Exercise 7.4 (Solutions)

Textbook of Algebra and Trigonometry for Class XI

Question # 1

Evaluate the following:

(i) ${}^{12}C_3$ (ii) ${}^{20}C_{17}$ (iii) ${}^{n}C_4$

Solution

(i) $${}^{12}C_3 = \frac{12!}{(12-3)!\,3!} = \frac{12!}{9!\,3!} = \frac{12\cdot 11\cdot 10\cdot 9!}{9!\,3!} = \frac{12\cdot 11\cdot 10}{3!} = \frac{1320}{6} = 220$$

(ii) $${}^{20}C_{17} = \frac{20!}{(20-17)!\,17!} = \frac{20!}{3!\,17!} = \frac{20\cdot 19\cdot 18\cdot 17!}{3!\,17!} = \frac{20\cdot 19\cdot 18}{3!} = \frac{6840}{6} = 1140$$

(iii) $${}^{n}C_4 = \frac{n!}{(n-4)!\,4!} = \frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!\,4!} = \frac{n(n-1)(n-2)(n-3)}{4!}$$

Question # 2

Find the value of n, when

(i) ${}^{n}C_5 = {}^{n}C_4$ (ii) ${}^{n}C_{10} = \dfrac{12\times 11}{2!}$ (iii) ${}^{n}C_{12} = {}^{n}C_6$

Solution

(i)

Since ${}^{n}C_5 = {}^{n}C_4$

$\Rightarrow\ {}^{n}C_{n-5} = {}^{n}C_4$ $\quad\because\ {}^{n}C_r = {}^{n}C_{n-r}$

$\Rightarrow\ n-5 = 4 \quad \Rightarrow\ n = 4+5 \quad \Rightarrow\ \boxed{n=9}$

(ii) $${}^{n}C_{10} = \frac{12\times 11}{2!}$$

$$\Rightarrow\ {}^{n}C_{10} = \frac{12\cdot 11\cdot 10!}{2!\,10!}$$

$$\Rightarrow\ {}^{n}C_{10} = \frac{12!}{(12-10)!\,10!}$$

$$\Rightarrow\ {}^{n}C_{10} = {}^{12}C_{10}$$

$$\Rightarrow\ \boxed{n=12}.$$

(iii) *Do yourself as Q # 2* (i)

Question # 3

Find the values of n and r, when

(i) ${}^{n}C_r = 35$ and ${}^{n}P_r = 210$ (ii) ${}^{n-1}C_{r-1} : {}^{n}C_r : {}^{n+1}C_{r+1} = 3:6:11$

Solution

(i) $\quad {}^nC_r = 35 \quad \text{and} \quad {}^nP_r = 210$

Since ${}^nC_r = 35 \quad \Rightarrow \dfrac{n!}{(n-r)!\,r!} = 35 \quad \Rightarrow \dfrac{n!}{(n-r)!} = 35\cdot r!$ ………. (i)

Also ${}^nP_r = 210 \quad \Rightarrow \dfrac{n!}{(n-r)!} = 210$ ………… (ii)

Comparing (i) and (ii)

$$35\cdot r! = 210$$

$$\Rightarrow r! = \frac{210}{35} \quad \Rightarrow r! = 6 \quad \Rightarrow r! = 3! \quad \Rightarrow \boxed{r=3}$$

Putting value of r in equation (ii)

$$\frac{n!}{(n-3)!} = 210$$

$$\Rightarrow \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} = 210$$

$$\Rightarrow n(n-1)(n-2) = 210$$

$$\Rightarrow n(n-1)(n-2) = 7\cdot 6\cdot 5$$

$$\Rightarrow \boxed{n=7}$$

(ii) $\quad {}^{n-1}C_{r-1} : {}^nC_r : {}^{n+1}C_{r+1} = 3:6:11$

First consider

$${}^{n-1}C_{r-1} : {}^nC_r = 3:6$$

$$\Rightarrow \frac{(n-1)!}{(n-1-r+1)!\,(r-1)!} : \frac{n!}{(n-r)!\,r!} = 3:6$$

$$\Rightarrow \frac{(n-1)!}{(n-r)!\,(r-1)!} : \frac{n!}{(n-r)!\,r!} = 3:6$$

$$\Rightarrow \frac{\dfrac{(n-1)!}{(n-r)!\,(r-1)!}}{\dfrac{n!}{(n-r)!\,r!}} = \frac{3}{6}$$

$$\Rightarrow \frac{(n-1)!}{(n-r)!\,(r-1)!} \times \frac{(n-r)!\,r!}{n!} = \frac{1}{2}$$

$$\Rightarrow \frac{(n-1)!}{(r-1)!} \times \frac{r!}{n!} = \frac{1}{2}$$

$$\Rightarrow \frac{r}{n} = \frac{1}{2} \quad \Rightarrow n = 2r \text{ ………. (i)}$$

Now consider ${}^{n}C_r : {}^{n+1}C_{r+1} = 6:11$

$$\Rightarrow \frac{n!}{(n-r)!\,r!} : \frac{(n+1)!}{(n+1-r-1)!\,(r+1)!} = 6:11$$

$$\Rightarrow \frac{n!}{(n-r)!\,r!} : \frac{(n+1)!}{(n-r)!\,(r+1)!} = 6:11$$

$$\Rightarrow \frac{\frac{n!}{(n-r)!\,r!}}{\frac{(n+1)!}{(n-r)!\,(r+1)!}} = \frac{6}{11}$$

$$\Rightarrow \frac{n!}{(n-r)!\,r!} \times \frac{(n-r)!\,(r+1)!}{(n+1)!} = \frac{6}{11}$$

$$\Rightarrow \frac{n!}{r!} \times \frac{(r+1)!}{(n+1)!} = \frac{6}{11}$$

$$\Rightarrow \frac{n!}{r!} \times \frac{(r+1)\,r!}{(n+1)\,n!} = \frac{6}{11}$$

$$\Rightarrow \frac{(r+1)}{(n+1)} = \frac{6}{11}$$

$\Rightarrow 11(r+1) = 6(n+1)$

$\Rightarrow 11(r+1) = 6(2r+1)$ $\quad\quad \because\ n = 2r$

$\Rightarrow 11r + 11 = 12r + 6$

$\Rightarrow 11r - 12r = 6 - 11 \Rightarrow \ -r = -5 \ \ \Rightarrow \boxed{r = 5}$

Putting value of r in equation (ii)

$\Rightarrow \boxed{n = 10}$

Question # 4

How many (a) diagonals and (b) triangles can be formed by joining the vertices of the polygon having

(i) 5 sides (ii) 8 sides (iii) 12 sides

Solution

(i)

(*a*) 5 sided polygon has 5 vertices,

so joining two vertices we have line segments = ${}^{5}C_2 = 10$

Number of sides = 5

So number of diagonals = 10 – 5 = 5

(*b*) 5 sided polygon has 5 vertices,

so joining any three vertices we have triangles = ${}^{5}C_3 = 10$

(ii)

(a) 8 sided polygon has 8 vertices
So joining any two vertices we have line segments = ${}^{8}C_2 = 28$
Number of sides = 8
So number of diagonals = 28 – 8 = 20

(b) 8 sided polygon has 8 vertices,
so joining any three vertices we have triangles = ${}^{8}C_3 = 56$.

(iii) *Do yourself as above.*

Question # 5
The members of a club are 12 boys and 8 girls. In how many ways can a committee of 3 boys and 2 girls be formed?

Solution

Number of boys = 12
So committees formed taking 3 boys $= {}^{12}C_3 = 220$
Number of girls = 8
So committees formed by taking 2 girls = $= {}^{8}C_2 = 28$
Now total committees formed including 3 boys and 2 girls = 220×28
$= 6160$

Question # 6
How many committees of 5 members can be chosen from a group of 8 persons when each committee must include 2 particular persons?

Solution

Number of persons $= 8$
Since two particular persons are included in every committee so we have to find combinations of 6 persons 3 at a time = ${}^{6}C_3 = 20$
Hence number of committees = 20

Question # 7
In how many ways can a hockey team of 11 players be selected out of 15 players? How many of them will include a particular player?

Solution

The number of player = 15
So combination, taking 11 player at a time = ${}^{15}C_{11} = 1365$
Now if one particular player is in each collection
then number of combination = ${}^{14}C_{10} = 1001$

Question # 8
Show that: ${}^{16}C_{11} + {}^{16}C_{10} = {}^{17}C_{11}$

Solution

L.H.S = ${}^{16}C_{11} + {}^{16}C_{10}$

$$= \frac{16!}{(16-11)!\,11!} + \frac{16!}{(16-10)!\,10!} = \frac{16!}{5!\,11!} + \frac{16!}{6!\,10!}$$

$$=\frac{16!}{5!\ 11\cdot 10!}+\frac{16!}{6\cdot 5!\ 10!}=\frac{16!}{10!\ 5!}\left(\frac{1}{11}+\frac{1}{6}\right)$$
$$=\frac{16!}{10!\ 5!}\left(\frac{6+11}{66}\right)=\frac{16!}{10!\ 5!}\left(\frac{17}{66}\right)=\frac{16!}{10!\ 5!}\left(\frac{17}{11\cdot 6}\right)$$
$$=\frac{17\cdot 16!}{11\cdot 10!\ 6\cdot 5!}=\frac{17!}{11!\ 6!}=\frac{17!}{11!\ (17-11)!}={}^{17}C_{11}=\text{R.H.S}$$

Alternative

L.H.S = ${}^{16}C_{11}+{}^{16}C_{10}=4368+8008=12276$ ………. (i)

R.H.S = ${}^{17}C_{11}=12376$ ……… (ii)

From (i) and (ii)

L.H.S = R.H.S

Question # 9

There are 8 men and 10 women members of a club. How many committees of numbers can be formed, having;

(i) 4 women (ii) at the most 4 women (iii) at least 4 women

Solution

Number of men = 8

Number of women = 10

(i) We have to form combination of 4 women out of 10 and 3 men out o

$$={}^{10}C_4\times{}^{8}C_3=210\times 56=11760$$

(ii) At the most 4 women means that women are less than or equal to 4, which implies the following possibilities $(1W,6M),(2W,5M),(3W,4M),(4W,3M),(7M)$

$$={}^{10}C_1\times{}^{8}C_6+{}^{10}C_2\times{}^{8}C_5+{}^{10}C_3\times{}^{8}C_4+{}^{10}C_4\times{}^{8}C_3+{}^{8}C_7$$
$$=(10)(28)+(45)(56)+(120)(70)+(210)(56)+(8)$$
$$=280+2520+8400+11760+8=22968$$

(iii) At least 4 women means that women are greater than or equal to 4, which implies the following possibilities $(4W,3M),(5W,2M),(6W,1M),(7W)$

$$={}^{10}C_4\times{}^{8}C_3+{}^{10}C_5\times{}^{8}C_2+{}^{10}C_6\times{}^{8}C_1+{}^{10}C_7$$
$$=(210)(56)+(252)(28)+(210)(8)+120$$
$$=11760+7056+1680+120$$
$$=20616$$

Question # 10

Prove that; ${}^{n}C_r+{}^{n}C_{r-1}={}^{n+1}C_r$

Solution

$$\text{L.H.S}={}^{n}C_r+{}^{n}C_{r-1}=\frac{n!}{(n-r)!\ r!}+\frac{n!}{(n-(r-1))!\ (r-1)!}$$

$$= \frac{n!}{(n-r)!\ r!} + \frac{n!}{(n-r+1)!\ (r-1)!}$$

$$= \frac{n!}{(n-r)!\ r(r-1)!} + \frac{n!}{(n-r+1)(n-r)!\ (r-1)!}$$

$$= \frac{n!}{(n-r)!\ (r-1)!}\left(\frac{1}{r} + \frac{1}{(n-r+1)}\right)$$

$$= \frac{n!}{(n-r)!\ (r-1)!}\left(\frac{n-r+1+r}{r(n-r+1)}\right)$$

$$= \frac{n!}{(n-r)!\ (r-1)!}\left(\frac{n+1}{r(n-r+1)}\right)$$

$$= \frac{(n+1)n!}{(n-r+1)(n-r)!\ \ r(r-1)!}$$

$$= \frac{(n+1)!}{(n-r+1)!\ \ r!} = \frac{(n+1)!}{(n+1-r)!\ \ r!}$$

$$= {}^{n+1}C_r = \text{R.H.S}$$