

Remainder Theorem:-

If a polynomial $f(x)$ of degree $n \geq 1$, n is non negative integer is divided by $(x-a)$ till no x term exists in the remainder. Then $f(a)$ is a remainder.

PROOF: Suppose a polynomial $f(x)$ is divided by $(x-a)$. Then there exists a unique quotient $q(x)$ and a unique remainder R .

$$\text{Dividend} = \text{Divisor} \times \text{quotient} + \text{Remainder}$$

$$f(x) = (x-a)q(x) + R$$

putting $x=a$ we get

$$f(a) = (a-a)q(a) + R = 0 + R = R$$

Factor Theorem:-

The polynomial $(x-a)$ is a factor of polynomial $f(x)$ if and only if $f(a) = 0$ i.e. $(x-a)$ is a factor of $f(x)$ if and only if $x=a$ is a root of polynomial equation $f(x) = 0$.

PROOF: Suppose $q(x)$ is the quotient and R is remainder when a polynomial $f(x)$ is divided by $(x-a)$.

Then by remainder theorem

$$f(x) = (x-a)q(x) + R$$

$$\text{Since } f(a) = 0 \Rightarrow R = 0$$

$$\therefore f(x) = (x-a)q(x)$$

$(x-a)$ is a factor of $f(x)$

Conversely if $(x-a)$ is a factor of $f(x)$ then

$$R = f(a) = 0$$

which proves the theorem

EXERCISE 4.5

Use the remainder theorem to find the remainder

1. Let $f(x) = x^2 + 3x + 7$

Sol. $x-a = x+1 \Rightarrow a = -1$

Remainder = $f(a)$ By Remainder Theorem

$$R = f(-1) = (-1)^2 + 3(-1) + 7 = 1 - 3 + 7 = 5$$

2. Let $f(x) = x^3 - x^2 + 5x + 4$

$$x-a = x-2 \Rightarrow a = 2$$

Remainder = $f(a)$

$$R = f(2) = (2)^3 - (2)^2 + 5(2) + 4 = 8 - 4 + 10 + 4 = 18$$

3. Let $f(x) = 3x^4 + 4x^3 + x - 5$

$$x-a = x+1 \Rightarrow a = -1$$

$$R = f(-1) = 3(-1)^4 + 4(-1)^3 - 1 - 5 = 3 - 4 - 1 - 5 = -7$$

4. $f(x) = x^3 - 2x^2 + 3x + 3$

$$x-a = x-3 \Rightarrow a = 3$$

$$R = f(a) = f(3) = 3^3 - 2(3)^2 + 3(3) + 3 = 27 - 18 + 9 + 3 = 21$$

Use factor theorem.....

5. $x-1, x^2 + 4x - 5$

Sol. Let $f(x) = x^2 + 4x - 5$

$$\text{and } x-a = x-1 \Rightarrow a = 1$$

Remainder = $f(a) = f(1)$

$$R = (1)^2 + 4(1) - 5 = 5 - 5 = 0$$

Hence $(x-1)$ is a factor of $f(x)$ by factor theorem.

6. Let $f(x) = x^3 + x^2 - 7x + 1$

$$x-a = x-2 \Rightarrow a = 2$$

$$R = f(a) = f(2) = 2^3 + 2^2 - 7(2) + 1 = 8 + 4 - 14 + 1 = -1 \neq 0$$

$\Rightarrow (x-2)$ is not a factor of $f(x)$

7. Let $f(w) = 2w^3 + w^2 - 4w + 7$

$$wx-a = w+2 \Rightarrow a = -2$$

MEGA LECTURE

Remainder = $f(w) = f(-2)$
 $R = 2(-2)^3 + (-2)^2 - 4(-2) + 7$
 $= -16 + 4 + 8 + 7 = 3 \neq 0$
 $\Rightarrow (w+2)$ is not a factor of $f(w)$

8. let $f(x) = x^n - a^n$
 where n is +ve integer
 $x - a = x - a \Rightarrow a = a$
 $R = f(x) = f(a) = a^n - a^n = 0$
 $\Rightarrow (x - a)$ is a factor of $f(x)$

9. let $f(x) = x^n + a^n$
 where n is odd integer
 $x - a = x + a \Rightarrow a = -a$
 $R = f(x) = f(-a) = (-a)^n + a^n$
 $= -a^n + a^n = 0$
 $\Rightarrow (x + a)$ is a factor of $f(x)$

10. let $f(x) = x^4 + 2x^3 + kx^2 + 3$
 $k = ? \quad R = 1$
 $x - a = x - 2 \Rightarrow a = 2$
 $R = f(x) = f(2)$
 $1 = (2)^4 + 2(2)^3 + k(2)^2 + 3$
 $1 = 16 + 16 + 4k + 3$
 $\Rightarrow 4k = -34 \Rightarrow k = -\frac{17}{2}$

11. $k = ? \quad R = 14$
Sol. $f(x) = x^3 + 2x^2 + kx + 4$
 $x - a = x - 2 \Rightarrow a = 2$
 $R = f(x) = f(a) = f(2)$
 $14 = (2)^3 + 2(2)^2 + 2k + 4$
 $14 = 8 + 8 + 2k + 4$
 $\Rightarrow 2k = -6 \Rightarrow k = -3$

Use Synthetic division....

12. factorize the polynomial

Sol $f(x) = x^3 + 0x^2 - 7x + 6 \quad x=2$

2	1	0	-7	6
	0	2	4	-6
	1	2	-3	0

Quotient = $x^2 + 2x - 3 \quad R = 0$
 $= x^2 + 3x - x - 3$
 $= x(x+3) - 1(x+3)$
 $= (x+3)(x-1)$

Hence $x^3 - 7x + 6 = (x-2)(x+3)(x-1)$

13. let $f(x) = x^3 + 0x^2 - 28x - 48$
 $x = -4$

-4	1	0	-28	-48
	0	-4	16	48
	1	-4	-12	0

Quotient = $x^2 - 4x - 12$
 $= x^2 - 6x + 2x - 12$
 $= x(x-6) + 2(x-6)$
 $= (x-6)(x+2)$

Hence $x^3 - 28x - 48 = (x+4)(x-6)(x+2)$

14. let $f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$
 $x = 2, \quad x = -3$

2	2	7	-4	-27	-18
	0	4	22	36	-18
	2	11	18	7	0
-3	2	11	18	7	0
	0	-6	-15	-9	
	2	5	3	0	

Quotient = $2x^2 + 5x + 3$
 $= 2x^2 + 2x + 3x + 3$
 $= 2x(x+1) + 3(x+1)$
 $= (x+1)(2x+3)$

Hence $2x^4 + 7x^3 - 4x^2 - 27x - 18 = (x-2)(x+3)(x+1)(2x+3)$

15. Use Synthetic division to find the values of p and q if $x+1$ and $x-2$ are the factors of $f(x) = x^3 + px^2 + qx + 6$

Sol $x - a = x + 1 \Rightarrow a = -1$
 $x - a = x - 2 \Rightarrow a = 2$

By Synthetic division

-1	1	p	q	6
	0	-1	-p+1	-q+p-1
2	1	p-1	q-p+1	-q+p+5
	0	2	2p+2	
	1	p+1	q+p+3	

Since $(x+1)$ and $(x-2)$ are the factors of $f(x)$
 Then Remainder = 0

MEGA LECTURE

$$\begin{aligned}
 -q + p + 5 &= 0 \quad \text{--- (1)} \\
 q + p + 3 &= 0 \quad \text{--- (2)} \\
 \hline
 2p + 8 &= 0 \Rightarrow p = -4
 \end{aligned}$$

put $p = -4$ in (2)
 $\Rightarrow q - 4 + 3 = 0 \Rightarrow q = 1$
 $p = -4 \quad q = 1$

16. Find the values of a and b if -2 and 2 are the roots of $f(x) = x^3 - 4x^2 + ax + b$

Sol. $a = -2 \quad a = 2$

-2	1	-4	a	b
	0	-2	12	-2a-24
2	1	-6	a+12	b-2a-24
	0	2	-8	
	1	-4	a+4	

Since $-2, 2$ are the roots of $f(x)$ then Remainder = 0

$$a + 4 = 0 \Rightarrow a = -4$$

$$b - 2a - 24 = 0$$

$$\Rightarrow b - 2(-4) - 24 = 0$$

$$\Rightarrow b + 8 - 24 = 0$$

$$\Rightarrow b - 16 = 0 \Rightarrow b = 16$$

$$a = -4 \quad b = 16$$

Relation Between Roots &

Coefficients of Quadratic Eq

Quadratic eq, $ax^2 + bx + c = 0$
 formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

let α, β be the roots of the eq

$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha + \beta = \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2b}{2a} = -\frac{b}{a} = \text{Sum of roots}$$

$$\alpha\beta = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$

$$= \frac{(-b)^2 - (\sqrt{b^2 - 4ac})^2}{4a^2}$$

$$= \frac{b^2 - (b^2 - 4ac)}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$$

Product of roots = $\frac{c}{a}$

Formation of an Eq

whose roots are given:

If α, β be the roots of the quadratic equation

then $x = \alpha \quad x = \beta$

$$\Rightarrow x - \alpha = 0 \quad x - \beta = 0$$

Equation becomes

$$(x - \alpha)(x - \beta) = 0$$

$$\Rightarrow x^2 - \beta x - \alpha x + \alpha\beta = 0$$

$$\Rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\Rightarrow x^2 - Sx + P = 0$$

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