

Q_{No}1 3, 6, 12, ...

$$a_1 = 3, \quad r = \frac{a_2}{a_1} = \frac{6}{3} = 2, \quad n = 5$$

Since $a_n = a_1 r^{n-1}$

$$\Rightarrow a_5 = (3)(2)^{5-1}$$

$$= (3)(2)^4 = (3)(16)$$

$$= 48 \quad \text{Answer}$$

Q_{No}2 $1+i, 2, \frac{4}{1+i}, \dots$

$$a_1 = 1+i, \quad r = \frac{a_2}{a_1} = \frac{2}{1+i}, \quad n = 11$$

Since $a_n = a_1 r^{n-1}$

$$\Rightarrow a_{11} = (1+i) \left(\frac{2}{1+i} \right)^{11-1}$$

$$= (1+i) \left(\frac{2}{1+i} \right)^{10}$$

$$= (1+i) \left(\frac{2}{1+i} \cdot \frac{1-i}{1-i} \right)^{10}$$

$$= (1+i) \left(\frac{2(1-i)}{(1)^2 - (i)^2} \right)^{10}$$

$$= (1+i) \left(\frac{2(1-i)}{1+1} \right)^{10} \quad \because i^2 = -1$$

$$= (1+i) \left(\frac{2(1-i)}{2} \right)^{10}$$

$$= (1+i)(1-i)^{10}$$

$$= (1+i) [(1-i)^2]^5$$

$$= (1+i) [(1)^2 - 2(1)(i) + (i)^2]^5$$

$$= (1+i) [1 - 2i - 1]^5 \quad \because i^2 = -1$$

$$= (1+i)(-2i)^5$$

$$= (1+i)(-2)^5(i)^5$$

$$= (1+i)(-32)(i^4 \cdot i)$$

$$= (1+i)(-32)(i^2)^2 \cdot i$$

$$= (1+i)(-32)(-1)^2 \cdot i$$

$$= (1+i)(-32)(1) \cdot i$$

$$= -32i(1+i)$$

$$= -32i - 32i^2$$

$$= -32i - 32(-1) = -32i + 32$$

$$= 32(-i+1) = 32(1-i) \quad \text{Ans.}$$

Q_{No}3 $1+i, 2i, -2+2i, \dots$

$$a_1 = 1+i, \quad r = \frac{a_2}{a_1} = \frac{2i}{1+i}, \quad n = 12$$

Since $a_n = a_1 r^{n-1}$

$$\Rightarrow a_{12} = (1+i) \left(\frac{2i}{1+i} \right)^{12-1}$$

$$= (1+i) \left(\frac{2i}{1+i} \cdot \frac{1-i}{1-i} \right)^{11}$$

$$= (1+i) \left(\frac{2i^2 - 2i^2}{(1)^2 - (i)^2} \right)^{11}$$

$$= (1+i) \left(\frac{2i^2 + 2}{1+1} \right)^{11} \quad \because i^2 = -1$$

$$= (1+i) \left(\frac{2(i+1)}{2} \right)^{11}$$

$$= (1+i)(1+i)^{11} = (1+i)^{12}$$

$$= [(1+i)^2]^6 = [1+2i+i^2]^6$$

$$= [1+2i-1]^6 = (2i)^6$$

$$= 2^6 \cdot (i)^6 = 64(i^2)^3$$

$$= 64(-1)^3 = -64 \quad \text{Answer}$$

Q_{No}4 Do yourself as Q_{No}2 & 3.

Q_{No}5 Here $a_1 = 12000$
depreciation = 5%
therefore $r = 1 - \frac{5}{100} = 1 - 0.05$
 $= 0.95$
 $n = 5$
Since $a_n = a_1 r^{n-1}$

$$\Rightarrow a_5 = (12000)(0.95)^{5-1}$$

$$= (12000)(0.95)^4$$

$$= (12000)(0.8145)$$

$$= 9774.08$$

Thus value of automobile
at the end of 4 year is 9774.08



No. 6 $x^2 - y^2, x+y, \frac{x+y}{x-y}, \dots, \frac{x+y}{(x-y)^9}$

Here $a_1 = x^2 - y^2$

$$r = \frac{x+y}{x^2-y^2} = \frac{x+y}{(x-y)(x+y)} = \frac{1}{x-y}$$

$n = ? \rightarrow a_n = \frac{x+y}{(x-y)^9}$

Since $a_n = a_1 r^{n-1}$

$$\Rightarrow \frac{x+y}{(x-y)^9} = (x^2 - y^2) \left(\frac{1}{x-y} \right)^{n-1}$$

$$\Rightarrow \frac{x+y}{(x-y)^9} = (x+y)(x-y) \frac{1}{(x-y)^{n-1}}$$

$$\Rightarrow \frac{1}{(x-y)^9} = \frac{1}{(x-y)^{n-1-1}}$$

$$\Rightarrow \frac{1}{(x-y)^9} = \frac{1}{(x-y)^{n-2}}$$

$$\Rightarrow \left(\frac{1}{x-y} \right)^9 = \left(\frac{1}{x-y} \right)^{n-2}$$

$$\Rightarrow 9 = n-2 \Rightarrow 9+2 = n$$

$$\Rightarrow \boxed{n = 11} \text{ Answer}$$

No. 7 Since a, b, c, d are in G.P

herefore $\frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

So $\frac{b}{a} = \frac{c}{b} \Rightarrow b^2 = ac$ — (i)

$\frac{c}{b} = \frac{d}{c} \Rightarrow c^2 = bd$ — (ii)

$\frac{b}{a} = \frac{d}{c} \Rightarrow bc = ad$ — (iii)

i) To show $a-b, b-c, c-d$ are in G.P.

Let $r = \frac{b-c}{a-b}$ — (1)

Also $r = \frac{c-d}{b-c}$

$$= \frac{c-d}{b-c} \cdot \frac{a-b}{a-b}$$

$$= \frac{ac - ad - bc + bd}{(b-c)(a-b)}$$

$$= \frac{b^2 - bc - bc + c^2}{(b-c)(a-b)} \quad \because ac = b^2$$

$$= \frac{b^2 - 2bc + c^2}{(b-c)(a-b)}$$

$$= \frac{(b-c)^2}{(b-c)(a-b)} = \frac{b-c}{a-b} \text{ — (2)}$$

from (1) & (2)
 $r = r'$

therefore $a-b, b-c, c-d$ are in G.P.

ii) To show $a^2 - b^2, b^2 - c^2, c^2 - d^2$ are in G.P.

Let $r = \frac{b^2 - c^2}{a^2 - b^2}$ — (1)

Also $r = \frac{c^2 - d^2}{b^2 - c^2}$

$$= \frac{c^2 - d^2}{b^2 - c^2} \cdot \frac{a^2 - b^2}{a^2 - b^2}$$

$$= \frac{a^2 c^2 - a^2 d^2 - b^2 c^2 + b^2 d^2}{(b^2 - c^2)(a^2 - b^2)}$$

$$= \frac{(ac)^2 - (ad)^2 - (bc)^2 + (bd)^2}{(b^2 - c^2)(a^2 - b^2)}$$

$$= \frac{(b^2)^2 - (bc)^2 - (bc)^2 + (c^2)^2}{(b^2 - c^2)(a^2 - b^2)} \text{ From (i) \& (ii)}$$

$$= \frac{b^4 - 2b^2 c^2 + c^4}{(b^2 - c^2)(a^2 - b^2)}$$

$$= \frac{(b^2 - c^2)^2}{(b^2 - c^2)(a^2 - b^2)}$$

$$= \frac{b^2 - c^2}{a^2 - b^2} \text{ — (2)}$$

From (1) and (2)

$r = r'$

hence $a^2 - b^2, b^2 - c^2, c^2 - d^2$ are in G.P.

iii) Do Yourself

Hint: $r = \frac{b^2 + c^2}{a^2 + b^2}$ — (1)

Also $r = \frac{c^2 + d^2}{b^2 + c^2} = \frac{c^2 + d^2}{b^2 + c^2} \cdot \frac{a^2 + b^2}{a^2 + b^2}$
(Same as ii)

Q No. 8 $a_1, a_1 r^2, a_1 r^4, \dots$

The sequence of reciprocal of the term is

$$\frac{1}{a_1}, \frac{1}{a_1 r^2}, \frac{1}{a_1 r^4}, \dots$$

To show this is in G.P. let

$$r_1 = \frac{a_2}{a_1} = \frac{1/a_1 r^2}{1/a_1} = \frac{1}{r^2} \quad a_1 = 1$$

Also

$$r_1' = \frac{a_3}{a_2} = \frac{1/a_1 r^4}{1/a_1 r^2} = \frac{1}{r^2} \quad a_1 r^2$$

$$= \frac{1}{r^2} \quad \text{--- (ii)}$$

From (i) & (ii)

$$r_1' = r_1$$

therefore the sequence of reciprocal of the term of G.P. is also in G.P.

Q No. 9 Let a_1 be the first term and r be the common difference ratio.

Since $\frac{a_5}{a_3} = \frac{4}{9}$

$$\Rightarrow \frac{a_1 r^{5-1}}{a_1 r^{3-1}} = \frac{4}{9} \Rightarrow \frac{r^4}{r^2} = \frac{4}{9}$$

$$\Rightarrow r^2 = \frac{4}{9} \Rightarrow r = \pm \frac{2}{3}$$

Also

$$a_2 = \frac{4}{9}$$

$$\Rightarrow a_1 r^{2-1} = \frac{4}{9} \Rightarrow a_1 r = \frac{4}{9}$$

When $r = \frac{2}{3}$

$$a_1 \left(\frac{2}{3}\right) = \frac{4}{9} \Rightarrow a_1 = \frac{2 \cdot 4}{9 \cdot \frac{2}{3}} = \frac{4}{3}$$

$$\Rightarrow a_1 = \frac{4}{3}$$

When $r = -\frac{2}{3}$

$$a_1 \left(-\frac{2}{3}\right) = \frac{4}{9} \Rightarrow a_1 = \frac{4}{9} \left(-\frac{3}{2}\right)$$

$$\Rightarrow a_1 = -\frac{2}{3}$$

Since $a_n = a_1 r^{n-1}$

When $a_1 = \frac{2}{3}$ and $r = \frac{2}{3}$

$$a_n = \left(\frac{2}{3}\right) \left(\frac{2}{3}\right)^{n-1} = \left(\frac{2}{3}\right)^n$$

When $a_1 = -\frac{2}{3}$ and $r = -\frac{2}{3}$

$$a_n = \left(-\frac{2}{3}\right) \left(-\frac{2}{3}\right)^{n-1} = \left(-\frac{2}{3}\right)^n \text{ or } (-1)^n \left(\frac{2}{3}\right)^n$$

Q No. 10 Consider $\frac{a_1}{r}, a_1, a_1 r$

are three consecutive terms

in G.P. by given condition.

$$\frac{a_1}{r} + a_1 + a_1 r = 26$$

$$\Rightarrow a_1 \left(\frac{1}{r} + 1 + r\right) = 26$$

$$\Rightarrow a_1 (1 + r + r^2) = 26r \quad \text{--- (i) \quad xing by } r$$

Also we have given

$$\left(\frac{a_1}{r}\right) (a_1) (a_1 r) = 216$$

$$\Rightarrow a_1^3 = 216$$

$$\Rightarrow a_1^3 = (6)^3 \Rightarrow a_1 = 6$$

putting in eq (i)

$$6(1 + r + r^2) = 26r$$

$$\Rightarrow 6 + 6r + 6r^2 - 26r = 0$$

$$\Rightarrow 6r^2 - 20r + 6 = 0$$

$$\Rightarrow 2(3r^2 - 10r + 3) = 0$$

$$\Rightarrow 3r^2 - 10r + 3 = 0$$

$$r = \frac{10 \pm \sqrt{100 - 4(3)(3)}}{2(3)}$$

$$= \frac{10 \pm \sqrt{64}}{6} = \frac{10 \pm 8}{6}$$

$$r = \frac{10+8}{6} \text{ or } r = \frac{10-8}{6}$$

$$= \frac{18}{6} = 3, \quad = \frac{2}{6} = \frac{1}{3}$$

When $a_1 = 6, r = 3$

$$\frac{a_1}{r} = \frac{6}{3} = 2$$

$$a_1 = 6$$

$$a_1 r = (6)(3) = 18$$

When $a_1 = 6, r = \frac{1}{3}$

$$\frac{a_1}{r} = \frac{6}{\frac{1}{3}} = 6 \times 3 = 18$$

$$a_1 = 6$$

$$a_1 r = 6 \times \frac{1}{3} = 2$$

Hence 2, 6, 18 OR 18, 6, 2 are required number in G.P.

Q.No.11 Let the four terms in G.P. are $a_1, a_1 r, a_1 r^2, a_1 r^3$

By given condition,

$$a_1 + a_1 r + a_1 r^2 + a_1 r^3 = 80$$

$$\Rightarrow a_1(1+r+r^2+r^3) = 80$$

$$\Rightarrow a_1[1(1+r)+r^2(1+r)] = 80$$

$$\Rightarrow a_1(1+r)(1+r^2) = 80 \quad \text{--- (i)}$$

Also we have given

$$\frac{a_1 r + a_1 r^3}{2} = 30$$

$$\Rightarrow \frac{a_1 r(1+r^2)}{2} = 30$$

$$\Rightarrow a_1 r(1+r^2) = 60 \quad \text{--- (ii)}$$

From eq (i)

$$1+r^2 = \frac{80}{a_1(1+r)} \quad \text{putting in (ii)}$$

$$a_1 r \cdot \frac{80}{a_1(1+r)} = 60$$

$$\Rightarrow \frac{80r}{1+r} = 60 \Rightarrow 80r = 60(1+r)$$

$$\Rightarrow 80r = 60 + 60r$$

$$\Rightarrow 80r - 60r = 60 \Rightarrow 20r = 60$$

$$\Rightarrow r = \frac{60}{20} \Rightarrow \boxed{r=3}$$

putting value of r in eq (i)

$$a_1(1+3)(1+(3)^2) = 80$$

$$\Rightarrow a_1(4)(10) = 80 \Rightarrow 40a_1 = 80$$

$$\Rightarrow a_1 = \frac{80}{40} \Rightarrow \boxed{a_1=2}$$

So

$$a_1 r = (2)(3) = 6$$

$$a_1 r^2 = (2)(3)^2 = 2 \times 9 = 18$$

$$a_1 r^3 = (2)(3)^3 = 2 \times 27 = 54$$

hence 2, 6, 18, 54 are required number

Q.No.12 Since $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in G.P. therefore $r = \frac{\frac{1}{b}}{\frac{1}{a}} = \frac{a}{b}$ --- (i)

$$\text{Also } r = \frac{\frac{1}{c}}{\frac{1}{b}} = \frac{b}{c} \quad \text{--- (ii)}$$

xing (i) & (ii)

$$r \cdot r = \frac{a}{b} \cdot \frac{b}{c} \Rightarrow r^2 = \frac{a}{c}$$

$$\Rightarrow r = \pm \sqrt{\frac{a}{c}} \quad \text{proved}$$

Q.No.13 Let $a-d, a, a+d$ are three numbers in A.P.

By given condition

$$a-d + a + a+d = 21$$

$$\Rightarrow 3a = 21 \Rightarrow \boxed{a=7}$$

Now $a-d-1, a-4, a+d-3$ are in G.P. therefore

$$r = \frac{a-4}{a-d-1} = \frac{a+d-3}{a-4}$$

$$\Rightarrow (a-4)^2 = (a+d-3)(a-d-1)$$

put $a=7$

$$(7-4)^2 = (7+d-3)(7-d-1)$$

$$\Rightarrow (3)^2 = (d+4)(6-d)$$

$$\Rightarrow 9 = 6d + 24 - d^2 - 4d$$

$$\Rightarrow 9 - 6d - 24 + d^2 + 4d = 0$$

$$\Rightarrow d^2 - 2d - 15 = 0$$

$$\Rightarrow d^2 - 5d + 3d - 15 = 0$$

$$\Rightarrow d(d-5) + 3(d-5) = 0$$

$$\Rightarrow (d-5)(d+3) = 0$$

$$d-5=0, \quad d+3=0$$

$$d=5, \quad d=-3$$

When $a=7, d=5$; When $a=7, d=3$

$$a-d = 7-5 = 2$$

$$a-d = 7+3 = 10$$

$$a_1 = 7$$

$$a_1 = 7$$

$$a+d = 7+5 = 12$$

$$a+d = 7+3 = 10$$

hence 2, 7, 12 or 10, 7, 4 are required number

Q.No.14 Hint, Consider number

$a-d, a, a+d$, then

$a-d+1, a+1, a+d+1$ are in G.P.

END

***** The End *****