Exercise 5.3 (Solutions)

Textbook of Algebra and Trigonometry for Class XI

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Question # 1

$$\frac{9x-7}{(x^2+1)(x+3)}$$

Solution $\quad \dfrac{9x-7}{(x^2+1)(x+3)}$

Resolving it into partial fraction.

$$\frac{9x-7}{(x^2+1)(x+3)} = \frac{Ax+B}{(x^2+1)} + \frac{C}{(x+3)}$$

Multiplying both sides by $(x^2+1)(x+3)$.

$9x-7 = (Ax+B)(x+3) + C(x^2+1)$ ....(i)

Put $x+3=0 \;\Rightarrow\; x=-3$ in equation (i).

$$9(-3)-7 = \big(A(-3)+B\big)(0) + C\left((-3)^2+1\right) \quad \Rightarrow -27-7 = 0 + C(9+1)$$

$$\Rightarrow -34 = 10C \qquad \Rightarrow C = -\frac{34}{10} \qquad \Rightarrow \boxed{C = -\frac{17}{5}}$$

Now equation (i) can be written as

$$9x-7 = A(x^2+3x) + B(x+3) + C(x^2+1)$$

Comparing the coefficients of x^2, x and x^0.

$0 = A + C$ .... (ii)

$9 = 3A + B$ .... (iii)

$-7 = +3B + C$ .... (iv)

Putting value of C in equation (ii)

$$0 = A - \frac{17}{5} \qquad \Rightarrow \boxed{A = \frac{17}{5}}$$

Now putting value of A in equation (iii)

$$9 = 3\left(\frac{17}{5}\right) + B \qquad \Rightarrow 9 = \frac{51}{5} + B \qquad \Rightarrow 9 - \frac{51}{5} = B \qquad \Rightarrow \boxed{B = -\frac{6}{5}}$$

Hence

$$\frac{9x-7}{(x^2+1)(x+3)} = \frac{\frac{17}{5}x - \frac{6}{5}}{x^2+1} + \frac{-\frac{17}{5}}{(x+3)}$$

$$= \frac{\frac{17x-6}{5}}{x^2+1} - \frac{\frac{17}{5}}{(x+3)} \qquad = \frac{17x-6}{5(x^2+1)} - \frac{17}{5(x+3)} \quad \textit{Answer}$$

Question # 2 $\dfrac{1}{(x^2+1)(x+1)}$

Solution $\dfrac{1}{(x^2+1)(x+1)}$

Now Consider

$$\frac{1}{(x^2+1)(x+1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1}$$

Multiplying both sides by $(x^2+1)(x+1)$.

$$1 = (Ax+B)(x+1) + C(x^2+1)\text{.........(i)}$$

Put $x+1=0 \quad \Rightarrow \ x=-1$ in equation (i)

$$1 = 0 + C\left((-1)^2+1\right) \qquad \Rightarrow 1 = 2C \quad \Rightarrow \boxed{C = \frac{1}{2}}$$

Now eq. (i) can be written as

$$1 = A(x^2+x) + B(x+1) + C(x^2+1)$$

Comparing the coefficients of x^2, x and x^0.

$$0 = A + C \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$
$$0 = A + B \ \ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$
$$1 = A + C \ \ldots\ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

Putting value of C in equation (ii)

$$0 = A + \frac{1}{2} \quad \Rightarrow \boxed{A = -\frac{1}{2}}$$

Putting value of A in equation (iii)

$$0 = -\frac{1}{2} + B \ \Rightarrow \boxed{B = \frac{1}{2}}$$

Hence $$\frac{1}{(x^2+1)(x+1)} = \frac{-\frac{1}{2}x+\frac{1}{2}}{x^2+1} + \frac{\frac{1}{2}}{x+1} = \frac{\frac{-x+1}{2}}{x^2+1} + \frac{\frac{1}{2}}{x+1}$$

$$= \frac{-x+1}{2(x^2+1)} + \frac{1}{2(x+1)} = \frac{1-x}{2(x^2+1)} + \frac{1}{2(x+1)} \qquad \textit{Answer}$$

Question # 3 $\dfrac{3x+7}{(x^2+4)(x+3)}$

Solution $\dfrac{3x+7}{(x^2+4)(x+3)}$

Resolving it into partial fraction.

$$\frac{3x+7}{(x^2+4)(x+3)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+3}$$

$$\left[\begin{array}{c} \text{Now do yourself, you will get} \\ A = {}^{2}\!/_{13}\,,\ B = {}^{33}\!/_{13} \text{ and } C = -{}^{2}\!/_{13} \end{array}\right]$$

Question # 4 $\dfrac{x^2+15}{(x^2+2x+5)(x-1)}$

Solution $\dfrac{x^2+15}{(x^2+2x+5)(x-1)}$

Resolving it into partial fraction.

$$\frac{x^2+15}{(x^2+2x+5)(x-1)} = \frac{Ax+B}{x^2+2x+5} + \frac{C}{x-1}$$

$$\Rightarrow \quad x^2+15 = (Ax+B)(x-1) + C(x^2+2x+5) \ldots\ldots \text{(i)}$$

Put $x-1=0 \quad \Rightarrow x=1$ in equation (i)

$$(1)^2+15 = \big(A(1)+B\big)(0) + C\big((1)^2+2(1)+5\big) \qquad \Rightarrow \ 1+15 = 0 + C(1+2+5)$$

$$\Rightarrow \ 16 = 8C \quad \Rightarrow \ \frac{16}{8} = C \quad \Rightarrow \ \boxed{C = 2}$$

Now equation (i) can be written as

$$x^2+15 = A(x^2-x) + B(x-1) + C(x^2+2x+5)$$

Comparing the coefficients of x^2, x and x^0.

$1 = A + C$ .... (ii)

$0 = -A + B + 2C$ .... (iii)

$15 = -B + 5C$ .... (iv)

Putting value of C in equation (ii).

$$1 = A+2 \quad \Rightarrow 1-2 = A \Rightarrow \boxed{A=-1}$$

Putting value of A and C in equation (iii)

$$0 = -(-1) + B + 2(2) \quad \Rightarrow 0 = 1 + B + 4 \quad \Rightarrow 0 = B+5 \quad \Rightarrow \boxed{B=-5}$$

Hence $$\frac{x^2+15}{(x^2+2x+5)(x-1)} = \frac{(-1)x-5}{x^2+2x+5} + \frac{2}{x-1}$$

$$= \frac{-x-5}{x^2+2x+5} + \frac{2}{x-1} \qquad \textit{Answer}$$

Question # 5 $\dfrac{x^2}{(x^2+4)(x+2)}$

Solution $\dfrac{x^2}{(x^2+4)(x+2)}$

Resolving it into partial fraction.

$$\frac{x^2}{(x^2+4)(x+2)} = \frac{Ax+B}{x^2+4} + \frac{C}{x+2}$$

$$\left[\begin{array}{l} \text{Now do yourself, you will get} \\ A = \frac{1}{2},\ B = -1 \text{ and } C = -\frac{1}{2} \end{array}\right]$$

Question # 6 $\dfrac{x^2+1}{x^3+1}$

Solution $\dfrac{x^2+1}{x^3+1} = \dfrac{x^2+1}{(x+1)(x^2-x+1)}$ $\because x^3+1=(x+1)(x^2-x+1)$

Now consider

$$\frac{x^2+1}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$$

$$\left[\begin{array}{l} \text{Now do yourself, you will get} \\ A = \frac{2}{3},\ B = \frac{1}{3} \text{ and } C = \frac{1}{3} \end{array}\right]$$

Question # 7 $\dfrac{x^2+2x+2}{(x^2+3)(x+1)(x-1)}$

Solution $\dfrac{x^2+2x+2}{(x^2+3)(x+1)(x-1)}$

Consider

$$\frac{x^2+2x+2}{(x^2+3)(x+1)(x-1)} = \frac{Ax+B}{x^2+3} + \frac{C}{x+1} + \frac{D}{x-1}$$

$\Rightarrow\ x^2+2x+2 = (Ax+B)(x+1)(x-1) + C(x^2+3)(x-1) + D(x^2+3)(x+1)$......(i)

Put $x+1=0 \ \Rightarrow\ x=-1$ in equation (i)

$$(-1)^2+2(-1)+2 = 0 + C\big((-1)^2+3\big)\big((-1)-1\big)+0 \qquad \Rightarrow 1-2+2 = C(4)(-2)$$

$$\Rightarrow 1 = -8C \qquad \Rightarrow \boxed{C = -\frac{1}{8}}$$

Now put $x-1=0 \ \Rightarrow\ x=1$ in equation (i)

$$\Rightarrow\ (1)^2+2(1)+2 = 0+0+D\big((1)^2+3\big)\big((1)+1\big) \qquad \Rightarrow 1+2+2 = D(4)(2)$$

$$\Rightarrow 5 = 8D \qquad \Rightarrow \boxed{D = \frac{5}{8}}$$

Equation (i) can be written as

$$x^2+2x+2 = (Ax+B)(x^2-1) + C(x^3-x^2+3x-3) + D(x^3+x^2+3x+3)$$

$$\Rightarrow\ x^2+2x+2 = A(x^3-x)+B(x^2-1) + C(x^3-x^2+3x-3) + D(x^3+x^2+3x+3)$$

Comparing the coefficients of x^3, x^2, x and x^0.

$0 = A + C + D$ .... (ii)

$1 = B - C + D$ .... (iii)

$2 = -A + 3C + 3D$ .... (iv)

$2 = -B - 3C + 3D$ .... (v)

Putting values of C and D in (ii)

$$0 = A - \frac{1}{8} + \frac{5}{8} \quad \Rightarrow 0 = A + \frac{1}{2} \quad \Rightarrow \boxed{A = -\frac{1}{2}}$$

Putting values of C and D in (iii)

$$1 = B - \left(-\frac{1}{8}\right) + \frac{5}{8} \quad \Rightarrow 1 = B + \frac{1}{8} + \frac{5}{8} \quad \Rightarrow 1 = B + \frac{3}{4}$$

$$\Rightarrow 1 - \frac{3}{4} = B \quad \Rightarrow \boxed{B = \frac{1}{4}}$$

Hence

$$\frac{x^2 + 2x + 2}{(x^2+3)(x+1)(x-1)} = \frac{-\frac{1}{2}x + \frac{1}{4}}{x^2+3} + \frac{-\frac{1}{8}}{x+1} + \frac{\frac{5}{8}}{x-1}$$

$$= \frac{\frac{-2x+1}{4}}{x^2+3} + \frac{-\frac{1}{8}}{x+1} + \frac{\frac{5}{8}}{x-1} = \frac{-2x+1}{4(x^2+3)} + \frac{-1}{8(x+1)} + \frac{5}{8(x-1)}$$

$$= \frac{1-2x}{4(x^2+3)} - \frac{1}{8(x+1)} + \frac{5}{8(x-1)} \quad \textit{Answer}$$

Question # 8 $\quad \dfrac{1}{(x-1)^2(x^2+2)}$

Solution $\quad \dfrac{1}{(x-1)^2(x^2+2)}$

Resolving it into partial fraction.

$$\frac{1}{(x-1)^2(x^2+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+2}$$

$$\Rightarrow \quad 1 = A(x-1)(x^2+2) + B(x^2+2) + (Cx+D)(x-1)^2 \;.......(i)$$

Put $x - 1 = 0 \quad \Rightarrow x = 1$ in equation (i)

$$1 = 0 + B\left((1)^2 + 2\right) + 0 \quad \Rightarrow \quad 1 = 3B \quad \Rightarrow \boxed{B = \frac{1}{3}}$$

Now equation (i) can be written as

$$1 = A(x^3 - x^2 + 2x - 2) + B(x^2+2) + (Cx+D)(x^2-2x+1)$$

$$\Rightarrow 1 = A(x^3 - x^2 + 2x - 2) + B(x^2+2) + C(x^3 - 2x^2 + x) + D(x^2 - 2x + 1)$$

Comparing the coefficients of x^3, x^2, x and x^0.

$0 = A + C$ ……………………… (ii)

$0 = -A + B - 2C + D$ …..……… (iii)

$0 = 2A + C - 2D$ ……………..… (iv)

$1 = -2A + 2B + D$ ……………… (v)

Multiplying eq. (iii) by 2 and adding in (iv)

$$\begin{aligned} 0 &= -2A + 2B - 4C + 2D \\ 0 &= \;\;2A \qquad\;\; + \;\; C - 2D \\ \hline 0 &= \qquad\;\; 2B - 3C \end{aligned}$$

Putting value of B in above

$$0=2\left(\frac{1}{3}\right)-3C \quad\Rightarrow 0=\frac{2}{3}-3C \quad\Rightarrow 3C=\frac{2}{3} \quad\Rightarrow \boxed{C=\frac{2}{9}}$$

Putting value of C in eq. (ii)

$$0=A+\frac{2}{9} \quad\Rightarrow \boxed{A=-\frac{2}{9}}$$

Putting value of A and B in eq. (v)

$$1=-2\left(-\frac{2}{9}\right)+2\left(\frac{1}{3}\right)+D \;\Rightarrow 1=\frac{4}{9}+\frac{2}{3}+D \;\Rightarrow 1-\frac{4}{9}-\frac{2}{3}=D \;\Rightarrow \boxed{D=-\frac{1}{9}}$$

Hence

$$\frac{1}{(x-1)^2(x^2+2)}=\frac{-2/9}{x-1}+\frac{1/3}{(x-1)^2}+\frac{\left(2/9\right)x+\left(-1/9\right)}{x^2+2}$$

$$=\frac{-2/9}{x-1}+\frac{1/3}{(x-1)^2}+\frac{\frac{2x-1}{9}}{x^2+2} \quad =\frac{-2}{9(x-1)}+\frac{1}{3(x-1)^2}+\frac{2x-1}{9(x^2+2)}$$

Question # 9 $\quad \dfrac{x^4}{1-x^4}$

Solution $\quad \dfrac{x^4}{1-x^4}$

$$\begin{array}{r|l} & -1 \\ 1-x^4 & x^4 \\ & \underline{x^4-1} \\ & \quad 1 \end{array}$$

$$=-1+\frac{1}{1-x^4} = -1+\frac{1}{(1-x^2)(1+x^2)}$$

$$=-1+\frac{1}{(1-x)(1+x)(1+x^2)}$$

Now consider

$$\frac{1}{(1-x)(1+x)(1+x^2)}=\frac{A}{1-x}+\frac{B}{1+x}+\frac{Cx+D}{1+x^2}$$

$$\left[\begin{array}{c} \textit{Now find values of } A, B, C \textit{ and } D \textit{ yourself.} \\ \textit{You will get } A=1/4,\ B=1/4,\ C=0 \textit{ and } D=1/2 \end{array}\right]$$

So

$$\frac{1}{(1-x)(1+x)(1+x^2)}=\frac{1/4}{1-x}+\frac{1/4}{1+x}+\frac{(0)x+1/2}{1+x^2}$$

$$=\frac{1}{4(1-x)}+\frac{1}{4(1+x)}+\frac{1}{2(1+x^2)}$$

Hence

$$\frac{x^4}{1-x^4}=-1+\frac{1}{4(1-x)}+\frac{1}{4(1+x)}+\frac{1}{2(1+x^2)} \quad \textit{Answer}$$

Question # 10 $\dfrac{x^2-2x+3}{x^4+x^2+1}$

$\because\ x^4+x^2+1=x^4+2x^2+1-x^2$
$=(x^2+1)^2-x^2$
$=(x^2+1+x)(x^2+1-x)$
$=(x^2+x+1)(x^2-x+1)$

Solution $\dfrac{x^2-2x+3}{x^4+x^2+1}$

$$=\frac{x^2-2x+3}{(x^2+x+1)(x^2-x+1)}$$

Now Consider

$$\frac{x^2-2x+3}{(x^2+x+1)(x^2-x+1)}=\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{x^2-x+1}$$

$\Rightarrow\ x^2-2x+3=(Ax+B)(x^2-x+1)+(Cx+D)(x^2+x+1)$.......(i)

$\Rightarrow\ x^2-2x+3=A(x^3-x^2+x)+B(x^2-x+1)+C(x^3+x^2+x)+D(x^2+x+1)$

Comparing the coefficients of x^3, x^2, x and x^0.

$0=A+C$ .... (ii)

$1=-A+B+C+D$................ (iii)

$-2=A-B+C+D$ .... (iv)

$3=B+D$ .... (v)

Subtracting (ii) and (iv)

$$\begin{array}{rcccc} 0= & A & & +C & \\ -2= & A & -B & +C & +D \\ \hline 2= & & B & & -D \end{array}$$

$\Rightarrow 2=B-D$ .... (vi)

Adding (v) and (vi)

$$\begin{array}{l} 3=B+D \\ 2=B-D \\ \hline 5=2B \end{array}$$

$$\Rightarrow \boxed{B=\frac{5}{2}}$$

Putting value of B in (v)

$$3=\frac{5}{2}+D$$

$$\Rightarrow 3-\frac{5}{2}=D \qquad \Rightarrow \boxed{D=\frac{1}{2}}$$

Putting value of B and D in (iii)

$$1=-A+\frac{5}{2}+C+\frac{1}{2}$$

$$\Rightarrow 1-\frac{5}{2}-\frac{1}{2}=-A+C$$

$\Rightarrow -2=-A+C$ .... (vii)

Adding (ii) and (vii)

$$\begin{array}{rl} 0 = & A + C \\ -2 = & -A + C \\ \hline -2 = & 2C \end{array}$$

$$\Rightarrow \boxed{C = -1}$$

Putting value of C in equation (ii)

$$0 = A - 1$$

$$\Rightarrow \boxed{A = 1}$$

Hence

$$\frac{x^2 - 2x + 3}{(x^2 + x + 1)(x^2 - x + 1)} = \frac{(1)x + \frac{5}{2}}{x^2 + x + 1} + \frac{(-1)x + \frac{1}{2}}{x^2 - x + 1}$$

$$= \frac{\frac{2x+5}{2}}{x^2 + x + 1} + \frac{\frac{-2x+1}{2}}{x^2 - x + 1}$$

$$= \frac{2x+5}{2(x^2 + x + 1)} + \frac{-2x+1}{2(x^2 - x + 1)}$$

$$= \frac{2x+5}{2(x^2 + x + 1)} + \frac{1-2x}{2(x^2 - x + 1)} \qquad \textit{Answer}$$