Rehman
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Saturday

Q No 1 L.H.S $= \tan\theta + \cot\theta$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}$$

$$= \frac{1}{\cos\theta \cdot \sin\theta} \qquad \because \sin^2\theta + \cos^2\theta = 1$$

$$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta} = \sec\theta \cdot \operatorname{cosec}\theta$$

$$= \operatorname{cosec}\theta \cdot \sec\theta = \text{R.H.S} \quad \text{proved.}$$

Domain :- $\{\theta \mid \theta \in \mathbb{R} \text{ but } \theta \neq n\frac{\pi}{2}, n \in \mathbb{Z}\}$

Q No 2 L.H.S $= \sec\theta \operatorname{cosec}\theta \cdot \sin\theta \cos\theta$

$$= \frac{1}{\cancel{\cos\theta}} \cdot \frac{1}{\cancel{\sin\theta}} \cdot \cancel{\sin\theta} \cdot \cancel{\cos\theta}$$

$$= 1 = \text{R.H.S}$$

Domain $= \mathbb{R} - \{n\frac{\pi}{2}\}, n \in \mathbb{Z}$

Q No. 3 L.H.S $= \cos\theta + \tan\theta \cdot \sin\theta$

$$= \cos\theta + \frac{\sin\theta}{\cos\theta} \cdot \sin\theta$$

$$= \cos\theta + \frac{\sin^2\theta}{\cos\theta} = \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} = \sec\theta = \text{R.H.S} \quad \text{proved.}$$

Domain $= \mathbb{R} - \{(2n+1)\frac{\pi}{2}\} \; ; \; n \in \mathbb{Z}$

Q No 4 L.H.S $= \operatorname{cosec}\theta + \tan\theta \sec\theta$

$$= \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos^2\theta} = \frac{\cos^2\theta + \sin^2\theta}{\sin\theta \cos^2\theta}$$

$$= \frac{1}{\sin\theta \cdot \cos^2\theta} = \frac{1}{\sin\theta} \cdot \frac{1}{\cos^2\theta}$$

$$= \operatorname{cosec}\theta \cdot \sec^2\theta = \text{R.H.S} \quad \text{proved.}$$

Domain $= \{\theta \mid \theta \in \mathbb{R} \text{ but } \theta \neq n\frac{\pi}{2}, n \in \mathbb{Z}\}$.

Q No 5 L.H.S $= \sec^2\theta - \csc^2\theta$ $\quad \because 1+\cot^2\theta = \csc^2\theta$, $1+\tan^2\theta = \sec^2\theta$

$= (1+\tan^2\theta) - (1+\cot^2\theta)$

$= \cancel{1} + \tan^2\theta - \cancel{1} - \cot^2\theta$

$= \tan^2\theta - \cot^2\theta =$ R.H.S proved

Q No 6 L.H.S $= \cot^2\theta - \cos^2\theta$

$= \dfrac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta = \cos^2\theta\left(\dfrac{1}{\sin^2\theta} - 1\right)$

$= \cos^2\theta\left(\dfrac{1 - \sin^2\theta}{\sin^2\theta}\right)$

$= \cos^2\theta\left(\dfrac{\cos^2\theta}{\sin^2\theta}\right)$ $\quad \because 1 - \sin^2\theta = \cos^2\theta$

$= \cos^2\theta \cdot \cot^2\theta =$ R.H.S

Q No 7 L.H.S $= (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)$

$= \sec^2\theta - \tan^2\theta$

$= (1+\tan^2\theta) - \tan^2\theta$ $\quad \because 1+\tan^2\theta = \sec^2\theta$

$= 1 + \cancel{\tan^2\theta} - \cancel{\tan^2\theta}$

$= 1 =$ R.H.S

Q No 8 L.H.S $= 2\cos^2\theta - 1$ $\quad \because \sin^2\theta + \cos^2\theta = 1$

$= 2(1 - \sin^2\theta) - 1$ $\quad \therefore \cos^2\theta = 1 - \sin^2\theta$

$= 2 - 2\sin^2\theta - 1$

$= 1 - 2\sin^2\theta =$ R.H.S

Q No 9 L.H.S $= \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$

$= \dfrac{1 - \dfrac{\sin^2\theta}{\cos^2\theta}}{1 + \dfrac{\sin^2\theta}{\cos^2\theta}} = \dfrac{\dfrac{\cos^2\theta - \sin^2\theta}{\cancel{\cos^2\theta}}}{\dfrac{\cos^2\theta + \sin^2\theta}{\cancel{\cos^2\theta}}}$

$= \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} = \dfrac{\cos^2\theta - \sin^2\theta}{1}$

$= \cos^2\theta - \sin^2\theta =$ R.H.S

Q No. 10 $\quad$ R.H.S $= \dfrac{\cot\theta - 1}{\cot\theta + 1}$

$$= \frac{\frac{\cos\theta}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} + 1} = \frac{\frac{\cos\theta - \sin\theta}{\cancel{\sin\theta}}}{\frac{\cos\theta + \sin\theta}{\cancel{\sin\theta}}}$$

$$= \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \text{R.H.S}$$

Q No. 11 $\quad$ L.H.S $= \dfrac{\sin\theta}{1+\cos\theta} + \cancel{\cos\theta}\ \cot\theta$. $\qquad$ * correction

$$= \frac{\sin\theta}{1+\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos\theta(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$$

$$= \frac{\sin^2\theta + \cos\theta + \cos^2\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta) + \cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{\cancel{1+\cos\theta}}{(\cancel{1+\cos\theta})\sin\theta} \qquad \because \sin^2\theta + \cos^2\theta = 1$$

$$= \frac{1}{\sin\theta} = \operatorname{cosec}\theta = \text{R.H.S}$$

Q No. 12 $\quad$ L.H.S $= \dfrac{\cot^2\theta - 1}{1 + \cot^2\theta}$

$$= \frac{\frac{\cos^2\theta}{\sin^2\theta} - 1}{1 + \frac{\cos^2\theta}{\sin^2\theta}} = \frac{\frac{\cos^2\theta - \sin^2\theta}{\cancel{\sin^2\theta}}}{\frac{\sin^2\theta + \cos^2\theta}{\cancel{\sin^2\theta}}}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta + \cos^2\theta} = \frac{\cos^2\theta - \sin^2\theta}{1}$$

$$= \cos^2\theta - \sin^2\theta = \cos^2\theta - (1 - \cos^2\theta)$$

$$= \cos^2\theta - 1 + \cos^2\theta$$

$$= 2\cos^2\theta - 1 = \text{R.H.S} \quad \text{proved}$$

Q.No.13 R.H.S $= (\operatorname{cosec}\theta + \cot\theta)^2$

$$= \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{1+\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1+\cos\theta)^2}{\sin^2\theta} = \frac{(1+\cos\theta)^2}{1-\cos^2\theta}$$

$$= \frac{(1+\cos\theta)\cancel{(1+\cos\theta)}}{(1-\cos\theta)\cancel{(1+\cos\theta)}}$$

$$= \frac{1+\cos\theta}{1-\cos\theta} = \text{R.H.S proved}$$

Q.No.14 L.H.S $= (\sec\theta - \tan\theta)^2$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1-\sin\theta}{\cos\theta}\right)^2$$

$$= \frac{(1-\sin\theta)^2}{\cos^2\theta} = \frac{(1-\sin\theta)^2}{1-\sin^2\theta}$$

$$= \frac{\cancel{(1-\sin\theta)}(1-\sin\theta)}{\cancel{(1-\sin\theta)}(1+\sin\theta)}$$

$$= \frac{1-\sin\theta}{1+\sin\theta} = \text{R.H.S proved.}$$

Q.No.15

$$\text{L.H.S} = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{2\tan\theta}{\sec^2\theta}$$

$$= 2\tan\theta\cos^2\theta$$

$$= 2\frac{\sin\theta}{\cancel{\cos\theta}}\cos^{\cancel{2}}\theta = 2\sin\theta\cos\theta = \text{RHS}$$

Q.No.16

$$\text{L.H.S} = \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta} \quad \text{Rationalizing}$$

$$= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)} = \frac{\cos^{\cancel{2}}\theta}{\cancel{\cos\theta}(1+\sin\theta)}$$

$$= \frac{\cos\theta}{1+\sin\theta} = \text{R.H.S proved}$$

Q.No.17

$$L.H.S = (\tan\theta + \cot\theta)^2$$

$$= \left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \left(\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)^2$$

$$= \left(\frac{1}{\cos\theta\sin\theta}\right)^2 \qquad \because \sin^2\theta + \cos^2\theta = 1$$

$$= (\sec\theta\ \operatorname{cosec}\theta)^2$$

$$= \sec^2\theta\ \operatorname{cosec}^2\theta = R.H.S \quad \text{proved}$$

Q.No. 18

See Alternative Solution at the end.

$$L.H.S = \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$

$$= \frac{\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} - 1}{\frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta} + 1} = \frac{\frac{\sin\theta + 1 - \cos\theta}{\cos\theta}}{\frac{\sin\theta - 1 + \cos\theta}{\cos\theta}}$$

$$= \frac{\sin\theta + 1 - \cos\theta}{\sin\theta - 1 + \cos\theta}$$

$$= \frac{\sin\theta + 1 - \cos\theta}{\sin\theta - 1 + \cos\theta} \cdot \frac{\sin\theta + 1 + \cos\theta}{\sin\theta + 1 + \cos\theta}$$

$$= \frac{(\sin\theta + 1) - \cos\theta}{(\sin\theta + \cos\theta) - 1} \cdot \frac{(\sin\theta + 1) + \cos\theta}{(\sin\theta + \cos\theta) + 1}$$

$$= \frac{(\sin\theta + 1)^2 - (\cos\theta)^2}{(\sin\theta + \cos\theta)^2 - (1)^2}$$

$$= \frac{\sin^2\theta + 2\sin\theta + 1 - \cos^2\theta}{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}$$

$$= \frac{\sin^2\theta + 2\sin\theta + \sin^2\theta}{\cancel{1} + 2\sin\theta\cos\theta \cancel{- 1}} = \frac{2\sin^2\theta + 2\sin\theta}{2\sin\theta\cos\theta}$$

$$= \frac{\cancel{2}\sin^{\cancel{2}}\theta}{\cancel{2}\cancel{\sin\theta}\cos\theta} + \frac{\cancel{2}\cancel{\sin\theta}}{\cancel{2}\cancel{\sin\theta}\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}$$

$$= \tan\theta + \sec\theta = R.H.S \quad \text{proved}$$

Q.No 19 L.H.S $= \dfrac{1}{\operatorname{cosec}\theta - \cot\theta} - \dfrac{1}{\sin\theta}$

$= \dfrac{1}{\operatorname{cosec}\theta - \cot\theta} \cdot \dfrac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}\theta + \cot\theta} - \dfrac{1}{\sin\theta}$

$= \dfrac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}^2\theta - \cot^2\theta} - \dfrac{1}{\sin\theta}$ $\quad \because 1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$= \dfrac{\operatorname{cosec}\theta + \cot\theta}{1} - \dfrac{1}{\sin\theta}$ $\quad \therefore 1 = \operatorname{cosec}^2\theta - \cot^2\theta$

$= \operatorname{cosec}\theta + \cot\theta - \dfrac{1}{\sin\theta} = \cancel{\dfrac{1}{\sin\theta}} + \dfrac{\cos\theta}{\sin\theta} - \cancel{\dfrac{1}{\sin\theta}}$

$= \dfrac{\cos\theta}{\sin\theta} = \cot\theta$ ——— (i)

R.H.S $= \dfrac{1}{\sin\theta} - \dfrac{1}{\operatorname{cosec}\theta + \cot\theta}$

$= \dfrac{1}{\sin\theta} - \dfrac{1}{\operatorname{cosec}\theta + \cot\theta} \cdot \dfrac{\operatorname{cosec}\theta - \cot\theta}{\operatorname{cosec}\theta - \cot\theta}$

$= \dfrac{1}{\sin\theta} - \dfrac{\operatorname{cosec}\theta - \cot\theta}{\operatorname{cosec}^2\theta - \cot^2\theta}$

$= \dfrac{1}{\sin\theta} - \dfrac{(\operatorname{cosec}\theta - \cot\theta)}{1}$ $\quad \because 1 + \cot^2\theta = \operatorname{cosec}^2\theta$, $\therefore 1 = \operatorname{cosec}^2\theta - \cot^2\theta$

$= \dfrac{1}{\sin\theta} - (\operatorname{cosec}\theta - \cot\theta) = \dfrac{1}{\sin\theta} - \operatorname{cosec}\theta + \cot\theta$

$= \cancel{\dfrac{1}{\sin\theta}} - \cancel{\dfrac{1}{\sin\theta}} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{\cos\theta}{\sin\theta} = \cot\theta$ ——— (ii)

From (i) & (ii) L.H.S = R.H.S proved

Q.No 20 L.H.S $= \sin^3\theta - \cos^3\theta$

$= (\sin\theta)^3 - (\cos\theta)^3$

$= (\sin\theta - \cos\theta)(\sin^2\theta + \cos^2\theta + \sin\theta\cos\theta)$

$= (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$

$=$ R.H.S

proved

Q.No.21 L.H.S $= \sin^6\theta - \cos^6\theta$

$= (\sin^2\theta)^3 - (\cos^2\theta)^3$

$= (\sin^2\theta - \cos^2\theta)\left((\sin^2\theta)^2 + (\cos^2\theta)^2 + \sin^2\theta\cos^2\theta\right)$

$= (\sin^2\theta - \cos^2\theta)\left((\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta\right)$

$= (\sin^2\theta - \cos^2\theta)\left[(\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta\cos^2\theta\right]$

$= (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

$=$ R.H.S proved

Q.No.22 L.H.S $= \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)\left((\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta\cos^2\theta\right)$

$= (1)\left((\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 3\sin^2\theta\cos^2\theta\right)$

$= \left((\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta\right)$

$= \left((1)^2 - 3\sin^2\theta\cos^2\theta\right) = 1 - 3\sin^2\theta\cos^2\theta = $ R.H.S

Q.No.23 L.H.S $= \dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta}$

$= \dfrac{1 - \cancel{\sin\theta} + 1 + \cancel{\sin\theta}}{(1+\sin\theta)(1-\sin\theta)} = \dfrac{2}{1-\sin^2\theta}$

$= \dfrac{2}{\cos^2\theta} = 2\sec^2\theta =$ R.H.S proved

Q.No 24 L.H.S $= \dfrac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

$= \dfrac{(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$

$= \dfrac{\cos^2\theta + \sin^2\theta + \cancel{2\cos^2\theta\sin^2\theta} + \cos^2\theta + \sin^2\theta - \cancel{2\cos^2\theta\sin^2\theta}}{\cos^2\theta - \sin^2\theta}$

$= \dfrac{2\cos^2\theta + 2\sin^2\theta}{\cos^2\theta - \sin^2\theta} = \dfrac{2(\cos^2\theta + \sin^2\theta)}{(1-\sin^2\theta) - \sin^2\theta}$

$= \dfrac{2(1)}{1 - \sin^2\theta - \sin^2\theta} = \dfrac{2}{1 - 2\sin^2\theta} =$ R.H.S proved

Rahman.

Question # 18 [by **Hafiz Syed Rizwan** (FSc, Session: 2007-2009) Punjab College of Science, Lahore.]

$$\text{L.H.S} = \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}$$

$\times$ing and $\div$ing by $\tan\theta + \sec\theta$

$$= \frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} \times \frac{\tan\theta + \sec\theta}{\tan\theta + \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta - 1)(\tan\theta + \sec\theta)}{\tan^2\theta - \tan\theta\sec\theta + \tan\theta + \tan\theta\sec\theta - \sec^2\theta + \sec\theta}$$

$$= \frac{(\tan\theta + \sec\theta - 1)(\tan\theta + \sec\theta)}{\tan^2\theta - \sec^2\theta + \tan\theta + \sec\theta} \qquad \because\ 1 + \tan^2\theta = \sec^2\theta \quad \therefore\ \tan^2\theta - \sec^2\theta = -1$$

$$= \frac{(\tan\theta + \sec\theta - 1)(\tan\theta + \sec\theta)}{-1 + \tan\theta + \sec\theta}$$

$$= \tan\theta + \sec\theta \qquad = \text{R.H.S}$$

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If you have any alternative solution then you can send us at www.megalecture.com