

Question # 1

- Verify following: (i) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin 30^\circ$
 (ii) $\sin^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{3} + \tan^2 \frac{\pi}{4} = 2$
 (iii) $2 \sin 45^\circ + \frac{1}{2} \csc 45^\circ = \frac{3}{\sqrt{2}}$
 (iv) $\sin^2 \frac{\pi}{6} : \sin^2 \frac{\pi}{4} : \sin^2 \frac{\pi}{3} : \sin^2 \frac{\pi}{2} = 1 : 2 : 3 : 4$

Solution

(i) L.H.S = $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$
 $= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2} \dots\dots\dots (i)$

R.H.S = $\sin 30^\circ = \frac{1}{2} \dots\dots\dots (ii)$

From (i) and (ii)

L.H.S = R.H.S

(ii) L.H.S = $\sin^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{3} + \tan^2 \frac{\pi}{4}$
 $= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2 = \frac{1}{4} + \frac{3}{4} + 1 = \frac{1+3+4}{4} = \frac{8}{4} = 2 = \text{R.H.S}$

(iii) L.H.S = $2 \sin 45^\circ + \frac{1}{2} \csc 45^\circ = 2 \sin 45^\circ + \frac{1}{2 \sin 45^\circ}$
 $= 2 \left(\frac{1}{\sqrt{2}}\right) + \frac{1}{2 \left(\frac{1}{\sqrt{2}}\right)} = \frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{2} = \frac{(\sqrt{2})^2}{\sqrt{2}} + \frac{\sqrt{2}}{(\sqrt{2})^2}$
 $= \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{2+1}{\sqrt{2}} = \frac{3}{\sqrt{2}} = \text{R.H.S}$

(iv) L.H.S = $\sin^2 \frac{\pi}{6} : \sin^2 \frac{\pi}{4} : \sin^2 \frac{\pi}{3} : \sin^2 \frac{\pi}{2}$
 $= \left(\frac{1}{2}\right)^2 : \left(\frac{1}{\sqrt{2}}\right)^2 : \left(\frac{\sqrt{3}}{2}\right)^2 : (1)^2 = \frac{1}{4} : \frac{1}{2} : \frac{3}{4} : 1$
 $= 1 : 2 : 3 : 4 = \text{R.H.S}$

×ing by 4

Question # 2

Evaluate the following

$$(i) \frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{6}}$$

$$(ii) \frac{1 - \tan^2 \frac{\pi}{3}}{1 + \tan^2 \frac{\pi}{3}}$$

Solution

$$(i) \frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{6}}{1 + \tan \frac{\pi}{3} \tan \frac{\pi}{6}} = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \cdot \frac{1}{\sqrt{3}}} = \frac{\frac{3-1}{\sqrt{3}}}{1+1} = \frac{\frac{2}{\sqrt{3}}}{2} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} \quad \text{Answer}$$

$$(ii) \frac{1 - \tan^2 \frac{\pi}{3}}{1 + \tan^2 \frac{\pi}{3}} = \frac{1 - (\sqrt{3})^2}{1 + (\sqrt{3})^2} = \frac{1-3}{1+3} = \frac{-2}{4} = -\frac{1}{2} \quad \text{Answer}$$

Question # 3

Verify the following when $\theta = 30^\circ, 45^\circ$

(i) $\sin 2\theta = 2 \sin \theta \cos \theta$

(ii) $\cos 2\theta = 2 \cos^2 \theta - 1$

(iii) $\cos 2\theta = 2 \cos^2 \theta - 1$

(iv) $\cos 2\theta = 1 - 2 \sin^2 \theta$

(v) $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

Solution

(i) When $\theta = 30^\circ$

L.H.S = $\sin 2\theta = \sin 2(30) = \sin 60 = \frac{\sqrt{3}}{2}$ (i)

R.H.S = $2 \sin \theta \cos \theta = 2 \sin 30^\circ \cos 30^\circ = 2 \left(\frac{1}{2}\right) \left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$ (ii)

From (i) and (ii)

L.H.S = R.H.S

When $\theta = 45^\circ$

L.H.S = $\sin 2\theta = \sin 2(45) = \sin 90 = 1$ (i)

R.H.S = $2 \sin \theta \cos \theta = 2 \sin 45^\circ \cos 45^\circ = 2 \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{\sqrt{2}}\right) = \frac{2}{2} = 1$ (ii)

From (i) and (ii)

L.H.S = R.H.S

(ii) When $\theta = 30^\circ$

$$\text{L.H.S} = \cos 2\theta = \cos 2(30) = \cos 60 = \frac{1}{2} \dots\dots\dots (i)$$

$$\begin{aligned} \text{R.H.S} &= \cos^2 \theta - \sin^2 \theta = \cos^2 30^\circ - \sin^2 30^\circ \\ &= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} \dots\dots\dots (ii) \end{aligned}$$

From (i) and (ii)
 L.H.S = R.H.S

When $\theta = 45^\circ$
 L.H.S = $\cos 2\theta = \cos 2(45) = \cos 90 = 0 \dots\dots\dots (i)$

$$\begin{aligned} \text{R.H.S} &= \cos^2 \theta - \sin^2 \theta = \cos^2 45^\circ - \sin^2 45^\circ \\ &= \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} - \frac{1}{2} = 0 \dots\dots\dots (ii) \end{aligned}$$

From (i) and (ii)
 L.H.S = R.H.S

(iii) and (iv) *Do yourself as above*

(v) When $\theta = 30^\circ$

$$\text{L.H.S} = \tan 2\theta = \tan 2(30) = \tan 60 = \sqrt{3} \dots\dots\dots (i)$$

$$\begin{aligned} \text{R.H.S} &= \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2 \left(\frac{1}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)^2} \\ &= \frac{2/\sqrt{3}}{1 - 1/3} = \frac{2/\sqrt{3}}{2/3} = \frac{2}{\sqrt{3}} \cdot \frac{3}{2} = \frac{2}{\sqrt{3}} \cdot \frac{(\sqrt{3})^2}{2} = \sqrt{3} \dots\dots\dots (ii) \end{aligned}$$

From (i) and (ii)
 L.H.S = R.H.S

When $\theta = 45^\circ$
 L.H.S = $\tan 2\theta = \tan 2(45) = \tan 90 = \infty \dots\dots\dots (i)$

$$\begin{aligned} \text{R.H.S} &= \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \tan 45^\circ}{1 - \tan^2 45^\circ} = \frac{2(1)}{1 - (1)^2} \\ &= \frac{2}{1 - 1} = \frac{2}{0} = \infty \dots\dots\dots (ii) \end{aligned}$$

From (i) and (ii)
 L.H.S = R.H.S

Question # 4

Find x, if $\tan^2 45^\circ - \cos^2 60^\circ = x \sin 45^\circ \cos 45^\circ \tan 60^\circ$

Solution

Since $\tan^2 45^\circ - \cos^2 60^\circ = x \sin 45^\circ \cos 45^\circ \tan 60^\circ$

This gives $(1)^2 - \left(\frac{1}{2}\right)^2 = x \left(\frac{1}{\sqrt{2}}\right) \left(\frac{1}{\sqrt{2}}\right) (\sqrt{3})$

$$\Rightarrow 1 - \frac{1}{4} = x \left(\frac{\sqrt{3}}{2}\right) \frac{3}{4} = x \frac{\sqrt{3}}{2} \Rightarrow \frac{6}{4\sqrt{3}} = x$$

$$\Rightarrow x = \frac{3}{2\sqrt{3}} \Rightarrow \boxed{x = \frac{\sqrt{3}}{2}} \text{ Answer}$$

Question # 5

Find the values of the trigonometric functions of the following

(i) $-\pi$ (ii) -3π (iii) $\frac{5}{2}\pi$ (iv) $-\frac{9}{2}\pi$ (v) -15π

(vi) 1530° (vii) -2430° (viii) $\frac{235}{2}\pi$ (ix) $\frac{407}{2}\pi$

Solutions

(i) Since $-\pi = -2\pi + \pi$, therefore

$$\sin(-\pi) = \sin(-2\pi + \pi) = \sin \pi = 0$$

$$\cos(-\pi) = \cos(-2\pi + \pi) = \cos \pi = -1$$

$$\tan(-\pi) = \tan(-2\pi + \pi) = \tan \pi = 0$$

$$\csc(-\pi) = \csc(-2\pi + \pi) = \csc \pi = \infty \text{ (undefined)}$$

$$\sec(-\pi) = \sec(-2\pi + \pi) = \sec \pi = -1$$

$$\cot(-\pi) = \cot(-2\pi + \pi) = \cot \pi = \infty \text{ (undefined)}$$

(ii) Since $-3\pi = -4\pi + \pi = -2(2\pi) + \pi$, therefore

$$\sin(-3\pi) = \sin(-2(2\pi) + \pi) = \sin \pi = 0$$

And now find other values yourself.

(iii) Since $\frac{5}{2}\pi = 2\frac{1}{2}\pi = 2\pi + \frac{\pi}{2}$, therefore

$$\sin\left(\frac{5}{2}\pi\right) = \sin\left(2\pi + \frac{\pi}{2}\right) = \sin \frac{\pi}{2} = 1, \quad \cos\left(\frac{5}{2}\pi\right) = \cos\left(2\pi + \frac{\pi}{2}\right) = \cos \frac{\pi}{2} = 0$$

$$\tan\left(\frac{5}{2}\pi\right) = \tan\left(2\pi + \frac{\pi}{2}\right) = \tan \frac{\pi}{2} = \infty, \quad \csc\left(\frac{5}{2}\pi\right) = \csc\left(2\pi + \frac{\pi}{2}\right) = \csc \frac{\pi}{2} = 1$$

$$\sec\left(\frac{5}{2}\pi\right) = \sec\left(2\pi + \frac{\pi}{2}\right) = \sec \frac{\pi}{2} = \infty, \quad \cot\left(\frac{5}{2}\pi\right) = \cot\left(2\pi + \frac{\pi}{2}\right) = \cot \frac{\pi}{2} = 0$$

(iv) Since $-\frac{9}{2}\pi = -4\frac{1}{2}\pi = -6\pi + \frac{3\pi}{2}$, therefore

$$\sin\left(-\frac{9}{2}\pi\right) = \sin\left(-6\pi + \frac{3\pi}{2}\right) = \sin \frac{3\pi}{2} = -1$$

And now find other values yourself.

(v) Since $-15\pi = -16\pi + \pi$, therefore

$$\sin(-15\pi) = \sin(-16\pi + \pi) = \sin \pi = 0$$

And now find other values yourself.

(vi) Since $1530^\circ = 1530 \times \frac{\pi}{180} = \frac{17}{2}\pi = 8\frac{1}{2}\pi = 8\pi + \frac{\pi}{2}$

$$\text{So } \sin(1530^\circ) = \sin\left(8\pi + \frac{\pi}{2}\right) = \sin \frac{\pi}{2} = 1$$

And now find other values yourself.

(vii) Since $-2430^\circ = -2430 \times \frac{\pi}{180} = -\frac{27}{2}\pi = -13\frac{1}{2}\pi = -14\pi + \frac{\pi}{2}$

Now do yourself

(viii) Since $\frac{235}{2}\pi = 116\frac{3}{2}\pi = 116\pi + \frac{3\pi}{2}$

Now do yourself

(ix) Since $\frac{407}{2}\pi = 202\frac{3}{2}\pi = 202\pi + \frac{3\pi}{2}$

Now do yourself

Question # 6

Find the values of the trigonometric functions of the following angles:

(i) 390°

(ii) -330°

(iii) 765°

(iv) -675°

(v) $-\frac{17}{3}\pi$

(vi) $\frac{13}{3}\pi$

(vii) $\frac{25}{6}\pi$

(viii) $-\frac{71}{6}\pi$

(ix) -1035°

Solutions

(i) Since $390^\circ = 360 + 30$

$$\text{So } \sin(390^\circ) = \sin(360 + 30) = \sin 30^\circ = \frac{1}{2}$$

$$\cos(390^\circ) = \cos(360 + 30) = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan(390^\circ) = \tan(360 + 30) = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\csc(390^\circ) = \csc(360 + 30) = \csc 30^\circ = 2$$

$$\sec(390^\circ) = \sec(360 + 30) = \sec 30^\circ = \frac{2}{\sqrt{3}}$$

$$\cot(390^\circ) = \cot(360 + 30) = \cot 30^\circ = \sqrt{3}$$

(ii) Since $-330^\circ = -360 + 30$

$$\text{So } \sin(-330^\circ) = \sin(-360 + 30) = \sin 30^\circ = \frac{1}{2}$$

And now find other values yourself.

(iii) Since $765^\circ = 720 + 45 = 2(360) + 45$

$$\text{So } \sin(760^\circ) = \sin(2(360) + 45) = \sin 45^\circ = \frac{1}{\sqrt{2}}$$

And now find other values yourself.

(iv) Since $-675^\circ = -720 + 45 = -2(360) + 45$ *Now do yourself*

(v) Since $-\frac{17}{3}\pi = -5\frac{2}{3}\pi = -6\pi + \frac{\pi}{3}$ *Now do yourself*

(vi) Since $\frac{13}{3}\pi = 4\frac{1}{3}\pi = 4\pi + \frac{\pi}{3}$ *Now do yourself.*

(vii) Since $\frac{25}{6}\pi = 4\frac{1}{6}\pi = 4\pi + \frac{\pi}{6}$

$$\text{So } \sin\left(\frac{25}{6}\pi\right) = \sin\left(4\pi + \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$$
 Now do yourself

(viii) Since $-\frac{71}{6}\pi = -11\frac{5}{6}\pi = -12\pi + \frac{\pi}{6}$ *Now do yourself.*

(ix) Since $-1035^\circ = -1035 \cdot \frac{\pi}{180} = -\frac{23\pi}{4} = -5\frac{3\pi}{4} = -6\pi + \frac{\pi}{4}$

$$\text{So } \sin(-1035^\circ) = \sin\left(-6\pi + \frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$
 Now do yourself

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