**Redical** Another word used for root, The sign $\sqrt{\ }$ is called redical sign. A number placed to the left of the sign shows the type of root

eg. $\sqrt[2]{\ }$, $\sqrt[3]{\ }$, $\sqrt[4]{\ }$, ........, $\sqrt[n]{\ }$

denote Square root, Cube roots, Fourth root, ........, nth roots respectively. Here numbers 2, 3, 4 and n are called index. If there is no number to the left of the sign $\sqrt{\ }$, Then root is called square root.

**Redical Equations** The equations involving redical expressions of the variable are called redical equations.

For example; $\sqrt{2x+8} + \sqrt{x+5} = 7$

$3x^2+2x-\sqrt{3x^2+2x-1}-7=0$

$\sqrt{x^2+2x-3} + \sqrt{x^2+x-8} = \sqrt{5x^2+15x-20}$

are redical equations.

**Extraneous Root** A number obtained in the process of solving an equation, which is actually not a root of the given equation. In other words, we may say that, a root that does not satisfy given equation is called an extraneous root.

★ There are some types of Redical Equations

**Type No. 1**

i) The equation of the form

$l(ax^2+bx)+m\sqrt{ax^2+bx+c}=0$

Questions of this type are

Example 1, Q. No. 1, 2, 10

## Example.1

$$3x^2+15x-2\sqrt{x^2+5x+1}=2$$

$$3(x^2+5x)-2\sqrt{x^2+5x+1}=2$$

put $\sqrt{x^2+5x+1}=y$

$\Rightarrow x^2+5x+1=y^2-1$

Then given equation becomes

$$3(y^2-1)-2y=2$$
$$3y^2-3-2y-2=0$$
$$3y^2-2y-5=0$$
$$3y^2+3y-5y-5=0$$
$$3y(y+1)-5(y+1)=0$$
$$(y+1)(3y-5)=0$$
$$y+1=0, \quad 3y-5=0$$
$$y=-1, \quad y=5/3$$

If $y=-1$ then, If $y=5/3$ then

$$\sqrt{x^2+5x+1}=-1, \quad \sqrt{x^2+5x+1}=5/3$$

Squaring both sides

$$x^2+5x+1=1, \quad x^2+5x+1=\frac{25}{9}$$
$$x^2+5x+1-1=0, \quad 9x^2+45x+9=25$$
$$x^2+5x=0, \quad 9x^2+45x+9-25=0$$
$$x(x+5)=0 \qquad 9x^2+45x-16=0$$
$$x=0,\ x+5=0$$
$$x=-5 \qquad 9x^2-3x+48x-16=0$$
$$3x(3x-1)+16(3x-1)=0$$
$$(3x-1)(3x+16)=0$$
$$3x-1=0, \quad 3x+16=0$$
$$x=1/3, \quad x=-16/3$$

On checking we found that 0 and $-5$ not satisfy given equation, so 0 and $-5$ are extraneous roots which can not be written in solution set.

So solution set is $\{1/3, -16/3\}$

### Type No.2

When given equation is of the form

$$\sqrt{x+a}+\sqrt{x+b}=\sqrt{x+c}$$

such as Example 2, Q. No. 3, 4, 5

## Example.2 $\sqrt{x+8}+\sqrt{x+3}=\sqrt{12x+13}$

Squaring both sides

$$\left(\sqrt{x+8}+\sqrt{x+3}\right)^2=\left(\sqrt{12x+13}\right)^2$$
$$\left(\sqrt{x+8}\right)^2+\left(\sqrt{x+3}\right)^2+2\sqrt{x+8}\sqrt{x+3}=12x+13$$
$$x+8+x+3+2\sqrt{(x+8)(x+3)}=12x+13$$
$$2x+11+2\sqrt{x^2+3x+8x+24}=12x+13$$
$$2\sqrt{x^2+11x+24}=12x-2x+13-11$$
$$2\sqrt{x^2+11x+24}=10x+2$$
$$2\sqrt{x^2+11x+24}=2(5x+1)$$
$$\sqrt{x^2+11x+24}=5x+1$$

Squaring again

$$\left(\sqrt{x^2+11x+24}\right)^2=(5x+1)^2$$
$$x^2+11x+24=25x^2+10x+1$$
$$25x^2-x^2+10x-11x+1-24=0$$
$$24x^2-x-23=0$$
$$24x^2-24x+23x-23=0$$
$$24x(x-1)+23(x-1)=0$$
$$(x-1)(24x+23)=0$$
$$x-1=0, \quad 24x+23=0$$
$$x=1, \quad x=-23/24$$

On checking we found that $-\frac{23}{24}$ is an extraneous root.

Hence solution set is $\{1\}$.

OR S.S. $\{1\}$.

**Type No .3**

When given equation is of the form

$$\sqrt{ax^2+bx+c} + \sqrt{px^2+qx+r} = \sqrt{lx^2+mx+n}$$

such as. Example 3, Q. No. 7,8,9

## Example.3

$$\sqrt{x^2+4x-21} + \sqrt{x^2-x-6} = \sqrt{6x^2-5x-39}$$

Factorizing the expression under radical sign.

$$\sqrt{x^2+7x-3x-21} + \sqrt{x^2+2x-3x-6} = \sqrt{6x^2-18x+13x-39}$$

$$\sqrt{x(x+7)-3(x-7)} + \sqrt{x(x+2)-3(x+2)} = \sqrt{6x(x-3)+13(x-3)}$$

$$\sqrt{(x-3)(x+7)} + \sqrt{(x-3)(x+2)} = \sqrt{(x-3)(6x+13)}$$

$$\sqrt{(x-3)(x+7)} + \sqrt{(x-3)(x+2)} - \sqrt{(x-3)(6x+13)} = 0$$

$$\sqrt{x-3}\left\{\sqrt{x+7}+\sqrt{x+2}-\sqrt{6x+13}\right\} = 0$$

$\sqrt{x-3}=0,\quad \sqrt{x+7}+\sqrt{x+2}-\sqrt{6x+13}=0$

$x-3=0,\quad \sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$

$x=3$ Squaring on both sides

$$\left(\sqrt{x+7}+\sqrt{x+2}\right)^2 = \left(\sqrt{6x+13}\right)^2$$

$$(\sqrt{x+7})^2+(\sqrt{x+2})^2+2\sqrt{x+7}\sqrt{x+2} = 6x+13$$

$$x+7+x+2+2\sqrt{(x+7)(x+2)} = 6x+13$$

$$2x+9+2\sqrt{x^2+2x+7x+14} = 6x+13$$

$$2\sqrt{x^2+9x+14} = 6x-2x+13-9$$

$$2\sqrt{x^2+9x+14} = 4x+4$$

$$\sqrt{x^2+9x+14} = 2x+2$$

Squaring again

$$\left(\sqrt{x^2+9x+14}\right)^2 = (2x+2)^2$$

$$x^2+9x+14 = 4x^2+8x+4$$

$$4x^2-x^2+8x-9x+4-14=0$$

$$3x^2-x-10=0$$

$$3x^2-6x+5x-10=0$$

$$3x(x-2)+5(x-2)=0$$

$$(x-2)(3x+5)=0$$

$x-2=0,\quad 3x+5=0$

$x=2,\quad x=-5/3$

On checking $-5/3$ is to be found as an extraneous root.

So S.S is $\{3,2\}$

**Type No.4**

When given equations are of the form

$$\sqrt{ax^2+bx+c} + \sqrt{px^2+qx+r} = mx+n$$

such as Example.4, Q.6, 11,12

## Example.4

$$\sqrt{3x^2-7x-30} - \sqrt{2x^2-7x-5} = x-5$$

Put $\sqrt{3x^2-7x-30}=a$, $\sqrt{2x^2-7x-5}=b$

Then given equation becomes

$$a-b=x-5 \rightarrow (1)$$

To find the values of $a$ and $b$ we find $a^2-b^2$ as;

$$a^2-b^2=\left(\sqrt{3x^2-7x-30}\right)^2-\left(\sqrt{2x^2-7x-5}\right)^2$$

$$a^2-b^2=3x^2-7x-30-(2x^2-7x-5)$$

$$a^2-b^2=3x^2-7x-30-2x^2+7x+5$$

$$a^2-b^2=x^2-25$$

$$a^2-b^2=(x)^2-(5)^2$$

So $\because a^2-b^2=(a-b)(a+b)$

$$(a-b)(a+b)=(x-5)(x+5) \rightarrow (2)$$

putting value $a-b=x-5$ from eq ① in eq ②

$(x-5)(a+b)=(x-5)(x+5)$

$\Rightarrow a+b=x+5 \rightarrow$ ③

Adding ① and ③

$$\begin{aligned} a-b &= x-5 \\ a+b &= x+5 \\ \hline 2a &= 2x \end{aligned}$$

$\Rightarrow a=x$

putting value of $a$ in equation ①

$x-b=x-5$

$\Rightarrow b=5$

| If $a=x$ Then | If $b=5$ Then |
|---|---|
| $\sqrt{3x^2-7x-30}=x$ | $\sqrt{2x^2-7x-5}=5$ |
| Squaring | Squaring |
| $3x^2-7x-30=x^2$ | $2x^2-7x-5=25$ |
| $3x^2-x^2-7x-30=0$ | $2x^2-7x-5-25=0$ |
| $2x^2-7x-30=0$ | $2x^2-7x-30=0$ |

Solving any one

$2x^2-7x-30=0$

$2x^2-12x+5x-30=0$

$2x(x-6)+5(x-6)=0$

$(x-6)(2x+5)=0$

$x-6=0,\quad 2x+5=0$

$x=6,\quad x=-5/3$

On checking we found $-5/3$ as an extraneous root

So S.S is $\{6\}$.

# EXERCISE 4.3

**Q.1** $3x^2+2x-\sqrt{3x^2+2x-1}=7$

put $\sqrt{3x^2+2x-1}=y$

$\Rightarrow 3x^2+2x-1=y^2$

$3x^2+2x=y^2+1$

Then given equation takes form

$y^2+1-y=7$

$y^2-y+1-7=0$

$y^2-y-6=0$

$y^2+2y-3y-6=0$

$y(y+2)-3(y+2)=0$

$(y+2)(y-3)=0$

$y+2=0,\quad y-3=0$

$y=-2,\quad y=3$

| If $y=-2$ Then, | If $y=3$ Then |
|---|---|
| $\sqrt{3x^2+2x-1}=-2$ | $\sqrt{3x^2+2x-1}=3$ |
| $3x^2+2x-1=4$ | $3x^2+2x-1=9$ |
| $3x^2+2x-1-4=0$ | $3x^2+2x-1-9=0$ |
| $3x^2+2x-5=0$ | $3x^2+2x-10=0$ |
| $3x^2-3x+5x-5=0$ | Using $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ |
| $3x(x-1)+5(x-1)=0$ | $x=\frac{-2\pm\sqrt{(2)^2-4(3)(-10)}}{2(3)}$ |
| $(x-1)(3x+5)=0$ | |
| $x-1=0,\ 3x+5=0$ | |
| $x=1,\ x=-5/3$ | $x=\frac{-2\pm\sqrt{4+120}}{6}$ |

$x=\frac{-2\pm\sqrt{124}}{6}$

$x=\frac{-2\pm\sqrt{4\times31}}{6} \Rightarrow x=\frac{-2\pm2\sqrt{31}}{6}$

$x=\frac{2\left(-1\pm\sqrt{31}\right)}{6} \rightarrow x=\frac{-1\pm\sqrt{31}}{3}$

On checking we found that 1 and $-5/3$ are extraneous roots. Hence

S.S is $\left\{\frac{-1\pm\sqrt{31}}{3}\right\}$.

**Q.2** $x^2 - \frac{x}{2} - 7 = x - 3\sqrt{2x^2-3x+2}$

Multiplying by 2

$2x^2 - x - 14 = 2x - 6\sqrt{2x^2-3x+2}$

$2x^2 - x - 2x - 14 + 6\sqrt{2x^2-3x+2} = 0$

$2x^2 - 3x - 14 + 6\sqrt{2x^2-3x+2} = 0$

put $\sqrt{2x^2-3x+2} = y$

$\Rightarrow 2x^2 - 3x + 2 = y^2$

$2x^2 - 3x = y^2 - 2$

Given equation takes form

$y^2 - 2 - 14 + 6y = 0$

$y^2 + 6y - 16 = 0$

$y^2 - 2y + 8y - 16 = 0$

$y(y-2) + 8(y-2) = 0$

$(y-2)(y+8) = 0$

$y - 2 = 0, \quad y + 8 = 0$

$y = 2, \quad y = -8$

If $y = 2$ then

$\sqrt{2x^2-3x+2} = 2$

Squaring

$2x^2 - 3x + 2 = 4$

$2x^2 - 3x + 2 - 4 = 0$

$2x^2 - 3x - 2 = 0$

$2x^2 + x - 4x - 2 = 0$

$x(2x+1) - 2(2x+1) = 0$

$(x-2)(2x+1) = 0$

$x - 2 = 0, \quad 2x + 1 = 0$

$x = 2, \quad x = -\frac{1}{2}$

If $y = -8$ then

$\sqrt{2x^2-3x+2} = -8$

Squaring

$2x^2 - 3x + 2 = 64$

$2x^2 - 3x + 2 - 64 = 0$

$2x^2 - 3x - 62 = 0$

Using $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-62)}}{2(2)}$

$x = \frac{3 \pm \sqrt{9+496}}{4}$

$x = \frac{3 \pm \sqrt{505}}{4}$

On checking we found $\frac{3 \pm \sqrt{505}}{4}$ is an extraneous root.

Hence S.S $\{2, -\frac{1}{2}\}$

**Q.3** $\sqrt{2x+8} + \sqrt{x+5} = 7$

Squaring both sides

$(\sqrt{2x+8} + \sqrt{x+5})^2 = (7)^2$

$(\sqrt{2x+8})^2 + (\sqrt{x+5})^2 + 2\sqrt{2x+8}\sqrt{x+5} = 49$

$2x + 8 + x + 5 + 2\sqrt{(2x+8)(x+5)} = 49$

$3x + 13 + 2\sqrt{2x^2+10x+8x+40} = 49$

$2\sqrt{2x^2+18x+40} = 49 - 13 - 3x$

$2\sqrt{2x^2+18x+40} = 36 - 3x$

Squaring both sides again

$4(2x^2+18x+40) = (36-3x)^2$

$8x^2 + 72x + 160 = 1296 + 9x^2 - 216x$

$9x^2 - 8x^2 - 216x - 72x + 1296 - 160 = 0$

$x^2 - 288x + 1136 = 0$

$x^2 - 4x - 284x + 1136 = 0$

$x(x-4) - 284(x-4) = 0$

$(x-4)(x-284) = 0$

$x - 4 = 0, \quad x - 284 = 0$

$x = 4, \quad x = 284$

On checking we found 284 is an extraneous root. So

S.S is $\{4\}$

**Q.4** $\sqrt{3x+4} = 2 + \sqrt{2x-4}$

$\sqrt{3x+4} - \sqrt{2x-4} = 2$

Squaring both sides

$(\sqrt{3x+4} - \sqrt{2x-4})^2 = (2)^2$

$(\sqrt{3x+4})^2 + (\sqrt{2x-4})^2 - 2\sqrt{3x+4}\sqrt{2x-4} = 4$

$3x + 4 + 2x - 4 - 2\sqrt{(3x+4)(2x-4)} = 4$

$5x - 2\sqrt{6x^2-12x+8x-16} = 4$

$$-2\sqrt{6x^2-4x-16} = 4-5x$$

Squaring again

$$4(6x^2-4x-16) = (4-5x)^2$$

$$24x^2-16x-64 = 16+25x^2-40x$$

$$25x^2-24x^2-40x+16x+16+64=0$$

$$x^2-24x+80=0$$

$$x^2-4x-20x+80=0$$

$$x(x-4)-20(x-4)=0$$

$$(x-4)(x-20)=0$$

$$x-4=0, \quad x-20=0$$

$$x=4, \quad x=20$$

On checking we found that no root is an extraneous root.

Hence S.S $\{4, 20\}$

**Q.5** $\sqrt{x+7} + \sqrt{x+2} = \sqrt{6x+13}$

Squaring on both sides.

$$\left(\sqrt{x+7}+\sqrt{x+2}\right)^2 = \left(\sqrt{6x+13}\right)^2$$

$$(\sqrt{x+7})^2 + (\sqrt{x+2})^2 + 2\sqrt{x+7}\sqrt{x+2} = 6x+13$$

$$x+7+x+2+2\sqrt{(x+7)(x+2)} = 6x+13$$

$$2x+9+2\sqrt{x^2+2x+7x+14} = 6x+13$$

$$2\sqrt{x^2+9x+14} = 6x-2x+13-9$$

$$2\sqrt{x^2+9x+14} = 4x+4$$

$$\sqrt{x^2+9x+14} = 2x+2$$

Squaring again

$$\left(\sqrt{x^2+9x+14}\right)^2 = (2x+2)^2$$

$$x^2+9x+14 = 4x^2+8x+4$$

$$4x^2-x^2+8x-9x+4-14=0$$

$$3x^2-x-10=0$$

$$3x^2-6x+5x-10=0$$

$$3x(x-2)+5(x-2)=0$$

$$(x-2)(3x+5)=0$$

$$x-2=0, \quad 3x+5=0$$

$$x=2, \quad x=-5/3$$

On checking we found that $-5/3$ is an extraneous root. Hence S.S $\{2\}$

**Q6** $\sqrt{x^2+x+1} - \sqrt{x^2+x-1} = 1$

put $\sqrt{x^2+x+1} = a$, $\sqrt{x^2+x-1} = b$

Given equation takes form

$$a-b=1 \rightarrow (1)$$

To find $a$ and $b$ we find $a^2-b^2$ as

$$a^2-b^2 = \left(\sqrt{x^2+x+1}\right)^2 - \left(\sqrt{x^2+x-1}\right)^2$$

$$a^2-b^2 = x^2+x+1-(x^2+x-1)$$

$$a^2-b^2 = x^2+x+1-x^2-x+1$$

$$a^2-b^2 = 2 \qquad \because a^2-b^2=(a-b)(a+b)$$

$$(a-b)(a+b)=2 \rightarrow (2)$$

putting value of $a-b$ from (1) in (2)

$$1(a+b)=2 \Rightarrow a+b=2 \rightarrow (3)$$

Adding (1) and (3)

$$a-b=1$$
$$a+b=2$$
$$2a=3 \Rightarrow a=3/2$$

putting values of $a$ in (1)

$$\frac{3}{2}-1=b \Rightarrow b=\frac{3-2}{2}=1/2$$

| If $a=3/2$ then | If $b=1/2$ then |
|---|---|
| $\sqrt{x^2+x+1}=3/2$ | $\sqrt{x^2+x-1}=1/2$ |
| $2\sqrt{x^2+x+1}=3$ | $2\sqrt{x^2+x-1}=1$ |
| Squaring | Squaring |
| $4(x^2+x+1)=9$ | $4(x^2+x-1)=1$ |
| $4x^2+4x+4-9=0$ | $4x^2+4x-4-1=0$ |
| $4x^2+4x-5=0$ | $4x^2+4x-5=0$ |

Solving any one

$$4x^2+4x-5=0$$

Using $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$$x=\frac{-4\pm\sqrt{(4)^2-4(4)(-5)}}{2(4)}$$

$$x=\frac{-4\pm\sqrt{16+80}}{8} \Rightarrow x=\frac{-4\pm\sqrt{96}}{8}$$

$$x=\frac{-4\pm\sqrt{16\times6}}{8} \rightarrow x=\frac{-4\pm4\sqrt{6}}{8}$$

$$x=\frac{4(-1\pm\sqrt{6})}{8} \rightarrow x=\frac{-1\pm\sqrt{6}}{2}$$

On checking we found that no root is an extraneous root. So

S.S $\left\{\dfrac{-1\pm\sqrt{6}}{2}\right\}$

## Q.7

$$\sqrt{x^2+2x-3}+\sqrt{x^2+7x-8}=\sqrt{5(x^2+3x-4)}$$

Factorizing the expressions under the radical sign.

$$\sqrt{x^2-x+3x-3}+\sqrt{x^2-x+8x-8}=\sqrt{5(x^2-x+4x-4)}$$

$$\sqrt{x(x-1)+3(x-1)}+\sqrt{x(x-1)+8(x-1)}=\sqrt{5[x(x-1)+4(x-1)]}$$

$$\sqrt{(x-1)(x+3)}+\sqrt{(x-1)(x+8)}=\sqrt{5(x-1)(x+4)}$$

$$\sqrt{(x-1)(x+3)}+\sqrt{(x-1)(x+8)}-\sqrt{(x-1)(x+4)5}=0$$

$$\sqrt{x-1}\left\{\sqrt{x+3}+\sqrt{x+8}-\sqrt{5(x+4)}\right\}=0$$

$\sqrt{x-1}=0$, $\quad \sqrt{x+3}+\sqrt{x+8}-\sqrt{5(x+4)}=0$

$x-1=0$, $\quad \sqrt{x+3}+\sqrt{x+8}=\sqrt{5(x+4)}$

$x=1$, $\quad$ Squaring both sides

$$\left(\sqrt{x+3}+\sqrt{x+8}\right)^2=\left(\sqrt{5(x+4)}\right)^2$$

$$(\sqrt{x+3})^2+(\sqrt{x+8})^2+2\sqrt{x+3}\sqrt{x+8}=5(x+4)$$

$$x+3+x+8+2\sqrt{(x+3)(x+8)}=5x+20$$

$$2x+11+2\sqrt{x^2+8x+3x+24}=5x+20$$

$$2\sqrt{x^2+11x+24}=5x-2x+20-11$$

$$2\sqrt{x^2+11x+24}=3x+9$$

Squaring again

$$4(x^2+11x+24)=(3x+9)^2$$

$$4x^2+44x+96=9x^2+54x+81$$

$$9x^2-4x^2+54x-44x+81-96=0$$

$$5x^2+10x-15=0$$

Dividing by 5

$$x^2+2x-3=0$$

$$x^2-x+3x-3=0$$

$$x(x-1)+3(x-1)=0$$

$$(x-1)(x+3)=0$$

$x-1=0$, $\quad x+3=0$

$x=1$, $\quad x=-3$

On checking we found that no root is extraneous root. So

S.S is $\{1,-3\}$.

## Q.8

$$\sqrt{2x^2-5x-3}+3\sqrt{2x+1}=\sqrt{2x^2+25x+12}$$

Factorizing the expressions under radical sign.

$$\sqrt{2x^2+x-6x-3}+3\sqrt{2x+1}=\sqrt{2x^2+x+24x+12}$$

$$\sqrt{x(2x+1)-3(2x+1)}+3\sqrt{2x+1}=\sqrt{x(2x+1)+12(2x+1)}$$

$$\sqrt{(2x+1)(x-3)}+3\sqrt{2x+1}-\sqrt{(2x+1)(x+12)}=0$$

$$\sqrt{2x+1}\left\{\sqrt{x-3}+3-\sqrt{x+12}\right\}=0$$

$\sqrt{2x+1}=0$, $\quad \sqrt{x-3}+3-\sqrt{x+12}=0$

$2x+1=0$, $\quad \sqrt{x-3}+3=\sqrt{x+12}$

$x=-\frac{1}{2}$, $\quad$ Squaring

$$\left(\sqrt{x-3}+3\right)^2=\left(\sqrt{x+12}\right)^2$$

$$(\sqrt{x-3})^2+(3)^2+2(3)\sqrt{x-3}=x+12$$

$$x-3+9+6\sqrt{x-3}=x+12$$

$$x+6+6\sqrt{x-3}=x+12$$

$$6\sqrt{x-3}=x-x+12-6$$

$$6\sqrt{x-3}=6 \rightarrow \sqrt{x-3}=1$$

$x-3=1 \Rightarrow x=4$

On checking we found no root is an extraneous root. Hence

S.S is $\{-\frac{1}{2}, 4\}$

**Q.9**

$\sqrt{3x^2-5x+2}+\sqrt{6x^2-11x+5}=\sqrt{5x^2-9x+4}$

Factorizing the expressions under radical sign.

$\sqrt{3x^2-3x-2x+2}+\sqrt{6x^2-6x-5x+5}=\sqrt{5x^2-5x-4x+4}$

$\sqrt{3x(x-1)-2(x-1)}+\sqrt{6x(x-1)-5(x-1)}=\sqrt{5x(x-1)-4(x-1)}$

$\sqrt{(x-1)(3x-2)}+\sqrt{(x-1)(6x-5)}=\sqrt{(x-1)(5x-4)}$

$\sqrt{(x-1)(3x-2)}+\sqrt{(x-1)(6x-5)}-\sqrt{(x-1)(5x-4)}=0$

$\sqrt{x-1}\{\sqrt{3x-2}+\sqrt{6x-5}-\sqrt{5x-4}\}=0$

$\sqrt{x-1}=0$, $\quad \sqrt{3x-2}+\sqrt{6x-5}-\sqrt{5x-4}=0$

$x-1=0$, $\quad \sqrt{3x-2}+\sqrt{6x-5}=\sqrt{5x-4}$

$x=1$, $\quad$ Squaring both sides

$(\sqrt{3x-2}+\sqrt{6x-5})^2=(\sqrt{5x-4})^2$

$(\sqrt{3x-2})^2+(\sqrt{6x-5})^2+2\sqrt{3x-2}\sqrt{6x-5}=5x-4$

$3x-2+6x-5+2\sqrt{(3x-2)(6x-5)}=5x-4$

$9x-7+2\sqrt{18x^2-15x-12x+10}=5x-4$

$2\sqrt{18x^2-27x+10}=5x-9x-4+7$

$2\sqrt{18x^2-27x+10}=3-4x$

Squaring again

$4(18x^2-27x+10)=(3-4x)^2$

$72x^2-108x+40=9+16x^2-24x$

$72x^2-16x^2-108x+24x+40-9=0$

$56x^2-84x+31=0$

Using $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{-(-84)\pm\sqrt{(-84)^2-4(56)(31)}}{2(56)}$

$x=\dfrac{84\pm\sqrt{7056-6944}}{112} \Rightarrow x=\dfrac{84\pm\sqrt{112}}{112}$

$x=\dfrac{84\pm\sqrt{16\times 7}}{112}$

$x=\dfrac{84\pm4\sqrt{7}}{112} \Rightarrow x=\dfrac{4(21\pm\sqrt{7})}{112}$

$x=\dfrac{21\pm\sqrt{7}}{28}$

On checking we found that no root is an extraneous root. So

S.S is $\left\{\dfrac{21\pm\sqrt{7}}{28}\right\}$

**Q.10**

$(x+4)(x+1)=\sqrt{x^2+2x-15}+3x+31$

$x^2+x+4x+4=\sqrt{x^2+2x-15}+3x+31$

$x^2+5x+4=\sqrt{x^2+2x-15}+3x+31$

$x^2+5x-3x+4-31-\sqrt{x^2+2x-15}=0$

$x^2+2x-27-\sqrt{x^2+2x-15}=0$

put $\sqrt{x^2+2x-15}=y$

$\Rightarrow x^2+2x-15=y^2$

$x^2+2x=y^2+15$

Given equation takes form

$y^2+15-27-y=0$

$y^2-y-12=0$

$y^2+3y-4y-12=0$

$y(y+3)-4(y+3)=0$

$(y-4)(y+3)=0$

$y-4=0$, $\quad y+3=0$

$y=4$, $\quad y=-3$

If $y=4$ then

$\sqrt{x^2+2x-15}=4$

$x^2+2x-15=16$

$x^2+2x-15-16=0$

$x^2+2x-31=0$

$a=1$, $b=2$, $c=-31$

Using $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

If $y=-3$ Then

$\sqrt{x^2+2x-15}=-3$

$x^2+2x-15=9$

$x^2+2x-15-9=0$

$x^2+2x-24=0$

$x^2-4x+6x-24=0$

$x(x-4)+6(x-4)=0$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-31)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4+124}}{2}$$

$$x = \frac{-2 \pm \sqrt{128}}{2}$$

$$x = \frac{-2 \pm 8\sqrt{2}}{2}$$

$(x-4)(x+6)=0$

$x-4=0,\ x+6=0$

$x=4,\ x=-6$

$\Rightarrow x = \frac{-2 \pm \sqrt{64\times 2}}{2}$

$\Rightarrow x = \frac{2(-1 \pm 4\sqrt{2})}{2}$

$$x = -1 \pm 4\sqrt{2}$$

On checking we found that 4 and $-6$ are extraneous roots. Hence

S.S is $\{-1 \pm 4\sqrt{2}\}$

## Q.11

$$\sqrt{3x^2-2x+9} + \sqrt{3x^2-2x-4} = 13$$

put $\sqrt{3x^2-2x+9} = a$, $\sqrt{3x^2-2x-4} = b$

Given equation takes form

$a+b = 13 \rightarrow (1)$

To find $a$ and $b$ we find $a^2-b^2$ as

$a^2-b^2 = (\sqrt{3x^2-2x+9})^2 - (\sqrt{3x^2-2x-4})^2$

$a^2-b^2 = 3x^2-2x+9-(3x^2-2x-4)$

$a^2-b^2 = 3x^2-2x+9-3x^2+2x+4$

$a^2-b^2 = 13 \qquad \because a^2-b^2=(a+b)(a-b)$

$(a+b)(a-b) = 13 \rightarrow (2)$

putting value of $a+b$ from (1) in (2)

$13(a-b) = 13 \Rightarrow a-b = 1 \rightarrow (3)$

Adding (1) and (3)

$$\begin{aligned} a+b &= 13 \\ a-b &= 1 \\ \hline 2a &= 14 \Rightarrow a = 7 \end{aligned}$$

putting value of $a$ in (1)

$7+b = 13 \Rightarrow b=6$

$a=7$ then $\qquad$ If $b=6$ then

$\sqrt{3x^2-2x+9} = 7 \qquad \sqrt{3x^2-2x-4} = 6$

$3x^2-2x+9 = 49 \qquad 3x^2-2x-4 = 36$

$3x^2-2x+9-49=0 \qquad 3x^2-2x-4-36=0$

$3x^2-2x-40=0 \qquad 3x^2-2x-40=0$

Solving any one

$3x^2-2x-40=0$

$3x^2-12x+10x-40=0$

$3x(x-4)+10(x-4)=0$

$(x-4)(3x+10)=0$

$x-4=0, \qquad 3x+10=0$

$x=4, \qquad x=-10/3$

On checking we found no root is extraneous root. i.e.

S.S is $\{4, -10/3\}$

## Q.12

$$\sqrt{5x^2+7x+2} - \sqrt{4x^2+7x+18} = x-4$$

put $\sqrt{5x^2+7x+2} = a$, $\sqrt{4x^2+7x+18} = b$

Given equation takes form

$a-b = x-4 \rightarrow (1)$

To find $a$ and $b$ we find $a^2-b^2$ as

$a^2-b^2 = (\sqrt{5x^2+7x+2})^2 - (\sqrt{4x^2+7x+18})^2$

$a^2-b^2 = 5x^2+7x+2-(4x^2+7x+18)$

$a^2-b^2 = 5x^2+7x+2-4x^2-7x-18$

$a^2-b^2 = x^2-16 \qquad \because a^2-b^2=(a-b)(a+b)$

$a^2-b^2 = (x)^2-(4)^2$

$(a-b)(a+b) = (x-4)(x+4) \rightarrow (2)$

putting value of $a-b$ from (1) in (2)

$(x-4)(a+b) = (x-4)(x+4)$

$\Rightarrow a+b = x+4 \rightarrow (3)$

Adding eq (1) and (3)

$$\begin{aligned} a-b &= x-4 \\ a+b &= x+4 \\ \hline 2a &= 2x \Rightarrow a = x \end{aligned}$$

putting value of $a$ in (1)

$x-b = x-4 \Rightarrow b=4$

If $a = x$ Then

$$\sqrt{5x^2+7x+2} = x$$
$$5x^2+7x+2 = x^2$$
$$5x^2-x^2+7x+2 = 0$$
$$4x^2+7x+2 = 0$$

If $b = 4$ Then

$$\sqrt{4x^2+7x+18} = 4$$
$$4x^2+7x+18 = 16$$
$$4x^2+7x+18-16 = 0$$
$$4x^2+7x+2 = 0$$

Solving any one

$$4x^2+7x+2 = 0$$

Using $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$x = \frac{-7 \pm \sqrt{(7)^2-4(4)(2)}}{2(4)}$$

$$x = \frac{-7 \pm \sqrt{49-32}}{8} \Rightarrow x = \frac{-7 \pm \sqrt{17}}{8}$$

On checking we found that no root is extraneous root. Hence

S.S is $\left\{ \dfrac{-7 \pm \sqrt{17}}{8} \right\}$