

QNo.1 i) b. -2 and 8

$$a = -2 \text{ and } b = 8$$

$$\begin{aligned} \text{Since } G.M &= \pm \sqrt{ab} \\ &= \pm \sqrt{(-2)(8)} \\ &= \pm \sqrt{-16} = \pm 4i \end{aligned}$$

ii) $-2i$ and $8i$

$$a = -2i, \quad b = 8i$$

$$\begin{aligned} \text{Since } G.M &= \pm \sqrt{ab} \\ &= \pm \sqrt{(-2i)(8i)} \\ &= \pm \sqrt{-16i^2} \\ &= \pm \sqrt{16} \quad \because i^2 = -1 \\ &= \pm 4 \end{aligned}$$

QNo.2 i) Let G_1, G_2 ~~etc~~ are two ~~three~~ G.Ms between 1 and 8.

then $1, G_1, G_2, 8$ are in G.P.

$$\begin{aligned} \text{here } a_1 &= 1, \quad a_4 = 8 \\ \Rightarrow a_1 r^3 &= 8 \\ \Rightarrow (1)r^3 &= 8 \\ \Rightarrow r^3 &= 8 \\ \Rightarrow r &= 2 \end{aligned}$$

Now

$$\begin{aligned} G_1 = a_2 &= a_1 r = (1)(2) = 2 \\ G_2 = a_3 &= a_1 r^2 = (1)(2)^2 = (1)(4) = 4 \\ \text{hence } 2, 4 &\text{ are two} \\ \text{G.Ms between } 1 &\text{ and } 8 \end{aligned}$$

ii) Do yourself as (i)

QNo.3 i) Let G_1, G_2 and G_3 are three G.Ms between 1 and 16.

then $1, G_1, G_2, G_3, 16$ are in G.P.

$$\begin{aligned} \text{here } a_1 &= 1, \quad a_5 = 16 \\ \Rightarrow a_1 r^4 &= 16 \\ \Rightarrow 1 \cdot r^4 &= 16 \\ \Rightarrow r^4 &= (2)^4 \\ \Rightarrow r &= 2 \end{aligned}$$

$$\text{Now } G_1 = a_2 = a_1 r = (1)(2) = 2$$

$$G_2 = a_3 = a_1 r^2 = (1)(2)^2 = 4$$

$$G_3 = a_4 = a_1 r^3 = (1)(2)^3 = 8$$

hence 2, 4, 8 are three G.Ms between 1 and 16.

QNo.3 (ii) Do yourself

QNo.4 Let G_1, G_2, G_3 and G_4 are four G.Ms between 3 and 96.

then $3, G_1, G_2, G_3, G_4, 96$ are in G.P.

$$\begin{aligned} \text{here } a_1 &= 3, \quad a_6 = 96 \\ \Rightarrow a_1 r^5 &= 96 \\ \Rightarrow 3 r^5 &= 96 \\ \Rightarrow r^5 &= \frac{96}{3} = 32 \\ \Rightarrow r^5 &= (2)^5 \Rightarrow r = 2 \end{aligned}$$

Now ~~Do yourself~~

$$\begin{aligned} G_1 = a_2 &= a_1 r = (3)(2) = 6 \\ G_2 = a_3 &= a_1 r^2 = (3)(2)^2 = 12 \\ G_3 = a_4 &= a_1 r^3 = (3)(2)^3 = 24 \\ G_4 = a_5 &= a_1 r^4 = (3)(2)^4 = 48 \end{aligned}$$

hence 6, 12, 24, 48 are the four G.Ms between 3 and 96.

QNo.5

Suppose $A > G$

$$\text{then } \frac{x+y}{2} > \pm \sqrt{xy}$$

$$\Rightarrow x+y > \pm 2\sqrt{xy}$$

$$\Rightarrow x+y \mp 2\sqrt{xy} > 0$$

$$\Rightarrow (\sqrt{x})^2 + (\sqrt{y})^2 \mp 2\sqrt{xy} > 0$$

$$\Rightarrow (\sqrt{x} \mp \sqrt{y})^2 > 0$$

which is true as square is always +ve hence $A > G$.

Q No 6 We know that

$$G.M = \sqrt{ab} \quad \text{--- (i)}$$

but we have given

$$G.M = \frac{a^n + b^n}{2^{n-1} + b^{n-1}} \quad \text{--- (ii)}$$

Comparing (i) and (ii)

$$\frac{a^n + b^n}{2^{n-1} + b^{n-1}} = \sqrt{ab}$$

$$\Rightarrow a^n + b^n = a^{\frac{1}{2}} b^{\frac{1}{2}} (2^{n-1} + b^{n-1})$$

$$\Rightarrow a^n + b^n = a^{n-\frac{1}{2}} b^{\frac{1}{2}} + a^{\frac{1}{2}} b^{n-\frac{1}{2}}$$

$$\Rightarrow a^n - a^{n-\frac{1}{2}} b^{\frac{1}{2}} = a^{\frac{1}{2}} b^{n-\frac{1}{2}} - b^n$$

$$\Rightarrow a^{n-\frac{1}{2}} (a^{\frac{1}{2}} - b^{\frac{1}{2}}) = b^{n-\frac{1}{2}} (a^{\frac{1}{2}} - b^{\frac{1}{2}})$$

$$\Rightarrow a^{n-\frac{1}{2}} = b^{n-\frac{1}{2}}$$

$$\Rightarrow \frac{a^{n-\frac{1}{2}}}{b^{n-\frac{1}{2}}} = 1$$

$$\Rightarrow \left(\frac{a}{b}\right)^{n-\frac{1}{2}} = \left(\frac{a}{b}\right)^0 \quad \because \left(\frac{a}{b}\right)^0 = 1$$

$$\Rightarrow n - \frac{1}{2} = 0 \Rightarrow \boxed{n = \frac{1}{2}}$$

Q No 7 Let a and b be two true integers then by given condition.

$$A.M = G.M + 2$$

$$\Rightarrow \frac{a+b}{2} = \sqrt{ab} + 2$$

$$\Rightarrow a+b = 2(\sqrt{ab} + 2) \quad \text{--- (i)}$$

Also we have given

$$a+b = 20 \quad \text{--- (ii)}$$

Comparing (i) & (ii)

$$2(\sqrt{ab} + 2) = 20 \Rightarrow 2\sqrt{ab} + 4 = 20$$

$$\Rightarrow 2\sqrt{ab} = 20 - 4$$

$$= 16$$

$$\Rightarrow \sqrt{ab} = \frac{16}{2} = 8$$

$$\Rightarrow ab = 64 \quad \text{by squaring}$$

$$\Rightarrow b = \frac{64}{a} \quad \text{--- (iii)}$$

putting in (ii)

$$a + \frac{64}{a} = 20$$

$$\Rightarrow a^2 + 64 = 20a$$

$$\Rightarrow a^2 - 20a + 64 = 0$$

$$\Rightarrow a^2 - 16a - 4a + 64 = 0$$

$$\Rightarrow a(a-16) - 4(a-16) = 0$$

$$\Rightarrow (a-16)(a-4) = 0$$

$$a-16=0, \quad a-4=0$$

$$a=16, \quad a=4$$

putting in (iii)

$$b = \frac{64}{16} = 4, \quad b = \frac{64}{4} = 16$$

hence 16, 4 OR 4, 16 are

required numbers

Q No 8 Let a, and b be two true required number

then A.M = 5

$$\Rightarrow \frac{a+b}{2} = 5$$

$$\Rightarrow a+b = 10 \quad \text{--- (i)}$$

Also

$$G.M = 4$$

$$\Rightarrow \sqrt{ab} = 4$$

$$\Rightarrow ab = 16 \quad \text{on squaring}$$

$$\Rightarrow b = \frac{16}{a} \quad \text{--- (ii)}$$

putting in (i)

$$a + \frac{16}{a} = 10$$

$$\Rightarrow a^2 + 16 = 10a$$

$$\Rightarrow a^2 - 10a + 16 = 0$$

$$\Rightarrow a^2 - 8a - 2a + 16 = 0$$

$$\Rightarrow a(a-8) - 2(a-8) = 0$$

$$\Rightarrow (a-8)(a-2) = 0$$

$$a-8=0, \quad a-2=0$$

$$a=8, \quad a=2$$

putting in (ii)

$$b = \frac{16}{8} = 2, \quad b = \frac{16}{2} = 8$$

hence 8, 2 OR 2, 8 are

required numbers

END

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