

Q.No.1

$$\frac{2x^2 - 3x + 4}{(x-1)^3}$$

Resolving it into partial fraction.

$$\frac{2x^2 - 3x + 4}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$$

Multiplying both sides by  $(x-1)^3$

$$2x^2 - 3x + 4 = A(x-1)^2 + B(x-1) + C \quad \text{--- (i)}$$

put  $x-1=0 \Rightarrow x=1$  in eq (i)

$$2(1)^2 - 3(1) + 4 = A(0)^2 + B(0) + C$$

$$\Rightarrow 2 - 3 + 4 = 0 + 0 + C \Rightarrow \boxed{C=3}$$

Now eq (i) implies

$$2x^2 - 3x + 4 = A(x^2 - 2x + 1) + B(x-1) + C$$

Comparing the coefficients of  $x^2$ ,  $x$  and  $x^0$

$$2 = A \quad \text{--- (ii)}$$

$$-3 = -2A + B \quad \text{--- (iii)}$$

$$4 = A - B + C \quad \text{--- (iv)}$$

From eq (ii)  $\boxed{A=2}$

putting value of  $A$  in eq (iii)

$$-3 = -2(2) + B \Rightarrow -3 = -4 + B$$

$$\Rightarrow -3 + 4 = B \Rightarrow \boxed{B=1}$$

So

$$\frac{2x^2 - 3x + 4}{(x-1)^3} = \frac{2}{x-1} + \frac{1}{(x-1)^2} + \frac{3}{(x-1)^3} \quad \text{Answer}$$

Q.No.2

$$\frac{5x^2 - 2x + 3}{(x+2)^3}$$

**\* Correction**

Resolving it into partial fraction.

$$\frac{5x^2 - 2x + 3}{(x+2)^3} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{(x+2)^3}$$

[ Do yourself as above, You will get  $A=5$ ,  $B=-22$ ,  $C=27$  ]

Q.No.3  $\frac{4x}{(x+1)^2(x-1)}$

Resolving it into partial fraction.

$$\frac{4x}{(x+1)^2(x-1)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x-1}$$

Multiplying both sides by  $(x+1)^2(x-1)$

$$4x = A(x+1)(x-1) + B(x-1) + C(x+1)^2 \quad \text{--- (i)}$$

put  $x+1=0 \Rightarrow x=-1$  in eq (i)

$$4(-1) = A(0) + B(-1-1) + C(0)^2$$

$$\Rightarrow -4 = 0 - 2B + 0 \Rightarrow -4 = -2B \Rightarrow \boxed{B=2}$$

put  $x-1=0 \Rightarrow x=1$  in eq (i)

$$4(1) = A(0) + B(0) + C(1+1)^2$$

$$\Rightarrow 4 = 0 + 0 + C(2)^2 \Rightarrow 4 = 4C \Rightarrow \boxed{C=1}$$

Now from eq (i)

$$4x = A(x^2-1) + B(x-1) + C(x^2+2x+1)$$

Comparing co-efficients of  $x^2$  only

$$0 = A + C \quad \text{--- (ii)}$$

put  $C=1$  in eq (ii)

$$0 = A + 1 \Rightarrow \boxed{A=-1}$$

So

$$\frac{4x}{(x+1)^2(x-1)} = \frac{-1}{x+1} + \frac{2}{(x+1)^2} + \frac{1}{x-1} \quad \text{Answer}$$

Q.No.4 Do yourself,  $A = -1, B = -3, C = 1$

Q.No.5 Do yourself,  $A = -\frac{1}{16}, B = \frac{1}{4}, C = \frac{1}{16}$

Q.No.6 Do yourself,  $A = 4, B = -3, C = 1$

Q.No.7 Do yourself,  $A = -\frac{1}{4}, B = \frac{1}{2}, C = \frac{1}{4}$

Q.No 8

$$\frac{x^2}{(x-1)^3(x+1)}$$

Resolving it into partial fraction.

$$\frac{x^2}{(x-1)^3(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} + \frac{D}{x+1}$$

xing both sides by  $(x-1)^3(x+1)$ 

$$x^2 = A(x-1)^2(x+1) + B(x-1)(x+1) + C(x+1) + D(x-1)^3 \quad \text{--- (i)}$$

put  $x-1=0 \Rightarrow x=1$  in eq (i)

$$(1)^2 = A(0) + B(0) + C(1+1) + D(0)^3$$

$$\Rightarrow 1 = 0 + 0 + C(2) + 0 \Rightarrow 1 = 2C \Rightarrow \boxed{C = \frac{1}{2}}$$

put  $x+1=0 \Rightarrow x=-1$  in eq (i)

$$(-1)^2 = A(0) + B(0) + C(0) + D(-1-1)^3$$

$$\Rightarrow 1 = 0 + 0 + 0 + D(-2)^3 \Rightarrow 1 = -8D \Rightarrow \boxed{D = -\frac{1}{8}}$$

From eq (i)

$$x^2 = A(x^2 - 2x + 1)(x+1) + B(x^2 - 1) + C(x+1) + D(x^3 - 3x^2 + 3x - 1)$$

$$\Rightarrow x^2 = A(x^3 - 2x^2 + x + x^2 - 2x + 1) + B(x^2 - 1) + C(x+1) + D(x^3 - 3x^2 + 3x - 1)$$

$$\Rightarrow x^2 = A(x^3 - x^2 - x + 1) + B(x^2 - 1) + C(x+1) + D(x^3 - 3x^2 + 3x - 1)$$

Comparing coefficients of  $x^3$ ,  $x^2$ ,  $x$  and  $x^0$ .

$$0 = A + D \quad \text{--- (ii)}$$

$$1 = -A + B - 3D \quad \text{--- (iii)}$$

$$0 = -A + C + 3D \quad \text{--- (iv)}$$

$$0 = A - B + C - D \quad \text{--- (v)}$$

put  $D = -\frac{1}{8}$  in eq (ii)

$$0 = A - \frac{1}{8} \Rightarrow \boxed{A = \frac{1}{8}}$$

put  $A = \frac{1}{8}$  and  $D = -\frac{1}{8}$  in eq (iii)

$$1 = -\frac{1}{8} + B - 3\left(-\frac{1}{8}\right) \Rightarrow 1 = -\frac{1}{8} + B + \frac{3}{8}$$

$$\Rightarrow 1 + \frac{1}{8} - \frac{3}{8} = B \Rightarrow \boxed{B = \frac{3}{4}}$$

So

$$\frac{x^2}{(x-1)^3(x+1)} = \frac{\frac{1}{8}}{x-1} + \frac{\frac{3}{4}}{(x-1)^2} + \frac{\frac{1}{2}}{(x-1)^3} + \frac{-\frac{1}{8}}{x+1}$$

$$= \frac{1}{8(x-1)} + \frac{3}{4(x-1)^2} + \frac{1}{2(x-1)^3} - \frac{1}{8(x+1)} \quad \text{Answer}$$

MEGA LECTURE

Q No.9 Do yourself  $A = \frac{1}{27}, B = -\frac{1}{27}, C = -\frac{1}{9}, D = \frac{2}{3}$

Q No.10 
$$\frac{4x^3}{(x^2-1)(x+1)^2} = \frac{4x^3}{(x-1)(x+1)(x+1)^2}$$
  

$$= \frac{4x^3}{(x-1)(x+1)^3}$$

Now Resolving it into partial fraction.

$$\frac{4x^3}{(x-1)(x+1)^3} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{(x+1)^3}$$

[ Now Do yourself you will get  
 $A = \frac{1}{2}, B = \frac{7}{2}, C = -5, D = 2$  ]

Q No.11 
$$\frac{2x+1}{(x+3)(x-1)(x+2)^2}$$

Resolving it into partial fraction.

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{x+2} + \frac{D}{(x+2)^2}$$

Multiplying both sides by  $(x+3)(x-1)(x+2)^2$

$$2x+1 = A(x-1)(x+2)^2 + B(x+3)(x+2)^2 + C(x+3)(x-1)(x+2) + D(x+3)(x-1) \quad (i)$$

put  $x+3=0 \Rightarrow x=-3$  in eq (i)

$$2(-3)+1 = A(-3-1)(-3+2)^2 + B(0) + C(0) + D(0)$$

$$\Rightarrow -6+1 = A(-4)(-1)^2 + 0 + 0 + 0 \Rightarrow -5 = -4A \Rightarrow \boxed{A = \frac{5}{4}}$$

Put  $x-1=0 \Rightarrow x=1$  in eq (i)

$$2(1)+1 = A(0) + B(1+3)(1+2)^2 + C(0) + D(0)$$

$$\Rightarrow 3 = 0 + B(4)(3)^2 + 0 + 0 \Rightarrow 3 = 36B$$

$$\Rightarrow B = \frac{3}{36} \Rightarrow \boxed{B = \frac{1}{12}}$$

Put  $x+2=0 \Rightarrow x=-2$  in eq (i)

$$2(-2)+1 = A(0) + B(0) + C(0) + D(-2+3)(-2-1)$$

$$\Rightarrow -4+1 = 0 + 0 + 0 + D(1)(-3) \Rightarrow -3 = -3D$$

$$\Rightarrow \boxed{D = 1}$$

Now from eq (i)

$$2x+1 = A(x-1)(x^2+4x+4) + B(x+3)(x^2+4x+4) + C(x+3)(x^2-x+2x-2) + D(x^2+3x-x-3)$$

$$\Rightarrow 2x+1 = A(x-1)(x^2+4x+4) + B(x+3)(x^2+4x+4)$$

$$+ C(x+3)(x^2+x-2) + D(x^2+2x-3)$$

$$\Rightarrow 2x+1 = A(x^3+4x^2+4x-x^2-4x-4) + B(x^3+4x^2+4x+3x^2+12x+12)$$

$$+ C(x^3+x^2-2x+3x^2+3x-6) + D(x^2+2x-3)$$

$$\Rightarrow 2x+1 = A(x^3+3x^2-4) + B(x^3+7x^2+16x+12)$$

$$+ C(x^3+4x^2+x-6) + D(x^2+2x-3)$$

Comparing the coefficients of  $x^3$  only.

$$0 = A + B + C \quad \text{--- (ii)}$$

put  $A = \frac{5}{4}$  and  $B = \frac{1}{12}$  in above

$$0 = \frac{5}{4} + \frac{1}{12} + C \Rightarrow 0 = \frac{4}{3} + C \Rightarrow \boxed{C = -\frac{4}{3}}$$

So

$$\frac{2x+1}{(x+3)(x-1)(x+2)^2} = \frac{5/4}{x+3} + \frac{1/12}{x-1} + \frac{-4/3}{x+2} + \frac{1}{(x+2)^2}$$

$$= \frac{5}{4(x+3)} + \frac{1}{2(x-1)} - \frac{4}{3(x+2)} + \frac{1}{(x+2)^2}$$

Answer

Q No 12

$$\frac{2x^4}{(x-3)(x+2)^2} = \frac{2x^4}{(x-3)(x^2+4x+4)}$$

$$= \frac{2x^4}{x^3+4x^2+4x-3x^2-12x-12}$$

$$= \frac{2x^4}{x^3+x^2-8x-12}$$

$$= 2x-2 + \frac{18x^2+8x-24}{x^3+x^2-8x-12}$$

$$= 2x-2 + \frac{18x^2+8x-24}{(x-3)(x+2)^2}$$

$$x^3+x^2-8x-12$$

$$\begin{array}{r} 2x-2 \\ \underline{2x^4} \\ -7x^4+2x^3-16x^2-24x \\ \underline{-2x^3+16x^2+24x} \\ +7x^3+2x^2+16x+2 \\ \underline{\phantom{+7x^3+2x^2+16x+2}} \\ 18x^2+8x-24 \end{array}$$

Now consider

$$\frac{18x^2+8x-24}{(x-3)(x+2)^2} = \frac{A}{x-3} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

ming both sides by  $(x-3)(x+2)^2$

MEGA LECTURE

$$18x^2 + 8x - 24 = A(x+2)^2 + B(x-3)(x+2) + C(x-3) \quad (i)$$

put  $x-3=0 \Rightarrow x=3$  in eq. (i)

$$18(3)^2 + 8(3) - 24 = A(3+2)^2 + B(0) + C(0)$$

$$\Rightarrow \frac{162}{1} + \frac{24}{1} - 24 = A(5)^2 + 0 + 0 \Rightarrow \frac{162}{1} = 25A \Rightarrow \boxed{A = \frac{162}{25}}$$

put  $x+2=0 \Rightarrow x=-2$  in eq. (i)

$$18(-2)^2 + 8(-2) - 24 = A(0)^2 + B(0) + C(-2-3)$$

$$\Rightarrow 72 - 16 - 24 = 0 + 0 + C(-5) \Rightarrow 32 = -5C \Rightarrow \boxed{C = -\frac{32}{5}}$$

Now eq. (i) can be written as

$$18x^2 + 8x - 24 = A(x^2 + 4x + 4) + B(x^2 - 3x + 2x - 6) + C(x-3)$$

$$\Rightarrow 18x^2 + 8x - 24 = A(x^2 + 4x + 4) + B(x^2 - x - 6) + C(x-3)$$

Comparing coefficients of  $x^2$  only.

$$18 = A + B \quad (ii)$$

put  $A = \frac{162}{25}$  in above

$$18 = \frac{162}{25} + B \Rightarrow 18 - \frac{162}{25} = B \Rightarrow \boxed{B = \frac{288}{25}}$$

$$\begin{aligned} \text{So } \frac{18x^2 + 8x - 24}{(x-3)(x+2)^2} &= \frac{162/25}{x-3} + \frac{288/25}{x+2} + \frac{-32/5}{(x+2)^2} \\ &= \frac{162}{25(x-3)} + \frac{288}{25(x+2)} - \frac{32}{5(x+2)^2} \end{aligned}$$

hence

$$\frac{18x^2 + 8x - 24}{(x-3)(x+2)^2} = \frac{162}{25(x-3)} + \frac{288}{25(x+2)} - \frac{32}{5(x+2)^2}$$

Answer