Question # 1

The probability that a person A will be alive 15 year hence is $\frac{5}{7}$ and the probability that another person B will be alive 15 years hence is $\frac{7}{9}$. Find the probability that both will be alive 15 year hence.

Solution Since $P(A)=\frac{5}{7}$

And $P(B)=\frac{7}{9}$

Then the probability that both will alive 15 year is

$$P(A\cap B)=P(A)\cdot P(B)$$
$$=\frac{5}{7}\cdot\frac{7}{9}$$
$$=\frac{5}{9} \qquad \textit{Answer}$$

Question # 2

A die is rolled twice: Event E_1 is the appearance of even number of dots and event E_2 is the appearance of more than 4 dots. Prove that:

$P(E_1\cap E_2)=P(E_1).P(E_2)$.

Solution When a die is rolled then possible outcomes are

1, 2 ,3 , 4, 5, 6

This shows that possible outcomes = $n(S)=6$

Since E_1 is the event that the dots on the die are even then favourable outcomes are 2, 4, 6

this shows $n(E_1)=3$

so probability = $P(E_1)$ = $\frac{n(E_1)}{n(S)}$

$$=\frac{3}{6} = \frac{1}{2}$$

Now since E_2 is the event that the dot appear are more than four then favourable outcomes are 5 and 6. This show $n(E_2)$ = 2

So probability = $P(E_2)$ = $\frac{n(E_2)}{n(S)}$

$$=\frac{2}{6}=\frac{1}{3}$$

Since E_1 and E_2 are not mutually exclusive

And the possible common outcome is 6 i.e. $n(E_1 \cap E_2) = 1$

So probability $P(E_1 \cap E_2) = \dfrac{n(E_1 \cap E_2)}{n(S)} = \dfrac{1}{6}$ ……….. (i)

Now $P(E_1) \cdot P(E_2) = \dfrac{1}{2} \cdot \dfrac{1}{3} = \dfrac{1}{6}$ ………… (ii)

Form (i) and (ii)

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2) \quad \text{Proved.}$$

Question # 3

Determine the probability of getting 2 heads in two successive tosses of a balanced coin.

Solution When two coins are tossed then possible outcomes are

HH, HT, TH, TT

i.e. $n(S) = 4$

Let A be the event of getting two heads then favourable outcome is HH.

so $n(A) = 1$

Now probability = $P(A) = \dfrac{n(A)}{n(S)}$

$= \dfrac{1}{4}$ *Answer*

Question # 4

Two coins are tossed twice each. Find the probability that the head appears on the first toss and the same faces appear in the two tosses.

Solution When the two coins are tossed then possible outcomes are

HH, HT, TH, TT

This shows $n(S) = 4$

Let A be the event that head appear in the first toss then favourable outcomes are HT, HH, i.e. $n(A) = 2$

Let B be the event that same face appear on the second toss then favourable outcomes are HH, TT. i.e. $n(B) = 2$

Now probability = $P(A \cap B) = P(A) \cdot P(B)$

$$= \frac{n(A)}{n(S)} \cdot \frac{n(B)}{n(S)} = \frac{2}{4} \cdot \frac{2}{4}$$

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

Question # 5

Two cards are drawn from a deck of 52 playing cards. If one card is drawn and replaced before drawing the second card, find the probability that both the cards are aces.

Solution Since there are 52 cards in the deck therefore $n(S) = 52$

Let A be the event that first card is an ace then $n(A)=4$

And let B be the event that the second card is also an ace then $n(B)=4$

Now probability = $P(A\cap B)=P(A)\cdot P(B)$

$$=\frac{n(A)}{n(S)}\cdot\frac{n(B)}{n(S)}$$

$$=\frac{4}{52}\cdot\frac{4}{52}=\frac{1}{169}$$

Question # 6

Two cards form a deck of 52 playing cards are drawn in such a way that the card is replaced after the first draw. Find the probability in the following cases:

i) first card is king and the second is queen.

ii) both the cards are faced cards i.e. king, queen, jack.

Solution Since there are 52 cards in the deck therefore $n(S)=52$

(i) Let A be the event that the first card is king then $n(A)=4$

and let B be the event that the second card is queen then $n(B)=4$

Now probability = $P(A\cap B)=P(A)\cdot P(B)$

$$=\frac{n(A)}{n(S)}\cdot\frac{n(B)}{n(S)}=\frac{4}{52}\cdot\frac{4}{52}=\frac{1}{169}$$ *Answer*

(ii) Let C be the event that first card is faced card.

Since there are 12 faced card in the deck therefore $n(C)=12$

and let D be the event that the second card is also faced card then $n(D)=12$

Now probability = $P(A\cap B)=P(A)\cdot P(B)$

$$=\frac{n(A)}{n(S)}\cdot\frac{n(B)}{n(S)}=\frac{12}{52}\cdot\frac{12}{52}$$

$$=\frac{3}{13}\cdot\frac{3}{13}=\frac{9}{169}$$ *Answer*

Question # 7

Two dice are thrown twice. What is probability that sum of dots shown in the first throw is 7 and that of the second throw is 11.

Solution When the two dice are thrown the possible outcomes are

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Which shows that $n(S)=36$

Let A be the event that the sum of dots in first throw is 7 then favourable outcomes are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) i.e. $n(A)=6$

Let B be the event that the sum of dots in second throw is 11 then favourable outcomes are (5, 6), (6, 5) i.e. $n(B)=2$

Now probability = $P(A\cap B)=P(A)\cdot P(B)$

$$=\frac{n(A)}{n(S)}\cdot\frac{n(B)}{n(S)}=\frac{6}{36}\cdot\frac{2}{36}$$

$$=\frac{1}{6}\cdot\frac{1}{18}=\frac{1}{108}$$

Question # 8

Find the probability that the sum of dots appearing in two successive throws of the dice is every time is 7.

Solution When the two dice are thrown the possible outcomes are

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Which shows that $n(S)=36$

Let A be the event that the sum of dots in first throw is 7 then favourable outcomes are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) i.e. $n(A)=6$

Let B be the event that the sum of dots in second throw is also 7 then similarly favourable outcomes = $n(B)=6$

Now probability = $P(A\cap B)=P(A)\cdot P(B)$

$$=\frac{n(A)}{n(S)}\cdot\frac{n(B)}{n(S)}=\frac{6}{36}\cdot\frac{6}{36}$$

$$=\frac{1}{6}\cdot\frac{1}{6}=\frac{1}{36}$$

Question # 9

A fair die is thrown twice. Find the probability that the prime number of dots appears in the first throw and the number of dots in the second throw is less than 5.

Solution When the die is thrown twice then the top may shows 1, 2, 3, 4, 5, 6

This shows possible outcomes = $n(S)=6$

Let A be the event that the number of the dots is prime then favourable outcomes are 2, 3 ,5 , i.e. $n(A)=3$

Let B be the event that the number of dots in second throw is less than 5 then favourable outcomes are 1 ,2 ,3 ,4 i.e. $n(B)=4$

Now probability = $P(A\cap B)=P(A)\cdot P(B)$

$$=\frac{n(A)}{n(S)}\cdot\frac{n(B)}{n(S)}=\frac{3}{6}\cdot\frac{4}{6}$$

$$= \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$$

Question # 10

A bag contain 8 red, 5 white and 7 black balls. 3 balls are drown from the bag. What is the probability that the first ball is red, the second ball is white and the third ball is black, when every time the ball is replaced?

Solution Since number of red balls = 8
Number of white ball = 5
Number of black ball = 7

Therefore total number of balls = 8 + 5 + 7 = 20 i.e. $n(S) = 20$

Let A be the event that the first ball is red then $n(A) = 8$

Let B be the event that the second ball is white then $n(B) = 5$

Let C be the event that the third ball is black then $n(C) = 7$

Now probability = $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

$$= \frac{n(A)}{n(S)} \cdot \frac{n(B)}{n(S)} \cdot \frac{n(C)}{n(S)}$$

$$= \frac{8}{20} \cdot \frac{5}{20} \cdot \frac{7}{20}$$