



$$y = \frac{1+\sqrt{33}}{2}, y = \frac{1-\sqrt{33}}{2}$$

$$\text{If } y = \frac{1+\sqrt{33}}{2} \text{ then}$$

$$x = \frac{1+\sqrt{33}}{2}$$

$$\text{If } y = \frac{1-\sqrt{33}}{2} \text{ then}$$

$$x = \frac{1-\sqrt{33}}{2}$$

$$x = 4$$

$$\text{If } y = -\frac{4}{3} \text{ then}$$

$$x = 2(-\frac{4}{3})$$

$$x = -\frac{8}{3}$$

$$2x(x-6y) - 3y(x-6y) = 0$$

$$(x-6y)(2x-3y) = 0$$

$$x-6y=0 \quad , \quad 2x-3y=0$$

$$x=6y \quad , \quad x=\frac{3}{2}y$$

If  $x=6y$  then from ① If  $x=\frac{3}{2}y$  then from ①

$$(6y)^2 - y^2 = 5$$

$$36y^2 - y^2 = 5$$

$$35y^2 = 5$$

$$y^2 = \frac{5}{35} \rightarrow y^2 = \frac{1}{7}$$

$$y = \pm \frac{1}{\sqrt{7}}$$

$$y = \frac{1}{\sqrt{7}}, y = -\frac{1}{\sqrt{7}}$$

$$\text{If } y = \frac{1}{\sqrt{7}} \text{ then}$$

$$x = 6(\frac{1}{\sqrt{7}}) = \frac{6}{\sqrt{7}}$$

$$\text{If } y = -\frac{1}{\sqrt{7}} \text{ then}$$

$$x = 6(-\frac{1}{\sqrt{7}}) = -\frac{6}{\sqrt{7}}$$

$$\left\{ \left( \frac{6}{\sqrt{7}}, \frac{1}{\sqrt{7}} \right), \left( -\frac{6}{\sqrt{7}}, -\frac{1}{\sqrt{7}} \right), (3, 2), (-3, -2) \right\}$$

(C) When both equations are quadratic but one of them involving  $xy$  in product form.

### Solution Method

In this type we balance constant terms of both equations, then subtracting each other we get Homogeneous Quadratic Equation and solve as homogeneous case.

Questions of such type are

Exp ③, Q. 7, 8, 9, 10

### Example.3

$$x^2 - y^2 = 5 \rightarrow ①$$

$$4x^2 - 3xy = 18 \rightarrow ②$$

To balance constant terms multiplying eq. ① by 18 and eq. ② by 5 we get

$$18x^2 - 18y^2 = 90 \rightarrow ③$$

$$20x^2 - 15xy = 90 \rightarrow ④$$

Subtracting eq. ③ from eq. ④

$$20x^2 - 15xy = 90$$

$$+ 18x^2 - 18y^2 = 90$$

$$\underline{2x^2 + 18y^2 - 15xy = 0}$$

$$2x^2 - 15xy + 18y^2 = 0$$

Factorizing it, we get

$$2x^2 - 12xy + 3xy - 18y^2 = 0$$

### EXERCISE.4.9

#### Q.1

$$2x^2 = 6 + 3y^2$$

$$\text{or } 2x^2 - 3y^2 = 6$$

$$\text{and } 3x^2 - 5y^2 = 7$$

Put  $x^2 = u$ ,  $y^2 = v$  thus we get

$$2u - 3v = 6 \rightarrow ①$$

$$3u - 5v = 7 \rightarrow ②$$

To balance coefficient of  $u$  multiplying eq. ① by 3 and eq. ② by 2 we get

$$6u - 9v = 18 \rightarrow ③$$

$$6u - 10v = 14 \rightarrow ④$$

Subtracting eq. ③ from eq. ④

$$6u - 10v = 14$$

$$\underline{6u + 9v = 18}$$

$$\underline{-v = -4} \rightarrow v = 4$$

Putting value of  $v$  in ① we get

$$24 - 3(4) = 6$$

$$2u - 12 = 6 \rightarrow 2u = 6 + 12$$

$$2u = 18 \rightarrow u = 9$$

If  $u = 9$  then

$$x^2 = 9 \rightarrow x = \pm 3$$

$$\{(\pm 3, \pm 2)\}$$

$$\text{or } \{(3, \pm 2), (-3, \pm 2)\}$$

$$\text{or } \{(3, 2), (3, -2), (-3, 2), (-3, -2)\}$$

If  $v = 4$  then

$$y^2 = 4 \rightarrow y = \pm 2$$

$$\{(\pm 3, \pm 2)\}$$

and  $x^2 - 13 = -2y^2$  or  $x^2 - 2y^2 = 13$

Put  $x^2 = u$  and  $y^2 = v$  we get

$$2u - 5v = 8 \rightarrow ①$$

$$u + 2v = 13 \rightarrow ②$$

To balance coefficient of  $u$  multiply

eq. ② by 2 we get

$$2u + 4v = 26 \rightarrow ③$$

Subtracting eq. ① from eq. ③

$$2u + 4v = 26$$

$$2u - 5v = -8$$

$$\underline{9v = 18} \rightarrow v = 2$$

put  $v = 2$  in eq. ② we get

$$u + 2(2) = 13 \rightarrow u = 13 - 4 \rightarrow u = 9$$

If  $u = 9$  then

$$x^2 = 9$$

$$x = \pm 3$$

$$y = 2$$

$$y = \pm \sqrt{2}$$

$$\{(\pm 3, \pm \sqrt{2})\}$$

**Q.2**

$$8x^2 = y^2$$

$$\text{or } 8x^2 - y^2 = 0$$

$$\text{and } x^2 + 2y^2 = 19$$

Put  $x^2 = u$  and  $y^2 = v$  we get

$$8u - v = 0 \rightarrow ①$$

$$u + 2v = 19 \rightarrow ②$$

To balance coefficient of  $v$  multiply  
eq. ① by 2 we get

$$16u - 2v = 0 \rightarrow ③$$

Adding ② and ③

$$u + 2v = 19$$

$$16u - 2v = 0$$

$$\underline{17u = 19}$$

$$\rightarrow u = \frac{19}{17}, \text{ Putting value of } u \text{ in}$$

① we get

$$8\left(\frac{19}{17}\right) - v = 0 \rightarrow v = \frac{152}{17}$$

If  $u = \frac{19}{17}$  then

$$x^2 = \frac{19}{17}$$

$$x = \pm \sqrt{\frac{19}{17}}$$

$$\{(\pm \sqrt{\frac{19}{17}}, \pm 2\sqrt{\frac{38}{17}})\}$$

**Q.3**  $2x^2 - 8 = 5y^2$  or  $2x^2 - 5y^2 = 8$

**Q.4**  $x^2 - 5xy + 6y^2 = 0 \rightarrow ①$

$$x^2 + y^2 = 45 \rightarrow ②$$

Factorizing homogeneous equation we get

$$x^2 - 2xy - 3xy + 6y^2 = 0$$

$$x(x-2y) - 3y(x-2y) = 0$$

$$(x-2y)(x-3y) = 0$$

$$x-2y=0, x-3y=0$$

$$x=2y, x=3y$$

If  $x = 2y$  then from ② If  $x = 3y$  then from ②

$$(2y)^2 + y^2 = 45$$

$$4y^2 + y^2 = 45$$

$$5y^2 = 45$$

$$y^2 = 9 \rightarrow y = \pm 3$$

$$y = 3, y = -3$$

$$\begin{aligned} x &= 2(-3) \\ x &= -6 \\ \{(6,3),(-6,-3),\left(\frac{9}{\sqrt{2}},\frac{3}{\sqrt{2}}\right),\left(\frac{-9}{\sqrt{2}},\frac{-3}{\sqrt{2}}\right)\} \end{aligned}$$

**Q.5**  $12x^2 - 25xy + 12y^2 = 0 \rightarrow ①$   
 $4x^2 + 7y^2 = 148 \rightarrow ②$

Factorizing homogeneous equation we get

$$\begin{aligned} 12x^2 - 16xy - 9xy + 12y^2 &= 0 \\ 4x(3x - 4y) - 3y(3x - 4y) &= 0 \\ (3x - 4y)(4x - 3y) &= 0 \\ 3x - 4y &= 0, \quad 4x - 3y = 0 \\ 3x &= 4y, \quad 4x = 3y \\ x = \frac{4}{3}y &\quad x = \frac{3}{4}y \end{aligned}$$

If  $x = \frac{4}{3}y$  then from ② If  $x = \frac{3}{4}y$  then from ②

$$\begin{aligned} 4\left(\frac{4}{3}y\right)^2 + 7y^2 &= 148 \\ \frac{64}{9}y^2 + 7y^2 &= 148 \\ 64y^2 + 63y^2 &= 1332 \\ 127y^2 &= 1332 \\ y^2 &= \frac{1332}{127} \end{aligned}$$

$$y = \pm \sqrt{\frac{1332}{127}}$$

$$y = \pm \sqrt{\frac{4 \times 333}{127}}$$

$$y = \pm 2\sqrt{\frac{333}{127}}$$

$$y = 2\sqrt{\frac{333}{127}}, y = -2\sqrt{\frac{333}{127}}$$

If  $y = 2\sqrt{\frac{333}{127}}$  then

$$x = \frac{4}{3}\left(2\sqrt{\frac{333}{127}}\right)$$

$$= \frac{8}{3}\sqrt{\frac{333}{127}}$$

If  $y = -2\sqrt{\frac{333}{127}}$  then

$$x = \frac{4}{3}\left(-2\sqrt{\frac{333}{127}}\right)$$

$$x = -\frac{8}{3}\sqrt{\frac{333}{127}}$$

If  $y = \frac{-3}{\sqrt{2}}$  then  
 $x = 3\left(\frac{-3}{\sqrt{2}}\right) = \frac{-9}{\sqrt{2}}$

$$\left\{\left(\frac{8}{3}\sqrt{\frac{333}{127}}, 2\sqrt{\frac{333}{127}}\right), \left(-\frac{8}{3}\sqrt{\frac{333}{127}}, -2\sqrt{\frac{333}{127}}\right), (3, 4), (-3, -4)\right\}$$

**Q.6**

$$12x^2 - 11xy + 2y^2 = 0 \rightarrow ①$$

$$2x^2 + 7xy = 60 \rightarrow ②$$

Factorizing homogeneous equation we get

$$12x^2 - 8xy - 3xy + 2y^2 = 0$$

$$4x(3x - 2y) - y(3x - 2y) = 0$$

$$(3x - 2y)(4x - y) = 0$$

$$3x - 2y = 0, \quad 4x - y = 0$$

$$3x = 2y, 4x = y \Rightarrow x = \frac{2}{3}y, x = \frac{1}{4}y$$

If  $x = \frac{2}{3}y$  then from ② If  $x = \frac{1}{4}y$  then from ②

$$2\left(\frac{2}{3}y\right)^2 + 7\left(\frac{2}{3}y\right)y = 60$$

$$\frac{8}{9}y^2 + \frac{14}{3}y^2 = 60$$

$$8y^2 + 42y^2 = 540$$

$$50y^2 = 540$$

$$y^2 = \frac{540}{50} \Rightarrow y = \frac{54}{5}$$

$$y = \pm \sqrt{\frac{54}{5}}$$

$$y = \pm \sqrt{\frac{9 \times 6}{5}}$$

$$y = \pm 3\sqrt{\frac{6}{5}}$$

$$y = 3\sqrt{\frac{6}{5}}, y = -3\sqrt{\frac{6}{5}}$$

If  $y = 3\sqrt{\frac{6}{5}}$  then

$$x = \frac{2}{3}(3\sqrt{\frac{6}{5}}) = 2\sqrt{\frac{6}{5}}$$

If  $y = -3\sqrt{\frac{6}{5}}$  then

$$x = \frac{2}{3}(-3\sqrt{\frac{6}{5}})$$

$$x = -2\sqrt{\frac{6}{5}}$$

$$\left\{\left(2\sqrt{\frac{6}{5}}, 3\sqrt{\frac{6}{5}}\right), \left(-2\sqrt{\frac{6}{5}}, -3\sqrt{\frac{6}{5}}\right)\right.$$

$$\left.\left(\sqrt{2}, 4\sqrt{2}\right), \left(-\sqrt{2}, -4\sqrt{2}\right)\right\}$$

**Q.7**  $x^2 - y^2 = 16 \rightarrow ①$   
 $xy = 15 \rightarrow ②$

To balance constant terms multiply eq. ① by 15 and eq. ② by 16 we get

$$15x^2 - 15y^2 = 240 \rightarrow ③$$

$$16xy = 240 \rightarrow ④$$

Subtracting eq. ④ from eq. ③

$$15x^2 - 15y^2 - 16xy = 0$$

$$15x^2 - 16xy - 15y^2 = 0$$

Factorizing

$$15x^2 - 25xy + 9xy - 15y^2 = 0$$

$$5x(3x - 5y) + 3y(3x - 5y) = 0$$

$$(3x - 5y)(5x + 3y) = 0$$

$$3x - 5y = 0, 5x + 3y = 0$$

$$3x = 5y, 5x = -3y$$

$$x = \frac{5}{3}y, x = -\frac{3}{5}y$$

If  $x = \frac{5}{3}y$  then from ②

$$(\frac{5}{3}y)y = 15$$

$$\frac{5}{3}y^2 = 15$$

$$y^2 = 15 \times \frac{3}{5}$$

$$y^2 = 9 \rightarrow y = \pm 3$$

$$y = 3, y = -3$$

If  $y = 3$  then

$$x = \frac{5}{3}(3) = 5$$

If  $y = -3$  then

$$x = \frac{5}{3}(-3)$$

$$x = -5$$

$$\{(5, 3), (-5, -3), (-3i, 3i), (3i, -3i)\}$$

**Q.8**  $x^2 + xy = 9 \rightarrow ①$   
 $x^2 - y^2 = 2 \rightarrow ②$

To balance constant term multiply eq.

① by 2 and eq. ② by 9 we get

$$2x^2 + 2xy = 18 \rightarrow ③$$

$$9x^2 - 9y^2 = 18 \rightarrow ④$$

Subtracting eq. ③ from ④

$$9x^2 - 9y^2 = 18$$

$$\underline{2x^2 + 2xy = 18}$$

$$\underline{\underline{7x^2 - 9y^2 - 2xy = 0}}$$

$$7x^2 - 2xy - 9y^2 = 0$$

$$7x^2 + 7xy - 9xy - 9y^2 = 0$$

$$7x(x+y) - 9y(x+y) = 0$$

$$(x+y)(7x - 9y) = 0$$

$$x+y = 0, 7x - 9y = 0$$

$$x = -y, x = \frac{9}{7}y$$

If  $x = -y$  then from ②

$$y^2 - y^2 = 2$$

$$0 = 2$$

which is impossible

If  $x = \frac{9}{7}y$  then from ②

$$(\frac{9}{7}y)^2 - y^2 = 2$$

$$\frac{81}{49}y^2 - y^2 = 2$$

$$81y^2 - 49y^2 = 98$$

$$32y^2 = 98$$

$$y^2 = \frac{98}{32} = \frac{49}{16}$$

$$y = \pm \frac{7}{4}$$

$$y = \frac{7}{4}, y = -\frac{7}{4}$$

If  $y = \frac{7}{4}$  then  $x = \frac{9}{7}(\frac{7}{4}) = \frac{9}{4}$

If  $y = -\frac{7}{4}$  then  $x = \frac{9}{7}(-\frac{7}{4}) = -\frac{9}{4}$

$$\{(\frac{-9}{4}, \frac{7}{4}), (\frac{9}{4}, \frac{7}{4})\}$$

**Q.9**

$$y^2 - 7 = 2xy \text{ or } y^2 - 2xy = 7 \rightarrow ①$$

$$2x^2 + 3 = xy \text{ or } 2x^2 - xy = -3 \rightarrow ②$$

To balance coefficients multiply eq. ①

by 3 and eq. ② by 7 we get

$$3y^2 - 6xy = 21 \rightarrow ③$$

$$14x^2 - 7xy = -21 \rightarrow ④$$

Adding. eq. ③ and eq. ④

$$3y^2 - 6xy = 21$$

$$14x^2 - 7xy = -21$$

$$\underline{14x^2 + 3y^2 - 13xy = 0}$$

$$14x^2 - 13xy + 3y^2 = 0$$

$$14x^2 - 7xy - 6xy + 3y^2 = 0$$

$$7x(2x-y) - 3y(2x-y) = 0$$

$$(2x-y)(7x-3y) = 0$$

$$2x-y = 0, 7x-3y = 0$$

$$x = \frac{1}{2}y, x = \frac{3}{7}y$$

If  $x = \frac{1}{2}y$  then from ①

$$y^2 - 2\left(\frac{1}{2}y\right)y = 7$$

$$y^2 - y^2 = 7$$

$$0 = 7$$

which is impossible

If  $x = \frac{3}{7}y$  then from ①

$$y^2 - 2\left(\frac{3}{7}y\right)y = 7$$

$$y^2 - \frac{6}{7}y^2 = 7$$

$$7y^2 - 6y^2 = 49$$

$$y^2 = 49$$

$$y = \pm 7$$

$$y = 7, y = -7$$

$$\text{If } y = 7 \text{ then } x = \frac{3}{7}(7) \rightarrow x = 3$$

$$\text{If } y = -7 \text{ then } x = \frac{3}{7}(-7) \rightarrow x = -3$$

$$\{(3, 7), (-3, -7)\}$$

## Q.10

$$x^2 + y^2 = 5 \rightarrow ①$$

$$xy = 2 \rightarrow ②$$

To balance coefficients multiply eq.

① by 2 and ② by 5 we get

$$2x^2 + 2y^2 = 10 \rightarrow ③$$

$$5xy = 10 \rightarrow ④$$

Subtracting eq. ④ from eq. ③

$$2x^2 + 2y^2 - 5xy = 0$$

$$2x^2 - 5xy + 2y^2 = 0$$

$$2x^2 - xy - 4xy + 2y^2 = 0$$

$$x(2x-y) - 2y(2x-y) = 0$$

$$(x-2y)(2x-y) = 0$$

$$x-2y = 0, 2x-y = 0$$

$$x = 2y, x = \frac{1}{2}y$$

If  $x = 2y$  then from ② If  $x = \frac{1}{2}y$  then from ②

$$(2y)y = 2$$

$$2y^2 = 2$$

$$y^2 = 1 \rightarrow y = \pm 1$$

$$y = 1, y = -1$$

If  $y = 1$  then

$$x = 2(1) = 2$$

If  $y = -1$  then

$$x = 2(-1) = -2$$

$$\{(2, 1), (-2, -1), (1, 2), (-1, -2)\}$$

## Theoretical Problems on Quadratic Equation:

To solve theoretical problems we should keep following steps in mind:-

(i) Read the problem carefully.

(ii) Suppose the unknown quantities.

as  $x, y, z$  C.L.C

(iii) Translate the problem in to symbols.

For example if we are given

(a) 5 is greater than  $x$ , we write it as  
 $5 > x$

(b)  $x$  is greater than  $y$ , we write it as  
 $x > y$

(c)  $x$  is greater than  $z$  by 5, we write  
it as  $x - z = 5$

(d) 5 is less than  $x$ , we write it as  
 $x < 5$

**Example.1** Let  $x$  be the one part. then  $12-x$  will be another part.

