

**Question # 1**

Evaluate the following:

(i)  ${}^{20}P_3$

(ii)  ${}^{16}P_4$

**Solution**

(i)

$${}^{20}P_3 = \frac{20!}{(20-3)!} = \frac{20!}{17!} = \frac{20 \cdot 19 \cdot 18 \cdot 17!}{17!} = 20 \cdot 19 \cdot 18 = 6840$$

(ii)

$${}^{16}P_4 = \frac{16!}{(16-4)!} = \frac{16!}{12!} = \frac{16 \cdot 15 \cdot 14 \cdot 13 \cdot 12!}{12!} = 16 \cdot 15 \cdot 14 \cdot 13 = 43680$$

*Others do yourself*

**Question # 2**

Find the value of  $n$  when:

(i)  ${}^nP_2 = 30$

(ii)  ${}^{11}P_n = 11 \cdot 10 \cdot 9$

(iii)  ${}^nP_4 : {}^{n-1}P_3 = 9 : 1$

**Solution**

(i)

$$\begin{aligned} {}^nP_2 = 30 &\Rightarrow \frac{n!}{(n-2)!} = 30 \Rightarrow \frac{n(n-1)(n-2)!}{(n-2)!} = 30 \\ \Rightarrow n(n-1) = 30 &\Rightarrow n(n-1) = 6 \cdot 5 \Rightarrow \boxed{n=6} \end{aligned}$$

(ii)

$$\begin{aligned} {}^{11}P_n = 11 \cdot 10 \cdot 9 &\Rightarrow \frac{11 \cdot 10 \cdot 9 \cdot 8!}{(11-n)!} = 11 \cdot 10 \cdot 9 \\ \Rightarrow \frac{8!}{(11-n)!} = 1 &\Rightarrow 8! = (11-n)! \Rightarrow 8 = 11 - n \Rightarrow n = 11 - 8 \end{aligned}$$

$\Rightarrow \boxed{n=3}$

(iii)

$$\begin{aligned} {}^nP_4 : {}^{n-1}P_3 = 9 : 1 &\Rightarrow \frac{{}^nP_4}{{}^{n-1}P_3} = \frac{9}{1} \Rightarrow {}^nP_4 = 9 \cdot {}^{n-1}P_3 \\ \Rightarrow \frac{n!}{(n-4)!} = 9 \frac{(n-1)!}{(n-1-3)!} &\Rightarrow \frac{n(n-1)!}{(n-4)!} = 9 \frac{(n-1)!}{(n-4)!} \Rightarrow \boxed{n=9} \end{aligned}$$

**Question # 3**

Prove from the first principle that:

(i)  ${}^nP_r = n \cdot {}^{n-1}P_{r-1}$

(ii)  ${}^nP_r = {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1}$

**Solution**

(i)

$$\text{R.H.S} = n \cdot {}^{n-1}P_{r-1} = n \cdot \frac{(n-1)!}{(n-1-(r-1))!} = \frac{n(n-1)!}{(n-1-r+1)!} = \frac{n!}{(n-r)!} = {}^nP_r = \text{L.H.S}$$

(ii)

$$\begin{aligned} \text{R.H.S} &= {}^{n-1}P_r + r \cdot {}^{n-1}P_{r-1} = \frac{(n-1)!}{(n-1-r)!} + r \cdot \frac{(n-1)!}{(n-1-r+1)!} \\ &= \frac{(n-1)!}{(n-r-1)!} + r \cdot \frac{(n-1)!}{(n-r)!} = \frac{(n-1)!}{(n-r-1)!} + r \cdot \frac{(n-1)!}{(n-r)(n-r-1)!} \\ &= \frac{(n-1)!}{(n-r-1)!} \left( 1 + r \cdot \frac{1}{(n-r)} \right) = \frac{(n-1)!}{(n-r-1)!} \left( \frac{n-r+r}{(n-r)} \right) \\ &= \frac{(n-1)!}{(n-r-1)!} \left( \frac{n}{(n-r)} \right) = \frac{n(n-1)!}{(n-r)(n-r-1)!} \\ &= \frac{n!}{(n-r)!} = {}^nP_r = \text{L.H.S} \end{aligned}$$

#### Question # 4

*Do yourself as Question # 5*

#### Question # 5

How many signals can be given by 6 flags of different colours, when any number of flags can be used at a time?

#### **Solution**

Total number of flags =  $n = 6$

Number of signal using one flag =  ${}^6P_1 = 6$

Number of signal using two flag =  ${}^6P_2 = 30$

Number of signal using three flag =  ${}^6P_3 = 120$

Number of signal using four flag =  ${}^6P_4 = 360$

Number of signal using five flag =  ${}^6P_5 = 720$

Number of signal using six flag =  ${}^6P_6 = 720$

Total number of signals =  $6 + 30 + 120 + 360 + 720 + 720 = 1956$

#### Question # 6

How many words can be formed from the letters of the following words using all letters when no letter is to be repeated:

(i) PLSNE

(ii) OBJECT

(iii) FASTING

#### **Solution**

(i)

Since number of letters in PLANE =  $n = 5$

Therefore total words form =  ${}^5P_5 = 120$

(ii)

Since number of letters in OBJECT =  $n = 6$

Therefore total words forms =  ${}^6P_6 = 720$

(iii)

Since number of letters in FASTING =  $n = 7$

Therefore total words forms =  ${}^7P_7 = 5040$

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### Question # 7

How many 3-digit numbers can be formed by using each one of the digits 2,3,5,7,9 only once?

#### Solution

Number of digits =  $n = 5$

So numbers forms taken 3 digits at a time =  ${}^5P_3 = 60$

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### Question # 8

Find the numbers greater than 23000 that can be formed from the digits 1,2,3,5,6 without repeating any digit.

#### Solution

Number greater than 23000 can be formed as

Number of numbers of the form 23\*\*\* =  ${}^3P_3 = 6$

Number of numbers of the form 25\*\*\* =  ${}^3P_3 = 6$

Number of numbers of the form 26\*\*\* =  ${}^3P_3 = 6$

Number of numbers of the form 3\*\*\*\* =  ${}^4P_4 = 24$

Number of numbers of the form 5\*\*\*\* =  ${}^4P_4 = 24$

Number of numbers of the form 6\*\*\*\* =  ${}^4P_4 = 24$

Thus the total number formed =  $6 + 6 + 6 + 24 + 24 + 24 = 90$

#### Alternative solution:

Permutation of 5 digits numbers =  ${}^5P_5 = 120$

Numbers less than 23000 are of the form 1\*\*\*\*

Then permutations =  ${}^4P_4 = 24$

If number less than 23000 are of the form 21\*\*\*

Then permutations =  ${}^3P_3 = 6$

Thus number greater than 23000 formed =  $120 - 24 - 6 = 90$

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### Question # 9

Find the number of 5 digits numbers that can be formed from the digits 1, 2, 4, 6, 8 (when no digit is repeated), but

(i) the digits 2 and 8 are next to each other.

(ii) the digits 2 and 8 are not next to each other.

#### Solution

Total number of digits = 5

(i) If we take 28 as a single digit then number of numbers =  ${}^4P_4 = 24$

If we take 82 as a single digit then number of numbers =  ${}^4P_4 = 24$

So the total numbers when 2 and 8 are next to each other =  $24 + 24 = 48$

- (ii) Number of total permutation =  ${}^5P_5 = 120$   
thus number of numbers when 2 and 8 are not next to each other =  $120 - 48 = 72$

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**Question # 10**

How many 6-digit numbers can be formed, without repeating any digit from the digits 0,1,2,3,4,5? In how many of them will 0 be at the ten places?

**Solution**

Since number of permutation of 6 digits =  ${}^6P_6 = 720$

But 0 at extreme left is meaning less

so number of permutation when 0 is at extreme left =  ${}^5P_5 = 120$

Thus the number formed by 6 digits =  $720 - 120 = 600$

Now if we fix 0 at ten place then number formed =  ${}^5P_5 = 120$

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**Question # 11**

How many 5-digit multiples of 5 can be formed from the digits 2,3,5,7,9, when no digit is repeated.

**Solution**

Number of digits = 5

For multiple of 5 we must have 5 at extreme right so number formed =  
 ${}^4P_4 = 24$

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**Question # 12**

In how many ways can 8 books including 2 books on English be arranged on a shelf in such a way that the English books are never together.

**Solution**

Total numbers of books = 8

Total number of permutation =  ${}^8P_8 = 40320$

Let  $E_1$  and  $E_2$  denotes two English books then

Number of permutation when  $E_1E_2$  place together =  ${}^7P_7 = 5040$

Number of permutation when  $E_2E_1$  place together =  ${}^7P_7 = 5040$

So total permutation when  $E_1$  and  $E_2$  together =  $5040 + 5040 = 10080$

Required permutation when English books are not together =  $40320 - 10080 = 30240$

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**Question # 13**

Find the number of arrangement of 3 books on English and 5 books on Urdu for placing them on a shelf such that the books on the same subjects are together.

**Solution**

Let  $E_1, E_2, E_3$  be the book on English and  $U_1, U_2, U_3, U_4, U_5$  be the book on Urdu

Then the permutation when

books are arranged as  $E_1, E_2, E_3, U_1, U_2, U_3, U_4, U_5 = {}^3P_3 \times {}^5P_5 = 6 \times 120 = 720$

books are arranged as  $U_1, U_2, U_3, U_4, U_5, E_1, E_2, E_3 = {}^5P_5 \times {}^3P_3 = 120 \times 6 = 720$   
so total permutation when books of same subject are together =  $720 + 720$   
= 1440

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**Question # 14**

In how many ways can 5 boys and 4 girls be seated on bench so that the girls and the boys occupy alternative seats?

**Solution**

Let the five boys be  $B_1, B_2, B_3, B_4, B_5$  and the four girls are  $G_1, G_2, G_3, G_4$  then there seats plane is  $B_1, G_1, B_2, G_2, B_3, G_3, B_4, G_4, B_5$

$$\begin{aligned} \text{Then the permutations} &= {}^5P_1 \times {}^4P_1 \times {}^4P_1 \times {}^3P_1 \times {}^3P_1 \times {}^2P_1 \times {}^2P_1 \times {}^1P_1 \times {}^1P_1 \\ &= 5 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1 = 2880 \end{aligned}$$

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*If you found any error, please report us at [www.megalecture@gmail.com](mailto:www.megalecture@gmail.com)*



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