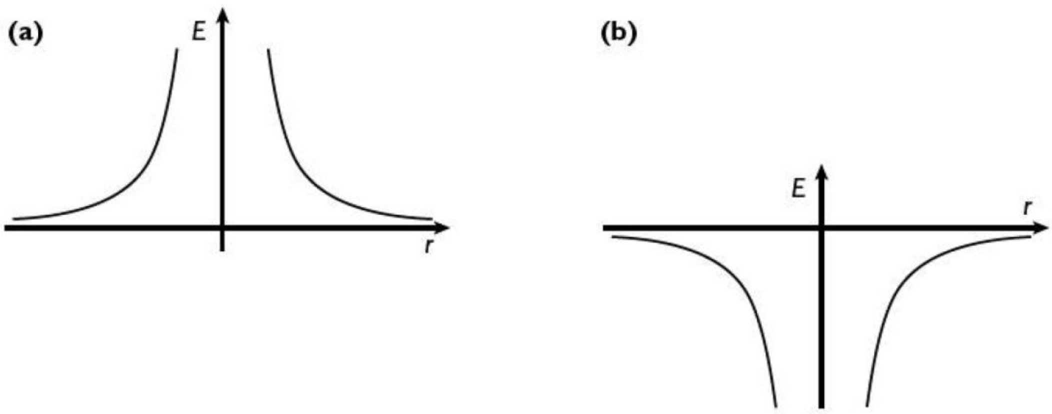
$$k = \frac{1}{4\pi\varepsilon_0}$$

and so

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$$

- When the charges are in a vacuum (or air), the quantity ε_0 is called the **permittivity of free space**
  - ➢ $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$
- **Electric field strength**: The force per unit positive charge at a point. Unit: Vm^{-1} or NC^{-1}
- The electric field E at the location of q is given by $E = F/q$, hence the electric field due to the isolated point charge is:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$



The electric field near (a) a positive point charge and (b) a negative point charge

- **Electric potential:** The energy per unit charge due to a charged body's position in an electric field. Unit: *V* (volt)
- Similar to electric field strength, electric potential is defined as the potential energy per unit positive charge (e.g. at point A, having potential energy E_{PA}):
  - ➢ $V_A = E_{PA}/Q$

- The electric field strength is equal to the negative of the potential gradient at that point:

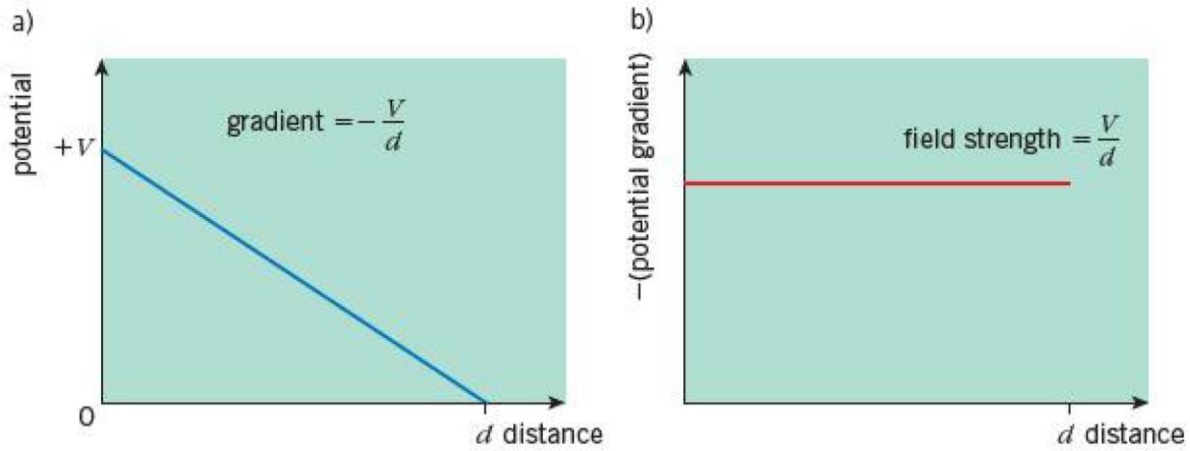


Figure 17.3 Graphs of the potential and the (negative of the) potential gradient for a uniform electric field.

- The potential *V* is given by:

$$V = \frac{kQ}{r} = \frac{Q}{4\pi\varepsilon_0 r}$$

- Electric fields and gravitational fields:
  - force ∝ 1 / r 2
  - potential ∝ 1 / r
  - gravitation force (always) attractive
  - electric force attractive or repulsive

Ideal gases (Chapter 10):

- **Boyle's law:** The pressure exerted by a fixed mass of gas is inversely proportional to its volume, provided the temperature of the gas remains constant
  - P1V1 = P2V2
- **Charles's law:** The volume occupied by a gas at constant pressure is directly proportional to its thermodynamic (absolute) temperature

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

- **Gay-Lussac's law:** $P_1/T_1 = P_2/T_2$
- Combining the three gas laws into a single relation:

$$pV \propto T$$

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

- Another series of experiments provide (using the number *n* of moles of the gas):

$pV \propto nT$ or $pV = nRT$

- The previous equation can be expressed in the form: $pV = NkT$
  - *N* is the number of molecules in the gas
  - *k* is the **Boltzmann constant** ($1.38 \times 10^{-23}\ \text{J K}^{-1}$)
- The molar gas constant *R* and the Boltzmann constant *k* are connected through N_A:

$k = R/N_A$

- **Avogadro constant**:
  - Amount of substance containing N_A particles/molecules/atoms
  - Amount of substance which contains the same number of particles/molecules/atoms as there are atoms in 12g of carbon-12
- *R*, **molar gas constant** or **universal gas constant** (same value for all gases), having the value of $8.3\ \text{J K}^{-1}\ \text{mol}^{-1}$
- An **ideal gas** is one which obeys the equation of state $pV = nRT$ or $pV = NkT$
- **Mole:** The amount of matter which contains the same number of atoms/nuclei as there are in 12 g of carbon-12
- Tiny pollen grains suspended in water shows the jerky, erratic, random motion (**Brownian motion**), due to the bombardment from all sides of the water molecules. Brownian motion can be reproduced by observing the motion of tiny soot particles in smoke, these particles move in a jerky motion too, proving the idea of rapid, random motion as required by the molecular model
- The **kinetic theory of gases** is a theory which links these microscopic properties of particles (atoms or molecules) to the macroscopic properties of a gas
- The assumptions of the kinetic theory of an ideal gas are:
  - Time of collisions negligible compared to time between collisions
  - No intermolecular forces except during collisions
  - Random motion of molecules
  - Large number of molecules
  - Total volume of molecules negligible compared to volume of containing vessel / Average separation large compared with the size of the molecules

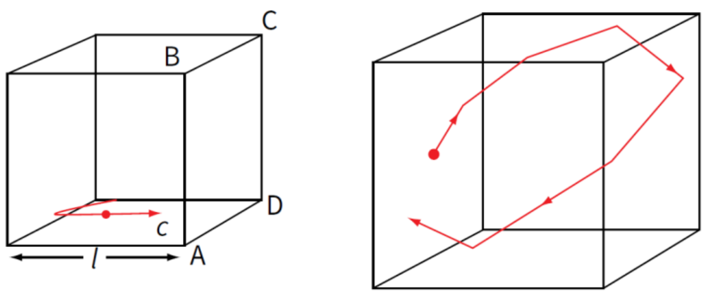


Figure 22.10 A single molecule of a gas, moving in a box.

- **Molecule in a box:**

- Consider a collision in which a single molecule with mass *m* moving with speed *c* parallel to one side of the box
- Collision striking side ABCD of the cube; elastically rebounded to the opposite direction (velocity is –*c*; momentum changes from *mc* to –*mc*), so momentum from the single collision is:

$$\text{change in momentum} = -mc - (+mc)$$
$$= -mc - mc = -2mc$$

- Between consecutive collisions with side ABCD, the molecule travels a distance of 2l at speed c. Hence:

$$\text{time between collisions with side ABCD} = \frac{2l}{c}$$

- Using Newton's second law:

$$\text{force} = \frac{\text{change in momentum}}{\text{time taken}} = \frac{2mc}{2l/c} = \frac{mc^2}{l}$$

- The area of side ABCD is l^2, hence pressure is:

$$\text{pressure} = \frac{\text{force}}{\text{area}} = \frac{mc^2/l}{l^2} = \frac{mc^2}{l^3}$$

- For the large number *N* of molecules, we write the average value of c^2 as $<c^2>$, and multiply by *N* to find the total pressure:

$$\text{pressure } p = \frac{Nm<c^2>}{l^3}$$

- Considering three dimensions, divide by 3 to find the pressure exerted:

$$\text{pressure } p = \frac{1}{3}\frac{Nm<c^2>}{l^3}$$

- l^3 is equal to the volume *V* of the cube, hence:

$$p = \frac{1}{3}\frac{Nm}{V}<c^2> \quad \text{or} \quad pV = \frac{1}{3}Nm<c^2>$$

- *Nm/V* is equal to the density *ρ* of the gas, and we can write:

$$p = \frac{1}{3}\rho <c^2>$$

- So pressure of gas depends only on its density and the mean square speed of its molecules

- The mean **translational kinetic energy** of an atom (or molecule) of an ideal gas is proportional to the thermodynamic temperature:

Since the average kinetic energy of a molecule is

$$\langle E_k \rangle = \tfrac{1}{2} m \langle c^2 \rangle$$

there seems to be a link between our kinetic theory equation for pV and energy. We can find this relation by re-writing the pV equation as

$$pV = \frac{2}{3} N \left(\frac{1}{2} m \langle c^2 \rangle \right) = \frac{2}{3} N \langle E_k \rangle$$

$$pV = \frac{2}{3} N \langle E_k \rangle = NkT$$

and

$$\langle E_k \rangle = \frac{3}{2} kT$$

The average speed of the molecules:

$$\langle E_k \rangle = \frac{1}{2} m \langle c^2 \rangle = \frac{3}{2} kT$$

$$\langle c^2 \rangle = 3kT/m$$

and

$$\sqrt{\langle c^2 \rangle} = \sqrt{3kT/m}$$

- The quantity $\sqrt{\langle c^2 \rangle}$ is called the **root-mean-square (r.m.s.)** speed **(c_{rms})** of the molecules

Alternating current (Chapter 24):

- The magnitude of the power dissipated in a resistor is given by the expression: I^2R or VI or V^2/R

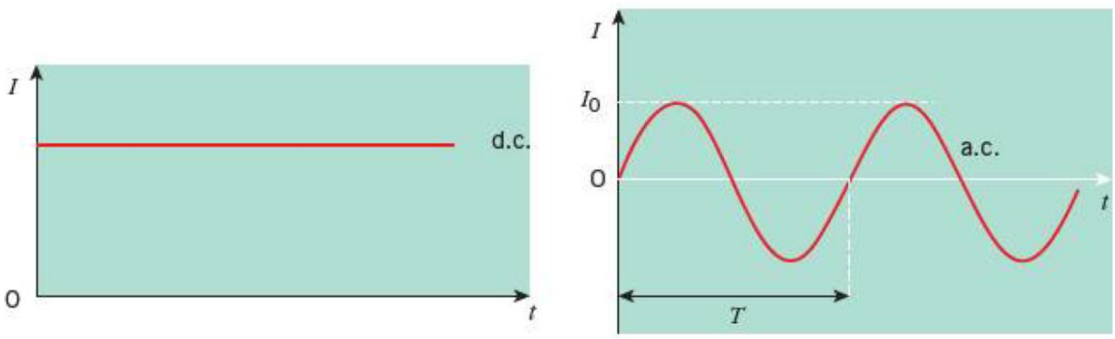


Figure 24.1 Direct and alternating currents

- Representing current and voltage:

$$I = I_0 \sin \omega t$$
$$V = V_0 \sin \omega t$$

- The **peak value** of the current or voltage is I_0 or V_0
- The time taken for one complete cycle of the a.c. is the period (T) of the current $T = 2\pi/\omega$
- The frequency is the number of complete cycles per unit time $f = \omega/2\pi$

The power generated in a resistance R is given by the usual formula

$$P = I^2R$$

but here the current I must be written as

$$I = I_0 \sin \omega t$$

Thus

$$P = I_0^2 R \sin^2 \omega t$$

- Since I_0 and R are constants, the average value of *P* will depend on the average value of $\sin^2 \omega t$, which is 1/2, hence the average power <*P*> delivered to the resistor is:

$$\langle P \rangle = \tfrac{1}{2} I_0^2 R = \tfrac{1}{2} V_0^2/R$$

  - This is half the maximum power
- Average value of the square of the current or voltage:

$$\langle I^2 \rangle = \tfrac{1}{2} I_0^2 \text{ and } \langle V^2 \rangle = \tfrac{1}{2} V_0^2$$

$$I_{rms} = \sqrt{\langle I^2 \rangle} = I_0/\sqrt{2} = 0.707 I_0$$
$$V_{rms} = \sqrt{\langle V^2 \rangle} = V_0/\sqrt{2} = 0.707 V_0$$

- The square root of $\langle I^2 \rangle$ is called the root-mean-square, or r.m.s.
- The r.m.s value of the alternating current or voltage is that value of the direct current or voltage that would produce thermal energy at the same rate in a resistor

Example

A 1.5 kW heater is connected to the domestic supply, which is quoted as 240V. Calculate the peak current in the heater, and its resistance.

The r.m.s version of the power/current/voltage equation is $I_{rms}V_{rms}$ = mean power.

This gives $I_{rms} = 1.5 \times 10^3/240 = 6.3$ A.

The peak current $I_0 = \sqrt{2}\, I_{rms} =$ **8.8 A**

The resistance $R = V_{rms}/I_{rms} = 240/6.3 =$ **38 Ω**

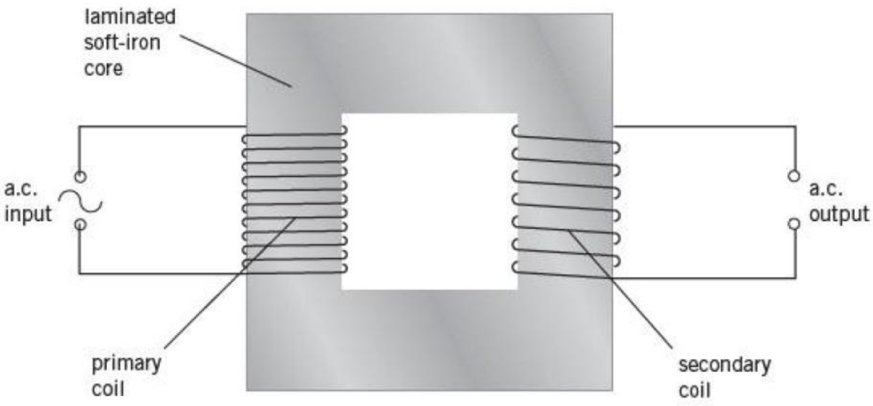


Figure 24.2 Simple transformer

- The alternating current in the primary coil gives rise to an in phase alternating magnetic flux, which threads through the secondary coil, in turn causes an induced e.m.f in the secondary coil – Faraday's law of electromagnetic induction
- For an ideal transformer (100% efficient):

input power = output power

$$V_pI_p = V_sI_s$$

and

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

- When $V_s > V_p$, $N_s > N_p$: step-up transformer
- When $V_s < V_p$, $N_s < N_p$: step-down transformer
- Sources of power losses in a transformer:
  - Loss of magnetic flux between the primary and secondary coils
  - Heating of the core due to eddy currents
  - Heating of the core due to repeated magnetisation and demagnetisation
- Soft iron core reduces heating due to repeated magnetisation and demagnetisation – not eddy currents – while laminating it reduces energy losses as thermal energy due to eddy currents
- Long-distance electrical transmission are prone to power losses due to heating of the cables (the I^2R effect); reduced if the power is transmitted at high voltage – done by using a.c. rather than d.c. for transformers to be usable
- **Rectification:** The process of converting alternating current (a.c.) into direct current (d.c.)

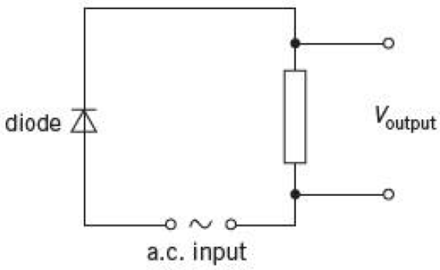


Figure 24.4 Single-diode circuit for half-wave rectification

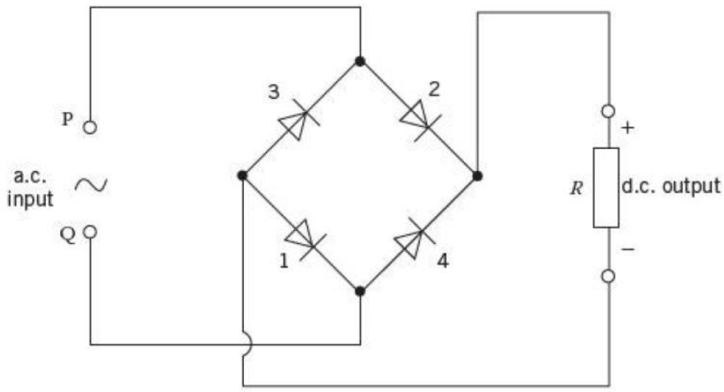


Figure 24.7 Four-diode (bridge) circuit for full-wave rectification

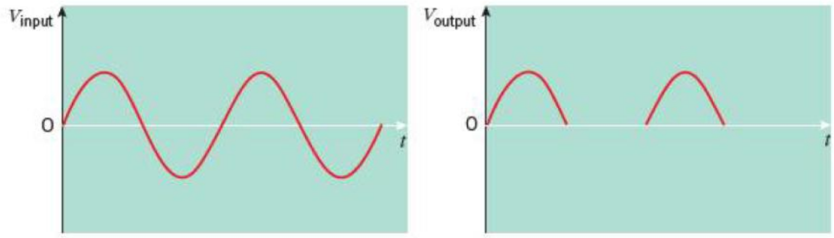


Figure 24.5 Half-wave rectification

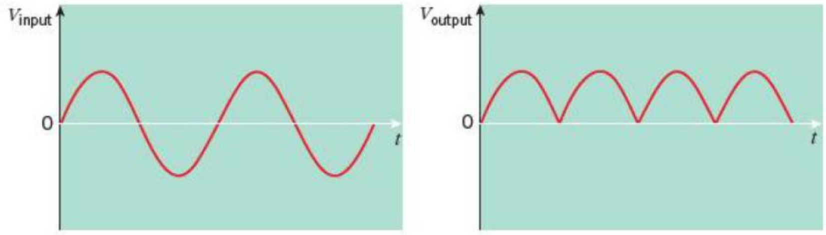


Figure 24.6 Full-wave rectification

- Full-wave rectification involves four diodes, and is referred to as a **bridge rectifier** circuit
- The inputs terminals are P and Q. If P is positive during the first half-cycle, diodes 1 and 2 will conduct; in the next half-cycle, Q is positive so diodes 3 and 4 will conduct. Thus the resistor will always have its upper terminal positive and its lower terminal negative. However, the output is not good a good approximation to the steady voltage, hence a capacitor across the output terminals is used:

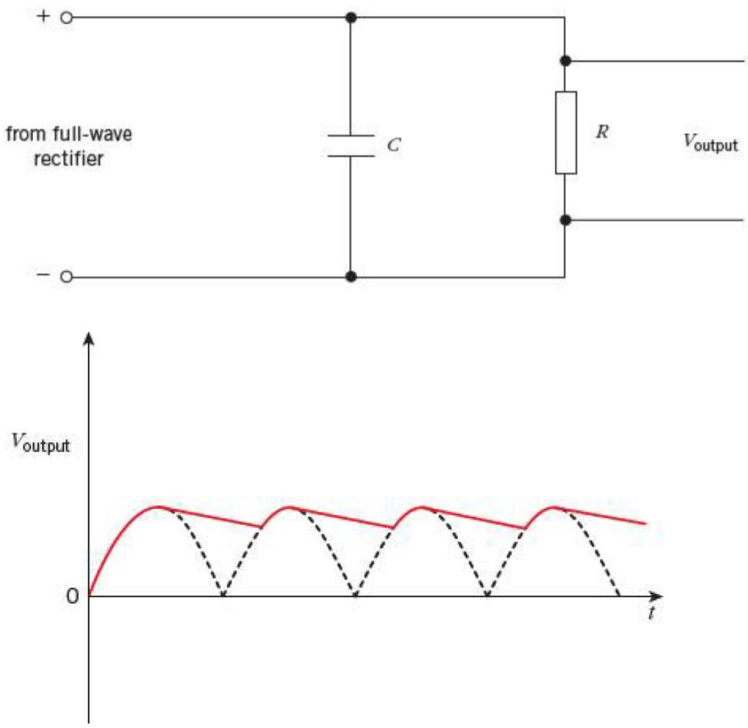


Figure 24.8 Smoothing by capacitor

- The capacitor charges up on the rising part of the half-cycle, then discharges through the resistor as the output voltage falls; is done to reduce the fluctuations in the unidirectional output, where the process is called **smoothing**
- The important factor is the time constant of the resistor-capacitor circuit. If the product of the capacitance *C* and the load *R* is much larger than the half-period of the original supply,

the ripple on the direct current or voltage will be small. Reducing the time constant will increase the ripple:

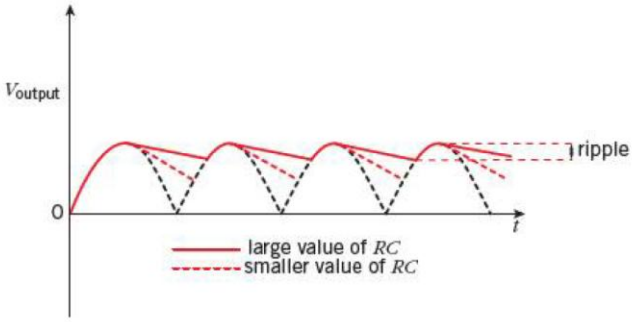


Figure 24.9 Magnitude of the ripple