Question # 1 & 2

Do yourself

Question # 3

In which quadrant are the terminal arms of the angle lie when

(i) $\sin\theta < 0$ and $\cos\theta > 0$

(ii) $\cot\theta > 0$ and $\operatorname{cosec}\theta > 0$

Solutions

(i) Since $\sin\theta < 0$ so θ lies in *IIIrd* or *IVth* quadrant.

Also $\cos\theta > 0$ so θ lies in *Ist* or *IVth* quadrant.

$\Rightarrow$ θ lies in *IVth* quadrant

(ii) Since $\cot\theta > 0$ so θ lies in *Ist* or *IIIrd* quadrant.

Also $\csc\theta > 0$ so θ lies in *Ist* or *IInd* quadrant

$\Rightarrow$ θ lies in *Ist* quadrant.

Question # 3 (iii), (iv) and ……

Do yourself as above

Question # 4

Find the values of the remaining trigonometric functions:

(i) $\sin\theta = \frac{12}{13}$ and the terminal arm of the angle is in quad. I.

(ii) $\cos\theta = \frac{9}{41}$ and the terminal arm of the angle is in quad. IV.

(iv) $\tan\theta = -\frac{1}{3}$ and the terminal arm of the angle is in quad. II.

Solutions

(i) Since $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$$

$$\Rightarrow \cos\theta = \pm\sqrt{1-\sin^2\theta}$$

As terminal ray lies in *Ist* quadrant so $\cos\theta$ is +ive.

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$\Rightarrow \cos\theta = \sqrt{1-\left(\frac{12}{13}\right)^2} \qquad \because \sin\theta = \frac{12}{13}$$

$$= \sqrt{1-\frac{144}{169}} = \sqrt{\frac{25}{169}} \qquad \Rightarrow \boxed{\cos\theta = \frac{5}{13}}$$

Now

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{12/13}{5/13} = \frac{12}{13}\cdot\frac{13}{5} \qquad \Rightarrow \boxed{\tan\theta = \frac{12}{5}}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{12/13} = \frac{13}{12} \qquad \Rightarrow \boxed{\csc\theta = \frac{13}{12}}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{5/13} = \frac{13}{5} \qquad \Rightarrow \boxed{\sec\theta = \frac{13}{5}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{12/5} = \frac{5}{12} \qquad \Rightarrow \boxed{\cot\theta = \frac{5}{12}}$$

(ii) Since $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow \sin\theta = \pm\sqrt{1-\cos^2\theta}$

As terminal ray lies in *IVth* quadrant so $\sin\theta$ is –ive .

$$\sin\theta = -\sqrt{1-\cos^2\theta}$$

$$\Rightarrow \sin\theta = -\sqrt{1-\left(\frac{9}{41}\right)^2}$$

$$= -\sqrt{1-\frac{81}{1681}} = -\sqrt{\frac{1600}{1681}} = -\frac{40}{41} \qquad \Rightarrow \boxed{\sin\theta = -\frac{40}{41}}$$

Now

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-40/41}{9/41} = -\frac{40}{41}\cdot\frac{41}{9} = -\frac{40}{9} \qquad \Rightarrow \boxed{\tan\theta = -\frac{40}{9}}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{-40/41} = -\frac{41}{40} \qquad \Rightarrow \boxed{\csc\theta = -\frac{40}{41}}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{9/11} = \frac{41}{9} \qquad \Rightarrow \boxed{\sec\theta = \frac{41}{9}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-40/9} = -\frac{9}{40} \qquad \Rightarrow \boxed{\cot\theta = -\frac{9}{40}}$$

(iv) Since $\sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \sec\theta = \pm\sqrt{1+\tan^2\theta}$

As terminal ray is in *IInd* quadrant so $\sec\theta$ is –ive.

$\Rightarrow \sec\theta = -\sqrt{1-\tan^2\theta}$

$$\Rightarrow \sec\theta = -\sqrt{1+\left(-\frac{1}{3}\right)^2} = -\sqrt{1+\frac{1}{9}} = -\sqrt{\frac{10}{9}}$$

$$\Rightarrow \boxed{\sec\theta = -\frac{\sqrt{10}}{3}}$$

Now $\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{1}{-\sqrt{10}/3} = -\dfrac{3}{\sqrt{10}}$ $\Rightarrow$ $\boxed{\cos\theta = -\dfrac{3}{\sqrt{10}}}$

Also $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$

$\Rightarrow \sin\theta = (\tan\theta)(\cos\theta) = \left(-\dfrac{1}{3}\right)\left(-\dfrac{3}{\sqrt{10}}\right)$ $\Rightarrow$ $\boxed{\sin\theta = \dfrac{1}{\sqrt{10}}}$

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{1}{1/\sqrt{10}}$ $\Rightarrow$ $\boxed{\csc\theta = \sqrt{10}}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{-1/3}$ $\Rightarrow$ $\boxed{\cot\theta = -3}$

Question # 4 (iii) and (v) Do yourself as above.

Question # 5

If $\cot\theta = \dfrac{15}{8}$ and terminal arm of the angle is not in quad. I, find the values of $\cos\theta$ and $\operatorname{cosec}\theta$.

Solution

As $\cot\theta$ is +ive and it is not in *Ist* quadrant so it is in *IIIrd* quadrant

($\cot\theta$ +ive in *Ist* and *IIIrd* quadrant)

Now $\csc^2\theta = 1 + \cot^2\theta$ $\Rightarrow \csc\theta = \pm\sqrt{1+\cot^2\theta}$

As terminal ray is in *IIIrd* quadrant so $\csc\theta$ is –ive.

$\csc\theta = -\sqrt{1+\cot^2\theta}$

$\Rightarrow \csc\theta = -\sqrt{1+\left(\dfrac{15}{8}\right)^2} = -\sqrt{1+\dfrac{225}{64}}$ $\because \cot\theta = \dfrac{15}{8}$

$= -\sqrt{\dfrac{289}{64}}$ $\Rightarrow$ $\boxed{\csc\theta = -\dfrac{17}{8}}$

$\sin\theta = \dfrac{1}{\csc\theta} = \dfrac{1}{-17/8}$ $\Rightarrow$ $\boxed{\sin\theta = -\dfrac{8}{17}}$

Now $\dfrac{\cos\theta}{\sin\theta} = \cot\theta$

$\Rightarrow \cos\theta = \cot\theta\,\sin\theta = \left(\dfrac{15}{8}\right)\left(-\dfrac{8}{17}\right)$ $\Rightarrow$ $\boxed{\cos\theta = -\dfrac{15}{17}}$

Question # 6

If $\operatorname{cosec}\theta = \dfrac{m^2+1}{2m}$ and $\left(0<\theta<\dfrac{\pi}{2}\right)$, find the values of the remaining trigonometric function.

Solution

Since $0<\theta<\frac{\pi}{2}$ therefore terminal ray lies in *Ist* quadrant.

Now $1+\cot^2\theta=\csc^2\theta$

$\Rightarrow \cot^2\theta=\csc^2\theta-1$

$\Rightarrow \cot\theta=\pm\sqrt{\csc^2\theta-1}$

As terminal ray of θ is in *Ist* quadrant so $\cot\theta$ is +ive.

$\cot\theta=\sqrt{\csc^2\theta-1}$

$$\Rightarrow \cot\theta=\sqrt{\left(\frac{m^2+1}{2m}\right)^2-1}=\sqrt{\frac{(m^2+1)^2}{(2m)^2}-1} \qquad \because \csc\theta=\frac{m^2+1}{2m}$$

$$=\sqrt{\frac{m^4+2m^2+1}{4m^2}-1}=\sqrt{\frac{m^4+2m^2+1-4m^2}{4m^2}}=\sqrt{\frac{m^4-2m^2+1}{4m^2}}$$

$$=\sqrt{\frac{(m^2-1)^2}{(2m)^2}}=\frac{m^2-1}{2m} \qquad \Rightarrow \boxed{\cot\theta=\frac{m^2-1}{2m}}$$

$$\sin\theta=\frac{1}{\csc\theta}=\frac{1}{\left(\frac{m^2+1}{2m}\right)}=\frac{2m}{(m^2+1)} \qquad \Rightarrow \boxed{\sin\theta=\frac{2m}{m^2+1}}$$

Now $\frac{\cos\theta}{\sin\theta}=\cot\theta \qquad \Rightarrow \cos\theta=(\cot\theta)(\sin\theta)$

$$\Rightarrow \cos\theta=\left(\frac{m^2-1}{2m}\right)\left(\frac{2m}{m^2+1}\right) \qquad \Rightarrow \boxed{\cos\theta=\left(\frac{m^2-1}{m^2+1}\right)}$$

$$\sec\theta=\frac{1}{\cos\theta}=\frac{1}{\frac{m^2-1}{m^2+1}} \qquad \Rightarrow \boxed{\sec\theta=\left(\frac{m^2+1}{m^2-1}\right)}$$

$$\tan\theta=\frac{1}{\cot\theta}=\frac{1}{\frac{m^2-1}{2m}} \qquad \Rightarrow \boxed{\tan\theta=\left(\frac{2m}{m^2-1}\right)}$$

Question # 7

If $\tan\theta=\frac{1}{\sqrt{7}}$ and the terminal arm of the angle is not is the II quad. Find the value of $\frac{\csc^2\theta-\sec^2\theta}{\csc^2\theta+\sec^2\theta}$

Solution

$\tan\theta$ is +ive and terminal arm is not in the *IIIrd* quadrant, therefore terminal arm lies in *Ist* quadrant.

Now $\sec^2\theta=1+\tan^2\theta$

$\Rightarrow \sec\theta=\pm\sqrt{1+\tan^2\theta}$

as terminal arm is in the first quadrant so $\sec\theta$ is +ive.

$$\sec\theta = \sqrt{1+\tan^2\theta}$$

$$\sec\theta = \sqrt{1+\left(\frac{1}{\sqrt{7}}\right)^2} = \sqrt{1+\frac{1}{7}} = \sqrt{\frac{8}{7}} \quad \Rightarrow \quad \boxed{\sec\theta = \frac{2\sqrt{2}}{\sqrt{7}}}$$

Now $$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{2\sqrt{2}/\sqrt{7}} \quad \Rightarrow \quad \boxed{\cos\theta = \frac{\sqrt{7}}{2\sqrt{2}}}$$

Now $$\frac{\sin\theta}{\cos\theta} = \tan\theta \quad \Rightarrow \quad \sin\theta = (\tan\theta)(\cos\theta)$$

$$\Rightarrow \sin\theta = \left(\frac{1}{\sqrt{7}}\right)\left(\frac{\sqrt{7}}{2\sqrt{2}}\right) \quad \Rightarrow \quad \boxed{\sin\theta = \frac{1}{2\sqrt{2}}}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{1/2\sqrt{2}} \quad \Rightarrow \quad \boxed{\csc\theta = 2\sqrt{2}}$$

Now $$\frac{\csc^2\theta - \sec^2\theta}{\csc^2\theta + \sec^2\theta} = \frac{(2\sqrt{2})^2 - \left(2\sqrt{2}/\sqrt{7}\right)^2}{(2\sqrt{2})^2 + \left(2\sqrt{2}/\sqrt{7}\right)^2} = \frac{8-\frac{8}{7}}{8+\frac{8}{7}}$$

$$= \frac{48/7}{64/7} = \frac{48}{7}\cdot\frac{7}{64} = \frac{4}{3} \quad Answer$$

Question # 8

If $\cot\theta = \frac{5}{2}$ and the terminal arm of the angle is in the I quad., find the value of

$$\frac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta}$$

Solution

Since $$\csc^2\theta = 1+\cot^2\theta$$

$$\Rightarrow \csc\theta = \pm\sqrt{1+\cot^2\theta}$$

As terminal ray is in *Ird* quadrant so $\csc\theta$ is +ive.

$$\csc\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+\left(\frac{5}{2}\right)^2} = \sqrt{1+\frac{25}{4}} = \sqrt{\frac{29}{4}} = \frac{\sqrt{29}}{2}$$

Now $$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{\sqrt{29}/2} \quad \Rightarrow \quad \boxed{\sin\theta = \frac{2}{\sqrt{29}}}$$

Now $$\frac{\cos\theta}{\sin\theta} = \cot\theta \quad \Rightarrow \quad \cos\theta = (\cot\theta)(\sin\theta)$$

$$\Rightarrow \cos\theta = \left(\frac{5}{2}\right)\left(\frac{2}{\sqrt{29}}\right) \quad \Rightarrow \quad \boxed{\cos\theta = \frac{5}{\sqrt{29}}}$$

Now $\dfrac{3\sin\theta + 4\cos\theta}{\cos\theta - \sin\theta} = \dfrac{3\left(2/\sqrt{29}\right) + 4\left(5/\sqrt{29}\right)}{5/\sqrt{29} - 2/\sqrt{29}} = \dfrac{6/\sqrt{29} + 20/\sqrt{29}}{5/\sqrt{29} - 2/\sqrt{29}}$

$$= \dfrac{\dfrac{6+20}{\sqrt{29}}}{\dfrac{5-2}{\sqrt{29}}} = \dfrac{26/\sqrt{29}}{3/\sqrt{29}} = \dfrac{26}{\sqrt{29}} \cdot \dfrac{\sqrt{2}}{3} = \dfrac{26}{3} \quad \textit{Answer}$$
