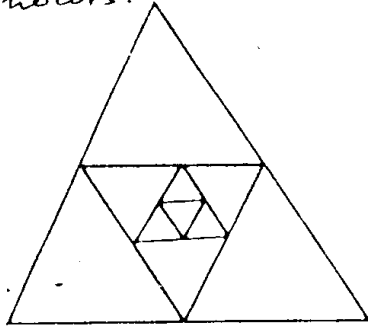


Here, $a_1 = 4A$, $r = \frac{16A}{4A} = 4$, $n = n$
 $\therefore a_n = a_1 r^{n-1}$
 $\rightarrow a_n = 4A(4)^{n-1}$
 $a_n = 4 \cdot 4^{n-1} \cdot A \rightarrow a_n = 4^n A$
 or $a_n = (2^2)^n A \rightarrow a_n = 2^{2n} A$
 which is required no. of bacteria in n hours.

Q.6



Perimeter of 1st equilateral triangle = $\frac{3}{2}$
 Perimeter of 2nd equilateral triangle = $\frac{1}{2}(\frac{3}{2}) = \frac{3}{4}$
 Perimeter of 3rd equilateral triangle = $\frac{1}{2}(\frac{3}{4}) = \frac{3}{8}$
 The sequence of Perimeters of the nested equilateral triangles is $\frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \dots$

To find its sum we have

$\frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \dots$
 Here, $a_1 = \frac{3}{2}$, $r = \frac{3/4}{3/2} = \frac{1}{2} < 1$

$\therefore S_\infty = \frac{a_1}{1-r}$
 $S_\infty = \frac{3/2}{1-1/2} = \frac{3/2}{1/2} = \frac{3}{2} \times \frac{2}{1} = 3$

$S_\infty = 3$. Hence total perimeter of all the triangles = 3.

HARMONIC SEQUENCE (H.P)

A sequence of numbers whose reciprocals form an A.P is called Harmonic sequence or Harmonic Progression (H.P).

For example,

$\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \dots$ is in H.P.
 where $3, 6, 9, \dots$ is in A.P.
 Generally, we represent A.P as $a_1, a_1+d, a_1+2d, \dots$

Similarly we represent H.P as $\frac{1}{a_1}, \frac{1}{a_1+d}, \frac{1}{a_1+2d}, \dots$

Example.1

Given that

$\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \dots$ is an H.P

$a_n = ?$ $a_8 = ?$

Since $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \dots$ is in H.P

So, $2, 5, 8, \dots$ is in A.P.

Here $a_1 = 2$, $d = 5 - 2 = 3$

$\therefore a_n = a_1 + (n-1)d$

$a_n = 2 + (n-1)3 = 2 + 3n - 3$

$a_n = 3n - 1$

Put $n = 8 \rightarrow a_8 = 3(8) - 1 = 23$

Hence in H.P

$a_n = \frac{1}{3n-1}$ and $a_8 = \frac{1}{23}$.

Example.2 Given that

$a_4 = \frac{2}{13}$, $a_7 = \frac{2}{25}$

Now in A.P.

$a_4 = \frac{13}{2}$, $a_7 = \frac{25}{2}$

$\therefore a_4 = a_1 + 3d \rightarrow a_1 + 3d = \frac{13}{2} \rightarrow \textcircled{1}$

$\therefore a_7 = a_1 + 6d \rightarrow a_1 + 6d = \frac{25}{2} \rightarrow \textcircled{2}$

Subtracting eq ① from eq ② we get

$3d = \frac{25}{2} - \frac{13}{2}$

$3d = \frac{12}{2} \rightarrow 3d = 6$ or $d = 2$

Put $d = 2$ in ①.

$a_1 + 3(2) = \frac{13}{2}$

$a_1 = \frac{13}{2} - 6 = \frac{13-12}{2} = \frac{1}{2}$

As $a_1 = \frac{1}{2}$, $d = 2$ Then A.P is

$a_1, a_1+d, a_1+2d, \dots$

$\frac{1}{2}, (\frac{1}{2}+2), (\frac{1}{2}+2(2)), \dots$

$\frac{1}{2}, \frac{5}{2}, \frac{9}{2}, \dots$

Hence required H.P will be

$\frac{2}{7}, \frac{2}{5}, \frac{2}{9}, \dots$ Ans.

HARMONIC MEAN(H.M)

A number H is said to be the harmonic mean (H.M) between two numbers a and b if

a, H, b are in H.P

→ $\frac{1}{a}, \frac{1}{H}, \frac{1}{b}$ are in A.P.

In this case,

$d = \frac{1}{H} - \frac{1}{a}$ also $d = \frac{1}{b} - \frac{1}{H}$

→ $\frac{1}{H} - \frac{1}{a} = \frac{1}{b} - \frac{1}{H}$

→ $\frac{1}{H} + \frac{1}{H} = \frac{1}{a} + \frac{1}{b}$

$\frac{2}{H} = \frac{b+a}{ab}$

$\frac{2}{H} = \frac{a+b}{ab}$

→ $(a+b)H = 2ab$

→ $H = \frac{2ab}{a+b}$

Example.3

Let H_1, H_2, H_3 be H.M's between $\frac{1}{5}$ and $\frac{1}{17}$, Then

$\frac{1}{5}, H_1, H_2, H_3, \frac{1}{17}$ are in H.P.

→ $5, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, 17$ are in A.P

$a_1 = 5, a_n = 17, n = 5$

$a_n = a_1 + (n-1)d$

$17 = 5 + (5-1)d$

$17-5 = 4d \Rightarrow 4d = 12 \Rightarrow d = 3$

$\frac{1}{H_1} = a_1 + d = 5 + 3 = 8$

→ $H_1 = \frac{1}{8}$

$\frac{1}{H_2} = \frac{1}{H_1} + d = 8 + 3 = 11$

→ $H_2 = \frac{1}{11}$

$\frac{1}{H_3} = \frac{1}{H_2} + d = 11 + 3 = 14$

→ $H_3 = \frac{1}{14}$ Hence required H.M's are $\frac{1}{8}, \frac{1}{11}, \frac{1}{14}$

Example.4 Let,

$H_1, H_2, H_3, \dots, H_n$ be n H.M's between a and b. Then

$a, H_1, H_2, H_3, \dots, H_n, b$ are in H.P

→ $\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \dots, \frac{1}{H_n}, \frac{1}{b}$ are in A.P.

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∴ $a_n = a_1 + (n-1)d$

→ $\frac{1}{b} = \frac{1}{a} + (n-1)d$

$\frac{1}{b} - \frac{1}{a} = (n-1)d$

$\frac{a-b}{ab} = (n-1)d$

$d = \frac{a-b}{ab(n-1)}$

$\frac{1}{H_1} = \frac{1}{a} + d = \frac{1}{a} + \frac{a-b}{ab(n-1)}$

$= \frac{b(n-1)+a-b}{ab(n-1)} = \frac{bn+b+a-b}{ab(n-1)}$

$= \frac{bn+a}{ab(n-1)}$

→ $H_1 = \frac{ab(n-1)}{bn+a}$

$\frac{1}{H_2} = \frac{1}{H_1} + d$

$= \frac{bn+a}{ab(n-1)} + \frac{a-b}{ab(n-1)}$

$= \frac{bn+a+a-b}{ab(n-1)}$

$= \frac{b(n-1)+2a}{ab(n-1)}$

→ $H_2 = \frac{ab(n-1)}{(n-1)b+2a}$

Similarly,

$H_3 = \frac{ab(n-1)}{(n-2)b+3a}$

$H_n = \frac{ab(n-1)}{[n-(n-1)]b+na}$

$H_n = \frac{ab(n-1)}{(n-n+1)b+na}$

$= \frac{ab(n-1)}{b+na}$

Hence n H.M's between a and b are

$\frac{ab(n-1)}{nb+a}, \frac{ab(n-1)}{(n-1)b+2a}, \frac{ab(n-1)}{(n-2)b+3a}, \dots, \frac{ab(n-1)}{b+na}$

RELATION BETWEEN A.M.G.M AND H.M

Q. Prove that A, G, H are in G.P

Proof:- We know that for any two numbers a and b.

$A = \frac{a+b}{2}, G = \sqrt{ab}, H = \frac{2ab}{a+b}$

Now, $AH = \left(\frac{a+b}{2}\right)\left(\frac{2ab}{a+b}\right) = ab$

also, $G^2 = (\pm\sqrt{ab})^2 = ab$

$\rightarrow G^2 = AH$

$\rightarrow G \cdot G = AH$

$\rightarrow \frac{G}{A} = \frac{H}{G}$

$\rightarrow A, G, H$ are in G.P

Q. Prove that $A > G > H$ if a, b are any two distinct real numbers and $G = \sqrt{ab}$

Proof: We know that,

$A = \frac{a+b}{2}, G = \pm\sqrt{ab}, H = \frac{2ab}{a+b}$

Let $A > G$

$\rightarrow \frac{a+b}{2} > \pm\sqrt{ab}$

Squaring both sides.

$\left(\frac{a+b}{2}\right)^2 > (\pm\sqrt{ab})^2$

$\frac{(a+b)^2}{4} > ab$

$a^2 + b^2 + 2ab > 4ab$

$a^2 + b^2 + 2ab - 4ab > 0$

$a^2 + b^2 - 2ab > 0$

$\rightarrow (a-b)^2 > 0$

Which is always true.

So $A > G \rightarrow \textcircled{1}$

Again let $G > H$

$\rightarrow \pm\sqrt{ab} > \frac{2ab}{a+b}$

Squaring both sides.

$ab > \frac{4a^2b^2}{(a+b)^2}$

$(a+b)^2 > \frac{4a^2b^2}{ab}$

$a^2 + b^2 + 2ab > 4ab$

$a^2 + b^2 + 2ab - 4ab > 0$

$a^2 + b^2 - 2ab > 0$

$\rightarrow (a-b)^2 > 0$

Which is always true.

So, $G > H \rightarrow \textcircled{2}$

From $\textcircled{1}$ & $\textcircled{2}$

$A > G > H$

Q. Prove that $A < G < H$ if a, b are any two distinct negative real numbers and $G = -\sqrt{ab}$

Proof: Suppose $a = -p, b = -q$, $p, q \in \mathbb{R}$

Let $A < G$

$\therefore A = \frac{a+b}{2} \rightarrow A = \frac{-p+(-q)}{2}$

$G = -\sqrt{ab}$ (given)

$= -\sqrt{(-p)(-q)} = -\sqrt{pq}$

So, $\frac{-p-q}{2} < -\sqrt{pq}$

$\rightarrow -\left(\frac{p+q}{2}\right) < -\sqrt{pq}$

$\rightarrow \frac{p+q}{2} > \sqrt{pq}$

As $2 > -1$ But $2 > 1$

$\rightarrow p+q > 2\sqrt{pq}$

Squaring both sides

$(p+q)^2 > 4pq$

$p^2 + q^2 + 2pq - 4pq > 0$

$\rightarrow (p-q)^2 > 0$

Which is always true.

Hence $A < G \rightarrow \textcircled{1}$

Again let $G < H$

$\therefore G = -\sqrt{ab} = -\sqrt{(-p)(-q)} = -\sqrt{pq}$

$H = \frac{2ab}{a+b} = \frac{2(-p)(-q)}{(-p)+(-q)} = \frac{2pq}{-p-q}$

So, $-\sqrt{pq} < \frac{2pq}{-p-q}$

$-\sqrt{pq} < -\frac{2pq}{p+q}$

$\rightarrow \sqrt{pq} > \frac{2pq}{p+q}$

Squaring both sides

$pq > \frac{(2pq)^2}{(p+q)^2}$

$\rightarrow (p+q)^2 > \frac{4p^2q^2}{pq}$

$\rightarrow p^2 + q^2 + 2pq > 4pq$

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$$p^2 + q^2 + 2pq - 4pq > 0$$

$$p^2 + q^2 - 2pq > 0$$

$$\text{or } (p-q)^2 > 0$$

which is always true

$$G < H \rightarrow \textcircled{2}$$

So from $\textcircled{1}$ and $\textcircled{2}$

$$A < G < H.$$

EXERCISE 6.10

Q.1 Given that $a_9 = ?$

(i) When $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \dots$ are in H.P

$\rightarrow 3, 5, 7, \dots$ are in A.P

Here $a_1 = 3, d = 5 - 3 = 2$

$$\because a_n = a_1 + (n-1)d$$

$$\rightarrow a_9 = a_1 + (9-1)d$$

$$a_9 = a_1 + 8d$$

So, $a_9 = 3 + 8(2) = 19$

Hence in H.P $a_9 = \frac{1}{19}$

(ii) Given that $a_9 = ?$

When $-\frac{1}{5}, -\frac{1}{3}, -1, \dots$ are in H.P

$\rightarrow -5, -3, -1, \dots$ are in A.P

Here $a_1 = -5, d = -3 - (-5) = -3 + 5 = 2$

$$\because a_n = a_1 + (n-1)d$$

$$\rightarrow a_9 = a_1 + (9-1)d$$

$$a_9 = a_1 + 8d$$

So, $a_9 = -5 + 8(2) = 11$

Hence in H.P

$$a_9 = \frac{1}{11}$$

Q.2 (i) Given that $a_{12} = ?$

When $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \dots$ are in H.P

$\rightarrow 2, 5, 8, \dots$ are in A.P

Here $a_1 = 2, d = 5 - 2 = 3$

$$\because a_n = a_1 + (n-1)d$$

$$\rightarrow a_{12} = a_1 + (12-1)d$$

$$a_{12} = a_1 + 11d$$

So, $a_{12} = 2 + 11(3) = 35$

Hence in H.P $a_{12} = \frac{1}{35}$

(ii) Given that $a_{12} = ?$

When, $\frac{1}{3}, \frac{2}{9}, \frac{1}{6}, \dots$ are in H.P

$\rightarrow 3, \frac{9}{2}, 6, \dots$ are in A.P

Here $a_1 = 3, d = \frac{9}{2} - 3 = \frac{9-6}{2} = \frac{3}{2}$

$$\because a_n = a_1 + (n-1)d$$

$$a_{12} = a_1 + (12-1)d$$

$$a_{12} = a_1 + 11d$$

So, $a_{12} = 3 + 11\left(\frac{3}{2}\right)$

$$a_{12} = 3 + \frac{33}{2} = \frac{6+33}{2}$$

$$a_{12} = \frac{39}{2}$$

Hence in H.P $a_{12} = \frac{2}{39}$

Q.3 (i) Let H_1, H_2, H_3, H_4 and H_5 be five H.M's between $-\frac{2}{5}$ and $\frac{2}{13}$

Then, $-\frac{2}{5}, H_1, H_2, H_3, H_4, H_5, \frac{2}{13}$ are in H.P

$\rightarrow -\frac{5}{2}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \frac{1}{H_5}, \frac{13}{2}$ are in A.P

Here $a_1 = -\frac{5}{2}, a_n = \frac{13}{2}, n = 7$

$$\because a_n = a_1 + (n-1)d$$

$$\rightarrow \frac{13}{2} = -\frac{5}{2} + (7-1)d$$

$$\frac{13}{2} + \frac{5}{2} = 6d$$

$$\frac{18}{2} = 6d \Rightarrow 6d = 9$$

$$\Rightarrow d = \frac{9}{6} = \frac{3}{2}$$

Now, $\frac{1}{H_1} = a_1 + d = -\frac{5}{2} + \frac{3}{2} = -\frac{2}{2} = -1$

$$\rightarrow H_1 = -1$$

$$\frac{1}{H_2} = \frac{1}{H_1} + d = -1 + \frac{3}{2} = \frac{-2+3}{2} = \frac{1}{2}$$

$$\rightarrow H_2 = 2$$

$$\frac{1}{H_3} = \frac{1}{H_2} + d = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$$

$$\rightarrow H_3 = \frac{1}{2}$$

$$\frac{1}{H_4} = \frac{1}{H_3} + d = 2 + \frac{3}{2} = \frac{4+3}{2} = \frac{7}{2}$$

$$\rightarrow H_4 = \frac{2}{7}$$

$$\frac{1}{H_5} = \frac{1}{H_4} + d = \frac{7}{2} + \frac{3}{2} = \frac{10}{2} = 5$$

$$\rightarrow H_5 = \frac{1}{5}$$

Hence required 5 H.M's are $-1, 2, \frac{1}{2}, \frac{2}{7}, \frac{1}{5}$

(ii) Let H_1, H_2, H_3, H_4 and H_5 be five H.M's between $\frac{1}{4}$ and $\frac{1}{24}$

Then, $\frac{1}{4}, H_1, H_2, H_3, H_4, H_5, \frac{1}{24}$ are in H.P
 $\rightarrow 4, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \frac{1}{H_5}, 24$ are in A.P

Here $a_1 = 4, a_n = 24, n = 7$

$$\because a_n = a_1 + (n-1)d$$

$$\rightarrow 24 = 4 + (7-1)d$$

$$24 - 4 = 6d \rightarrow 6d = 20$$

$$\rightarrow d = \frac{20}{6} = \frac{10}{3}$$

Now $\frac{1}{H_1} = a_1 + d = 4 + \frac{10}{3} = \frac{12+10}{3}$

$$\frac{1}{H_1} = \frac{22}{3} \rightarrow H_1 = \frac{3}{22}$$

$$\frac{1}{H_2} = \frac{1}{H_1} + d = \frac{22}{3} + \frac{10}{3} = \frac{32}{3}$$

$$\rightarrow H_2 = \frac{3}{32}$$

$$\frac{1}{H_3} = \frac{1}{H_2} + d = \frac{32}{3} + \frac{10}{3}$$

$$\frac{1}{H_3} = \frac{42}{3} = 14 \rightarrow H_3 = \frac{1}{14}$$

$$\frac{1}{H_4} = \frac{1}{H_3} + d = 14 + \frac{10}{3} = \frac{42+10}{3} = \frac{52}{3}$$

$$\rightarrow H_4 = \frac{3}{52}$$

$$\frac{1}{H_5} = \frac{1}{H_4} + d = \frac{52}{3} + \frac{10}{3} = \frac{52+10}{3}$$

$$\frac{1}{H_5} = \frac{62}{3} \rightarrow H_5 = \frac{3}{62}$$

Hence required five H.M's are

$$\frac{3}{22}, \frac{3}{32}, \frac{1}{14}, \frac{3}{52}, \frac{3}{62}$$

Q.4 (i) Let H_1, H_2, H_3, H_4 be four H.M's between $\frac{1}{3}$ and $\frac{1}{23}$

Then, $\frac{1}{3}, H_1, H_2, H_3, H_4, \frac{1}{23}$ are in H.P

$\rightarrow 3, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, 23$ are in A.P

Here, $a_1 = 3, a_n = 23, n = 6$

$$\because a_n = a_1 + (n-1)d$$

$$\rightarrow 23 = 3 + (6-1)d$$

$$23 - 3 = 5d \rightarrow 5d = 20$$

$$d = 4$$

Now, $\frac{1}{H_1} = a_1 + d = 3 + 4 = 7$

$$\rightarrow H_1 = \frac{1}{7}$$

$$\frac{1}{H_2} = \frac{1}{H_1} + d = 7 + 4 = 11$$

$$\frac{1}{H_2} = 11 \rightarrow H_2 = \frac{1}{11}$$

$$\frac{1}{H_3} = \frac{1}{H_2} + d = 11 + 4 = 15$$

$$\rightarrow H_3 = \frac{1}{15}$$

$$\frac{1}{H_4} = \frac{1}{H_3} + d = 15 + 4 = 19$$

$$\rightarrow H_4 = \frac{1}{19}$$

Hence four H.M's between $\frac{1}{3}$ and

$$\frac{1}{23} \text{ are } \frac{1}{7}, \frac{1}{11}, \frac{1}{15}, \frac{1}{19}$$

(ii) Let H_1, H_2, H_3, H_4 be four H.M's between $\frac{7}{3}$ and $\frac{7}{11}$

Then $\frac{7}{3}, H_1, H_2, H_3, H_4, \frac{7}{11}$ are in H.P

$\rightarrow \frac{3}{7}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \frac{11}{7}$ are in A.P

Here $a_1 = \frac{3}{7}, a_n = \frac{11}{7}, n = 6$

$$\because a_n = a_1 + (n-1)d$$

$$\frac{11}{7} = \frac{3}{7} + (6-1)d$$

$$\frac{11}{7} - \frac{3}{7} = 5d \rightarrow 5d = \frac{11-3}{7} = \frac{8}{7}$$

$$\rightarrow d = \frac{8}{35}$$

Now, $\frac{1}{H_1} = a_1 + d = \frac{3}{7} + \frac{8}{35} = \frac{3(5)+8}{35}$

$$\frac{1}{H_1} = \frac{15+8}{35} = \frac{23}{35} \rightarrow H_1 = \frac{35}{23}$$

$$\frac{1}{H_2} = \frac{1}{H_1} + d = \frac{23}{35} + \frac{8}{35} = \frac{31}{35} \rightarrow H_2 = \frac{35}{31}$$

$$\frac{1}{H_3} = \frac{1}{H_2} + d = \frac{31}{35} + \frac{8}{35} = \frac{39}{35} \rightarrow H_3 = \frac{35}{39}$$

$$\frac{1}{H_4} = \frac{1}{H_3} + d = \frac{39}{35} + \frac{8}{35} = \frac{47}{35} \rightarrow H_4 = \frac{35}{47}$$

Hence four H.M's between $\frac{7}{3}$ and

$$\frac{7}{11} \text{ are } \frac{35}{23}, \frac{35}{31}, \frac{35}{39}, \frac{35}{47}$$

(iii) Let H_1, H_2, H_3, H_4 be four H.M's between 4 and 20. Then

$4, H_1, H_2, H_3, H_4, 20$ are in H.P.

$\rightarrow \frac{1}{4}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \frac{1}{H_4}, \frac{1}{20}$ are in A.P.

Here $a_1 = \frac{1}{4}, a_n = \frac{1}{20}, n = 6$

$$\because a_n = a_1 + (n-1)d$$

$$\rightarrow \frac{1}{20} = \frac{1}{4} + (6-1)d$$

$$5d = \frac{1}{20} - \frac{1}{4} = \frac{1-5}{20} = \frac{-4}{20} = -\frac{1}{5}$$

$$\rightarrow d = -\frac{1}{25}$$

Now, $\frac{1}{H_1} = a_1 + d = \frac{1}{4} + (-\frac{1}{25}) = \frac{25-4}{100}$

$\frac{1}{H_1} = \frac{21}{100} \Rightarrow H_1 = \frac{100}{21}$

$\frac{1}{H_2} = \frac{1}{H_1} + d = \frac{21}{100} + (-\frac{1}{25}) = \frac{21-4}{100}$

$\frac{1}{H_2} = \frac{17}{100} \Rightarrow H_2 = \frac{100}{17}$

$\frac{1}{H_3} = \frac{1}{H_2} + d = \frac{17}{100} + (-\frac{1}{25}) = \frac{17-4}{100}$

$\frac{1}{H_3} = \frac{13}{100} \Rightarrow H_3 = \frac{100}{13}$

$\frac{1}{H_4} = \frac{1}{H_3} + d = \frac{13}{100} + (-\frac{1}{25}) = \frac{13-4}{100}$

$\frac{1}{H_4} = \frac{9}{100} \Rightarrow H_4 = \frac{100}{9}$

Hence four H.M's between 4 & 20

are $\frac{100}{21}, \frac{100}{17}, \frac{100}{13}, \frac{100}{9}$

Q.5 Given that $a_{14} = ?$

When in H.P

$a_7 = \frac{1}{3}, a_{10} = \frac{5}{21}$

\Rightarrow In A.P $a_7 = 3, a_{10} = \frac{21}{5}$

$\therefore a_7 = a_1 + 6d \Rightarrow a_1 + 6d = 3 \rightarrow \textcircled{1}$

Also, $a_{10} = a_1 + 9d \Rightarrow a_1 + 9d = \frac{21}{5} \rightarrow \textcircled{2}$

Subtracting eq ① from eq ②

$3d = \frac{21}{5} - 3 = \frac{21-15}{5} = \frac{6}{5}$

$\Rightarrow d = \frac{6}{5} \times \frac{1}{3} = \frac{2}{5}$

Put $d = \frac{2}{5}$ in eq ①

$a_1 + 6(\frac{2}{5}) = 3$

$a_1 = 3 - \frac{12}{5} = \frac{15-12}{5}$

$\Rightarrow a_1 = \frac{3}{5}$

$a_{14} = a_1 + 13d = \frac{3}{5} + 13(\frac{2}{5}) = \frac{3}{5} + \frac{26}{5} = \frac{29}{5}$

Hence in H.P $a_{14} = \frac{5}{29}$

Q.6 Given that $a_9 = ?$

When in H.P

\Rightarrow In A.P $a_1 = -3, a_5 = 5$

$\therefore a_5 = a_1 + 4d \Rightarrow a_1 + 4d = 5 \rightarrow \textcircled{1}$

Put $a_1 = -3$ in eq ①

$-3 + 4d = 5 \Rightarrow 4d = 5 + 3 = 8 \Rightarrow d = 2$

as $a_9 = a_1 + 8d$

$\Rightarrow a_9 = -3 + 8(2) = -3 + 16 = 13$

Hence in H.P

$a_9 = \frac{1}{13}$

Q.7

Here

$a = 2, b = b, H.M = 5$

$\therefore H.M = \frac{2ab}{a+b}$

$\Rightarrow 5 = \frac{(2)(2)b}{2+b}$

$5(2+b) = 4b$

$10 + 5b = 4b$

$5b - 4b = -10 \Rightarrow b = -10$

Q.8

Given that

$\frac{1}{k}, \frac{1}{2k+1}, \frac{1}{4k-1}$ are in H.P

$\Rightarrow k, 2k+1, 4k-1$ are in A.P

Now, $d = 2k+1 - k$ also $d = 4k-1 - (2k+1)$

$d = k+1$ also $d = 4k-1 - 2k-1$

$d = 2k-2$

$\Rightarrow k+1 = 2k-2$

$1+2 = 2k-k$

$\Rightarrow k = 3$

Q.9

We know that if H is H.M between two numbers a & b.

Then, $H = \frac{2ab}{a+b}$

So for given condition

$\frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \frac{2ab}{a+b}$

$(a+b)(a^{n+1} + b^{n+1}) = 2ab(a^n + b^n)$

$\checkmark a \cdot a^{n+1} + a b^{n+1} + a^{n+1} \cdot b + b \cdot b^{n+1} = 2a^{n+1} + 2ab^{n+1}$

$a \cdot a^{n+1} + b a^{n+1} - 2a^{n+1} \cdot b = 2a b^{n+1} - a b^{n+1} - b \cdot b^{n+1}$

$a \cdot a^{n+1} - b a^{n+1} = a b^{n+1} - b \cdot b^{n+1}$

$a^{n+1}(a-b) = (a-b)b^{n+1}$

$\Rightarrow (a-b)(a^{n+1} - b^{n+1}) = 0$

Either $a-b=0$ or $a^{n+1} - b^{n+1} = 0$

But $a-b \neq 0$ so, $a^{n+1} = b^{n+1}$

because $a \neq b$

$\frac{a^{n+1}}{b^{n+1}} = \frac{b^{n+1}}{b^{n+1}}$

$\Rightarrow (\frac{a}{b})^{n+1} = 1 \Rightarrow (\frac{a}{b})^{n+1} = (\frac{a}{b})^0$

$\Rightarrow n+1 = 0$

$\Rightarrow \boxed{n = -1}$ Ans.

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Q.10 Given that
 a^2, b^2, c^2 are in A.P
 $\rightarrow b^2 - a^2 = c^2 - b^2$
 $(b-a)(b+a) = (c-b)(c+b)$
 $-(a-b)(a+b) = -(b-c)(b+c)$
 $\rightarrow (a-b)(a+b) = (b-c)(b+c)$
 $\rightarrow \frac{a-b}{b+c} = \frac{b-c}{a+b} \rightarrow \textcircled{1}$

We are to prove
 $a+b, c+a, b+c$ are in H.P
 or $\frac{1}{a+b}, \frac{1}{c+a}, \frac{1}{b+c}$ are in A.P

$\rightarrow \frac{1}{c+a} - \frac{1}{a+b} = \frac{1}{b+c} - \frac{1}{c+a}$
 $\frac{a+b-(c+a)}{(c+a)(a+b)} = \frac{(c+a)-(b+c)}{(b+c)(c+a)}$
 $\frac{a+b-c-a}{a+b} = \frac{c+a-b-c}{b+c}$
 $\rightarrow \frac{b-c}{a+b} = \frac{a-b}{b+c}$

Which is already proved
 Thus, $a+b, c+a, b+c$ are in H.P

Q.11 Given that in H.P
 $a_1 + a_5 = \frac{4}{7}, a_1 = \frac{1}{2}$
 Put $a_1 = \frac{1}{2}$
 $\frac{1}{2} + a_5 = \frac{4}{7} \rightarrow a_5 = \frac{4}{7} - \frac{1}{2}$
 $a_5 = \frac{8-7}{14} \rightarrow a_5 = \frac{1}{14}$
 So in H.P
 $a_1 = \frac{1}{2}, a_5 = \frac{1}{14}$
 \rightarrow In A.P $a_1 = 2, a_5 = 14$
 $\because a_5 = a_1 + 4d$
 $\rightarrow a_1 + 4d = 14 \rightarrow 4d = 14 - a_1$
 $\rightarrow 4d = 14 - 2 = 12 \rightarrow d = \frac{12}{4} = 3$
 Now A.P is
 $a_1, a_1 + d, a_1 + 2d, a_1 + 3d, \dots$
 $2, (2+3), (2+2(3)), \dots$
 $2, 5, 8, 11, \dots$
 Now, $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \dots$ is required H.P

Q.12 We know that
 $A = \frac{a+b}{2}, G = \pm\sqrt{ab}, H = \frac{2ab}{a+b}$
 Now $G^2 = ab \rightarrow \textcircled{1}$
 $AH = \left(\frac{a+b}{2}\right)\left(\frac{2ab}{a+b}\right) = ab$
 or $AH = ab \rightarrow \textcircled{2}$
 From eq $\textcircled{1}$ & eq $\textcircled{2}$.
 $G^2 = AH$.

Q.13 (i) Given that
 $a = -2, b = -6$
 $\because A = \frac{a+b}{2} \rightarrow A = \frac{-2+(-6)}{2} = \frac{-8}{2} = -4$
 $\because G = \pm\sqrt{ab} \rightarrow G = \pm\sqrt{(-2)(-6)} = \pm\sqrt{12}$
 $\because H = \frac{2ab}{a+b} \rightarrow H = \frac{2(-2)(-6)}{-2+(-6)} = \frac{24}{-8} = -3$
 Now, $G^2 = (\pm\sqrt{12})^2 = 12$
 and $AH = (-4)(-3) = 12$
 $\therefore G^2 = AH$.

(ii) Given that
 $a = 2i, b = 4i$
 $\because A = \frac{a+b}{2} \rightarrow A = \frac{2i+4i}{2} = \frac{6i}{2} = 3i$
 $\because G = \pm\sqrt{ab} \rightarrow G = \pm\sqrt{(2i)(4i)} = \pm\sqrt{8i^2}$
 $= \pm\sqrt{8(-1)} = \pm\sqrt{-8}$
 $\because H = \frac{2ab}{a+b} \rightarrow H = \frac{2(2i)(4i)}{2i+4i} = \frac{16i^2}{6i} = \frac{8i}{3}$
 Now, $G^2 = (\pm\sqrt{-8})^2 = -8$
 $AH = (3i)\left(\frac{8i}{3}\right) = 8i^2 = -8$
 Hence, $G^2 = AH$.

(iii) Given that $G > 0$
 $a = 9, b = 4$
 $\because A = \frac{a+b}{2} \rightarrow A = \frac{9+4}{2} = \frac{13}{2}$
 $\because G = \sqrt{ab} \rightarrow G = \sqrt{(9)(4)} = \sqrt{36} = 6$
 $\because H = \frac{2ab}{a+b} = \frac{2(9)(4)}{9+4} = \frac{72}{13}$
 Now, $G^2 = (6)^2 = 36$
 $AH = \left(\frac{13}{2}\right)\left(\frac{72}{13}\right) = \frac{72}{2}$
 $AH = 36$
 $\therefore G^2 = AH$

Q.14 Given that $G > 0$

(i) $a=2, b=8$
 $\because A = \frac{a+b}{2} = \frac{2+8}{2} = \frac{10}{2} = 5$
 $\because G = \sqrt{ab} = \sqrt{2 \times 8} = \sqrt{16} = 4$
 $\because H = \frac{2ab}{a+b} = \frac{2(2)(8)}{2+8} = \frac{32}{10} = \frac{16}{5} = 3.2$
 $\rightarrow 5 > 4 > 3.2$
 $\rightarrow A > G > H$

(ii) Given that $G > 0$

$a = \frac{2}{5}, b = \frac{8}{5}$
 $\because A = \frac{a+b}{2} = \frac{1}{2}(\frac{2}{5} + \frac{8}{5}) = \frac{10}{10} = 1$
 $\because G = \sqrt{ab} = \sqrt{\frac{2}{5} \times \frac{8}{5}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$
 $\because H = \frac{2ab}{a+b} = \frac{2(\frac{2}{5})(\frac{8}{5})}{\frac{2}{5} + \frac{8}{5}} = \frac{32/25}{(2+8)/5} = \frac{32/25}{10/5} = \frac{32}{25} \times \frac{5}{10} = \frac{32}{50}$
 $\rightarrow 1 > \frac{4}{5} > \frac{32}{50}$
 $\rightarrow A > G > H$

Q.15 Given that $G < 0$

(i) $a=-2, b=-8$
 $\because A = \frac{a+b}{2} = \frac{(-2)+(-8)}{2} = \frac{-10}{2} = -5$
 $\because G = -\sqrt{ab} = -\sqrt{(-2)(-8)} = -\sqrt{16} = -4$
 $\because H = \frac{2ab}{a+b} = \frac{2(-2)(-8)}{-2+(-8)} = \frac{32}{-10} = -3.2$
 $\rightarrow -5 < -4 < -3.2$
 $\rightarrow A < G < H$

(ii) Given that $G < 0$

$a = -\frac{2}{5}, b = -\frac{8}{5}$
 $\because A = \frac{a+b}{2} \rightarrow A = \frac{1}{2}(-\frac{2}{5} - \frac{8}{5})$
 $A = \frac{1}{2}(-\frac{2-8}{5}) = \frac{1}{2}(-\frac{10}{5}) = -1$
 $G = -\sqrt{ab} = -\sqrt{(-\frac{2}{5})(-\frac{8}{5})} = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$
 $H = \frac{2ab}{a+b} = \frac{2(-\frac{2}{5})(-\frac{8}{5})}{-\frac{2}{5} + (-\frac{8}{5})} = \frac{32/25}{-10/5} = \frac{32}{-50} = -\frac{16}{25}$
 $\rightarrow -1 < -\frac{4}{5} < -\frac{16}{25}$
 $\rightarrow A < G < H$

Q.16 let a and b be numbers
 Given that

$H.M = 4, A.M = \frac{9}{2}$
 $\because H.M = \frac{2ab}{a+b} \rightarrow \frac{2ab}{a+b} = 4 \rightarrow \textcircled{1}$
 also, $A.M = \frac{a+b}{2} \rightarrow \frac{a+b}{2} = \frac{9}{2} \rightarrow \textcircled{2}$

From eq ② $a+b=9 \rightarrow \textcircled{3}$

Putting value of $a+b$ in ①

$\frac{2ab}{9} = 4 \rightarrow 2ab = 36$

$\rightarrow ab = 18$

$\because (a-b)^2 = (a+b)^2 - 4ab$

$\rightarrow (a-b)^2 = (9)^2 - 4(18) = 81 - 72 = 9$

$\rightarrow a-b = \pm 3$

$a-b = 3 \rightarrow \textcircled{4}$

$\therefore a-b = -3 \rightarrow \textcircled{5}$

Adding ③ & ④

$2a = 12$

$\rightarrow a = 6$

if $a=6$ then by ③

$6+b=9$

$b=9-6=3$

Adding ③ & ⑤

$2a = 6$

$\rightarrow a = 3$

if $a=3$ then by ③

$3+b=9$

$b=9-3=6$

Hence required numbers are

6, 3 or 3, 6

Q.17 let a and b be numbers

Given that

$G.M = 4, H.M = \frac{16}{5}$

$\because G.M \text{ (positive)} = \sqrt{ab} = 4$

$\rightarrow ab = 16 \rightarrow \textcircled{1}$

$\because H.M = \frac{2ab}{a+b} \rightarrow \frac{2ab}{a+b} = \frac{16}{5}$

$\rightarrow \frac{ab}{a+b} = \frac{8}{5} \rightarrow 5ab = 8(a+b)$

$\rightarrow a+b = \frac{5}{8}(ab) = \frac{5}{8}(16) = 10$

$a+b = 10 \rightarrow \textcircled{2}$

$\because (a-b)^2 = (a+b)^2 - 4ab$

$\rightarrow (a-b)^2 = (10)^2 - 4(16)$

$(a-b)^2 = 100 - 64 = 36$

$\rightarrow a-b = \pm 6$

$a-b = 6 \rightarrow \textcircled{3}, a-b = -6 \rightarrow \textcircled{4}$

Adding ② & ③

$$2a = 16 \Rightarrow a = 8$$

If $a = 8$ then by ②

$$8 + b = 10$$

$$\Rightarrow b = 10 - 8$$

$$\Rightarrow b = 2$$

Hence required numbers are
 8, 2 or 2, 8.

Adding ② & ④

$$2a = 4 \Rightarrow a = 2$$

If $a = 2$ then by ②

$$2 + b = 10$$

$$b = 10 - 2 = 8$$

$$\Rightarrow b = 8$$

$$25a^2 - 25a - 16a + 16 = 0$$

$$25a(a-1) - 16(a-1) = 0$$

$$(a-1)(25a-16) = 0$$

$$a-1 = 0$$

$$25a - 16 = 0$$

$$a = 1$$

$$25a = 16$$

$$\Rightarrow a = 16/25$$

If $a = 1$ then $r = \frac{1}{3(1)} = \frac{1}{3}$

If $a = \frac{16}{25}$ then $r = \frac{1}{3(16/25)} = \frac{25}{48}$

If $a = 1, r = \frac{1}{3}$, then numbers are

$$a, ar, ar^2$$

$$1, 1(\frac{1}{3}), 1(\frac{1}{3})^2$$

$$1, \frac{1}{3}, \frac{1}{9}$$

If $a = \frac{16}{25}, r = \frac{25}{48}$ then numbers

are a, ar, ar^2

$$\frac{16}{25}, \frac{16}{25}(\frac{25}{48}), \frac{16}{25}(\frac{25}{48})^2$$

$$\frac{16}{25}, \frac{1}{3}, \frac{25 \times 25 \times 16}{25 \times 48 \times 48}$$

$$\frac{16}{25}, \frac{1}{3}, \frac{25}{144}$$

Ans.

Q.18 let the three consecutive terms of A.P. be

$$a, ar, ar^2$$

According to given condition

$$a - \frac{1}{2}, ar - \frac{4}{21}, ar^2 - \frac{1}{36} \text{ are in H.P.}$$

$$\text{also } a(ar)(ar^2) = \frac{1}{27}$$

$$a^3 r^3 = (\frac{1}{3})^3$$

$$\Rightarrow ar = \frac{1}{3} \Rightarrow r = \frac{1}{3a}$$

Now putting $r = \frac{1}{3a}$ we have H.P. as

$$a - \frac{1}{2}, a(\frac{1}{3a}) - \frac{4}{21}, a(\frac{1}{3a})^2 - \frac{1}{36}$$

$$\frac{2a-1}{2}, \frac{1}{3} - \frac{4}{21}, \frac{1}{9a} - \frac{1}{36} \text{ are in H.P.}$$

$$\frac{2a-1}{2}, \frac{7-4}{21}, \frac{4-a}{36a} \text{ are in H.P.}$$

$$\frac{2a-1}{2}, \frac{3}{21}, \frac{4-a}{36a} \text{ are in H.P.}$$

$$\frac{2a-1}{2}, \frac{1}{7}, \frac{4-a}{36a} \text{ are in H.P.}$$

$$\Rightarrow \frac{2}{2a-1}, 7, \frac{36a}{4-a} \text{ are in A.P.}$$

$$\Rightarrow 7 - \frac{2}{2a-1} = \frac{36a}{4-a} - 7$$

$$\Rightarrow 7 + 7 = \frac{36a}{4-a} + \frac{2}{2a-1}$$

$$14 = \frac{36a(2a-1) + 2(4-a)}{(4-a)(2a-1)}$$

$$14 = \frac{72a^2 - 36a + 8 - 2a}{8a - 4 - 2a^2 + a}$$

$$14 = \frac{72a^2 - 38a + 8}{-2a^2 + 9a - 4}$$

$$14(-2a^2 + 9a - 4) = 72a^2 - 38a + 8$$

$$-28a^2 + 126a - 56 = 72a^2 - 38a + 8$$

$$72a^2 + 28a^2 - 38a - 126a + 8 + 56 = 0$$

$$100a^2 - 164a + 64 = 0$$

$$25a^2 - 41a + 16 = 0$$

SUMMATION NOTATION

The Greek letter Σ (sigma) is used to sum a sequence of numbers. We write the sum in sigma notation as,

$$a_1 + a_2 + a_3 + \dots + a_n = \sum_{k=1}^n a_k$$

- ① Here Σ indicates the sum and k is called index of summation.
- ② The summation begins from $k=1$ and ends with $k=n$.
- ③ $k=1$ is called lower limit while $k=n$ is called upper limit.

PROPERTIES OF SUMMATION :-

(i) $\sum_{k=1}^n (a_k + b_k) = \sum_{k=1}^n a_k + \sum_{k=1}^n b_k$

(ii) $\sum_{k=1}^n (a_k - b_k) = \sum_{k=1}^n a_k - \sum_{k=1}^n b_k$

(iii) $\sum_{k=1}^n c a_k = c \sum_{k=1}^n a_k$