

$$x = \frac{-(a+2b+c) \pm \sqrt{(a-c)^2}}{2(a+b)}$$

$$x = \frac{-(a+2b+c) \pm (a-c)}{2(a+b)}$$

$$x = \frac{-(a+2b+c)+a-c}{2(a+b)}, \quad x = \frac{-(a+2b+c)-a+c}{2(a+b)}$$

$$x = \frac{-a-2b-c+a-c}{2(a+b)}, \quad x = \frac{-a-2b-c-a+c}{2(a+b)}$$

$$x = \frac{-2b-2c}{2(a+b)}, \quad x = \frac{-2b-2a}{2(a+b)}$$

$$x = \frac{-2(b+c)}{2(a+b)}, \quad x = \frac{-2(a+b)}{2(a+b)}$$

$$x = \frac{-(b+c)}{a+b}, \quad x = -1$$

$$\left\{ -\frac{(b+c)}{a+b}, -1 \right\}$$

Equations Reducible to Quadratic Form

Some times we come across equations of type which do not look to be quadratic, but they can be reduced to the quadratic form. There are such five types.

Type No.1

When given equation is of the form

$$ax^{2n} + bx^n + c = 0$$

$$\text{OR. } a(x^n)^2 + bx^n + c = 0$$

We put $x^n = y$

Following questions are of Type No.1

Example. No. 1, Q. No. 1, 2, 3, 4, 5

Example 1

$$x^{1/2} - x^{1/4} - 6 = 0$$

We may write as,

$$x^{1/4 \times 2} - x^{1/4} - 6 = 0$$

$$(x^{1/4})^2 - x^{1/4} - 6 = 0 \quad \because a^{m \times n} = (a^m)^n$$

Put $x^{1/4} = y$ then

$$y^2 - y - 6 = 0$$

Factorizing,

$$y^2 + 2y - 3y - 6 = 0$$

$$y(y+2) - 3(y+2) = 0$$

$$(y+2)(y-3) = 0$$

$$y+2=0, \quad y-3=0$$

$$y = -2, \quad y = 3$$

If $y = -2$, If $y = 3$

Then $x^{1/4} = -2$ Then $x^{1/4} = 3$

$$(x^{1/4})^4 = (-2)^4, \quad (x^{1/4})^4 = (3)^4$$

$$x = 16, \quad x = 81$$

Type No.2

When given equation is of the form

$$(x+a)(x+b)(x+c)(x+d) = k$$

where $a+c = b+d$

Then we re-arrange given equation as

$$\{(x+a)(x+c)\} \cdot \{(x+b)(x+d)\} = k$$

In this way first two terms become same we put them y and solve further

Questions of such type are as;

Example. 2, Q. No. 6, 7, 8, 9, 10, 11, 12, 13

Example 2

$$(x-7)(x-3)(x+1)(x+5) - 1680 = 0$$

$$(x-7)(x-3)(x+1)(x+5) = 1680$$

Re-arranging

$$(x+1)(x-3) \cdot (x+5)(x-7) = 1680$$

$$(x^2 - 3x + x - 3) \cdot (x^2 - 7x + 5x - 35) = 1680$$

$$(x^2 - 2x - 3)(x^2 - 2x - 35) = 1680$$

Put $x^2 - 2x = y$

$$\text{Then } (y-3)(y-35) = 1680$$

$$y^2 - 35y - 3y + 105 = 1680$$

MEGA LECTURE

$$y^2 - 38y + 105 - 1680 = 0$$

$$y^2 - 38y - 1575 = 0$$

Using $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$y = \frac{-(-38) \pm \sqrt{(-38)^2 - 4(1)(-1575)}}{2(1)}$$

$$y = \frac{38 \pm \sqrt{1444 + 6300}}{2}$$

$$y = \frac{38 \pm \sqrt{7744}}{2} \Rightarrow y = \frac{38 \pm 88}{2}$$

$$y = \frac{38+88}{2}, \quad y = \frac{38-88}{2}$$

$$y = \frac{126}{2}, \quad y = \frac{-50}{2}$$

$$y = 63, \quad y = -25$$

if $y = 63$ Then, if $y = -25$ Then

$$x^2 - 2x = 63$$

$$x^2 - 2x - 63 = 0$$

$$x^2 + 7x - 9x - 63 = 0$$

$$x(x+7) - 9(x+7) = 0$$

$$(x+7)(x-9) = 0$$

$$x+7=0, \quad x-9=0$$

$$x = -7, \quad x = 9$$

$$x^2 - 2x = -25$$

$$x^2 - 2x + 25 = 0$$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(25)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 - 100}}{2}$$

$$x = \frac{2 \pm \sqrt{-96}}{2} \Rightarrow x = \frac{2 \pm \sqrt{16 \times 6 \times (-1)}}{2}$$

$$x = \frac{2 \pm 4\sqrt{6}\sqrt{-1}}{2} \Rightarrow x = \frac{2 \pm 2\sqrt{6}i}{2}$$

$$x = 1 \pm 2\sqrt{6}i \quad \because \sqrt{-1} = i$$

$$\{-7, 9, 1 \pm 2\sqrt{6}i\}$$

Type No.3

When given equation has variable as an exponent, just like Example 3, 4, Q No. 14

15, 16, 17.

Example 3

$$2^{2x} - 3 \cdot 2^{x+2} + 32 = 0$$

$$2^{2x} - 3 \cdot 2^x \cdot 2^2 + 32 = 0$$

$$(2^x)^2 - 3 \cdot 2^x \cdot 4 + 32 = 0$$

$$\left(\frac{x}{2}\right)^2 - 12 \cdot 2^x + 32 = 0$$

put $2^x = y$

Then $y^2 - 12y + 32 = 0$

$$y^2 - 4y - 8y + 32 = 0$$

$$y(y-4) - 8(y-4) = 0$$

$$(y-4)(y-8) = 0$$

$$\Rightarrow y-4=0, \quad y-8=0$$

$$y = 4, \quad y = 8$$

if $y = 4$ Then

$$2^x = 4$$

$$2^x = 2^2$$

$$\Rightarrow x = 2$$

if $y = 8$ Then

$$2^x = 8$$

$$2^x = 2^3$$

$$\Rightarrow x = 3$$

$$\{2, 3\}$$

Example 4

$$4^{1+x} + 4^{1-x} = 10$$

$$4 \cdot 4^x + 4 \cdot 4^{-x} = 10$$

$$4 \cdot 4^x + 4 \cdot \frac{1}{4^x} = 10$$

put $4^x = y$ Then

$$4y + 4 \cdot \frac{1}{y} = 10$$

$$[4y^2] + 4 = 10y$$

$$4y^2 - 10y + 4 = 0$$

$$2y^2 - 5y + 2 = 0$$

Factorizing $2y^2 - y - 4y + 2 = 0$

$$y(2y-1) - 2(2y-1) = 0$$

$$(y-2)(2y-1) = 0$$

$$y-2=0, \quad 2y-1=0$$

$$\Rightarrow y = 2, \quad y = \frac{1}{2}$$

if $y = 2$ Then

$$4^x = 2$$

$$\left(\frac{2}{2}\right)^x = 2$$

$$2^{2x} = 2^1$$

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

if $y = \frac{1}{2}$ Then

$$4^x = \frac{1}{2}$$

$$\left(\frac{2}{2}\right)^x = \frac{1}{2}$$

$$2^{2x} = 2^{-1}$$

$$\text{and } 2x = -1 \quad \left\{ \frac{1}{2}, -\frac{1}{2} \right\}$$

$$\text{and } x = -\frac{1}{2}$$

Type No.4

When given equation remains unchanged by replacing x with $1/x$, such equations are called reciprocal equations such as Example 5, Q. No. 18, 19, 20, 21, 22, 23, 24

Example 5

$$x^4 - 3x^3 + 4x^2 - 3x + 1 = 0$$

Dividing by x^2

$$\frac{x^4}{x^2} - \frac{3x^3}{x^2} + \frac{4x^2}{x^2} - \frac{3x}{x^2} + \frac{1}{x^2} = \frac{0}{x^2}$$

$$x^2 - 3x + 4 - \frac{3}{x} + \frac{1}{x^2} = 0$$

$$x^2 + \frac{1}{x^2} - 3x - \frac{3}{x} + 4 = 0$$

$$x^2 + \frac{1}{x^2} - 3\left(x + \frac{1}{x}\right) + 4 = 0$$

put $x + \frac{1}{x} = y$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = y^2 \text{ or } x^2 + \frac{1}{x^2} + 2 = y^2$$

$$\Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$$

Then given equation takes form

$$y^2 - 2 - 3y + 4 = 0$$

$$y^2 - 3y + 2 = 0$$

$$y^2 - y - 2y + 2 = 0$$

$$y(y-1) - 2(y-1) = 0$$

$$(y-1)(y-2) = 0$$

$$y-1=0, y-2=0$$

$$y=1, y=2$$

if $y=1$ then

$$x + \frac{1}{x} = 1$$

$$x^2 + 1 = x$$

$$x^2 - x + 1 = 0$$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1-4}}{2}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

if $y=2$ then

$$x + \frac{1}{x} = 2$$

$$x^2 + 1 = 2x$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x-1=0$$

$$x=1$$

$$\left\{ 1, \frac{1 \pm \sqrt{-3}}{2} \right\}$$

EXERCISE.4.2

Q.1

$$x^4 - 6x^2 + 8 = 0$$

$$(x^2)^2 - 6x^2 + 8 = 0$$

put $x^2 = y$ then

$$y^2 - 6y + 8 = 0$$

$$y^2 - 2y - 4y + 8 = 0$$

$$y(y-2) - 4(y-2) = 0$$

$$(y-2)(y-4) = 0$$

$$y-2=0, y-4=0$$

$$y=2, y=4$$

if $y=2$ then if $y=4$ then

$$x^2 = 2$$

$$x^2 = 4$$

$$x = \pm\sqrt{2}$$

$$x = \pm\sqrt{4}$$

$$\left\{ \pm\sqrt{2}, \pm 2 \right\}$$

$$x = \pm 2$$

Q.2

$$x^2 - 10 = 3x^{-1}$$

$$x^2 - 3x^{-1} - 10 = 0$$

$$(x^1)^2 - 3x^{-1} - 10 = 0$$

put $x^{-1} = y$ then

$$y^2 - 3y - 10 = 0$$

$$y^2 + 2y - 5y - 10 = 0$$

$$y(y+2) - 5(y+2) = 0$$

$$(y-5)(y+2) = 0$$

$$y-5=0, y+2=0$$

$$y=5, y=-2$$

if $y=5$ then

$$x^{-1} = 5$$

$$\frac{1}{x} = 5$$

$$\Rightarrow x = 1/5$$

if $y=-2$ then

$$x^{-1} = -2$$

$$\frac{1}{x} = -2$$

$$x = -1/2$$

$$\left\{ 1/5, -1/2 \right\}$$

Q.3 $x^6 - 9x^3 + 8 = 0$

$$(x^3)^2 - 9x^3 + 8 = 0$$

put $x^3 = y$ then

$$y^2 - 9y + 8 = 0$$

$$y^2 - y - 8y + 8 = 0$$

$$y(y-1) - 8(y-1) = 0$$

$$(y-1)(y-8) = 0$$

$$y-1=0, y-8=0$$

$$y=1, y=8$$

If $y=1$ then

$$x^3 = 1$$

$$x^3 - 1 = 0$$

$$x^3 - (1)^3 = 0$$

$$\therefore a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$(x-1)(x^2+x+1)=0$$

$$x-1=0, x^2+x+1=0$$

$$x=1, x^2+x+1=0$$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$x = \frac{-1 \pm \sqrt{-3}}{2}$$

If $y=8$ then

$$x^3 = 8$$

$$x^3 - 8 = 0$$

$$x^3 - (2)^3 = 0$$

$$(x-2)(x^2+2x+4)=0$$

$$x-2=0, x^2+2x+4=0$$

$$x=2, x^2+2x+4=0$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4-16}}{2}$$

$$x = \frac{-2 \pm \sqrt{-12}}{2}$$

$$x = \frac{-2 \pm \sqrt{4} \sqrt{-3}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{-3}}{2}$$

$$x = 2 \left(\frac{-1 \pm \sqrt{-3}}{2} \right)$$

$$x = -1 \pm \sqrt{-3}$$

$\left\{ 1, 2, \frac{-1 \pm \sqrt{-3}}{2}, -1 \pm \sqrt{-3} \right\}$

Q.4 $8x^6 - 19x^3 - 27 = 0$

$$8(x^3)^2 - 19x^3 - 27 = 0$$

put $x^3 = y$ Then

$$8y^2 - 19y - 27 = 0$$

$$8y^2 + 8y - 27y - 27 = 0$$

$$8y(y+1) - 27(y+1) = 0$$

$$(y+1)(8y-27) = 0$$

$$y+1=0, 8y-27=0$$

$$y=-1, y=27/8$$

If $y=-1$ then

$$x^3 = -1$$

$$x^3 + 1 = 0$$

$$x^3 + (1)^3 = 0$$

$$(x+1)(x^2-x+1)=0$$

$$\therefore a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$\therefore a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$(x+1)(x^2-x+1)=0$$

$$\text{Either } x+1=0$$

$$\Rightarrow x = -1$$

$$\text{OR } x^2 - x + 1 = 0$$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1-4}}{2}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

$$x = \frac{-6 \pm 6\sqrt{-3}}{8}$$

$$x = 6 \left(\frac{-1 \pm \sqrt{-3}}{8} \right)$$

$$\left\{ -1, \frac{3}{2}, \frac{1 \pm \sqrt{-3}}{2}, 3 \left(\frac{-1 \pm \sqrt{-3}}{4} \right) \right\}$$

$$(2x-3)(4x^2+6x+9)=0$$

$$\text{Either } 2x-3=0$$

$$\Rightarrow x = 3/2$$

$$\text{OR } 4x^2 + 6x + 9 = 0$$

$$x = \frac{-6 \pm \sqrt{(6)^2 - 4(4)(9)}}{2(4)}$$

$$x = \frac{-6 \pm \sqrt{36-144}}{8}$$

$$x = \frac{-6 \pm \sqrt{-108}}{8}$$

$$x = \frac{-6 \pm \sqrt{36 \times (-3)}}{8}$$

$$\Rightarrow x = 3 \left(\frac{-1 \pm \sqrt{-3}}{4} \right)$$

13

Q.5 $x^{1/5} + 8 = 6x^{1/5}$

$$x^{1/5} - 6x^{1/5} + 8 = 0$$

$$(x^{1/5})^2 - 6x^{1/5} + 8 = 0$$

put $x^{1/5} = y$ Then

$$y^2 - 6y + 8 = 0$$

$$y^2 - 2y - 4y + 8 = 0$$

$$y(y-2) - 4(y-2) = 0$$

$$(y-2)(y-4) = 0$$

$$y-2=0, y-4=0$$

if $y=2$ Then if $y=4$ Then

$$x^{1/5} = 2$$

$$(x^{1/5})^5 = 2^5$$

$$x = 32$$

$$x^{1/5} = 4$$

$$(x^{1/5})^5 = 4^5$$

$$x = 1024$$

$$\{32, 1024\}$$

Q6 $(x+1)(x+2)(x+3)(x+4) = 24$

Re-arranging it

$$(x+1)(x+4) \cdot (x+2)(x+3) = 24$$

$$(x^2 + 4x + x + 4) \cdot (x^2 + 3x + 2x + 6) = 24$$

$$(x^2 + 5x + 4) \cdot (x^2 + 5x + 6) = 24$$

put $x^2 + 5x = y$ Then

$$(y+4)(y+6) = 24$$

$$y^2 + 6y + 4y + 24 = 24$$

$$y^2 + 10y + 24 - 24 = 0$$

$$y^2 + 10y = 0$$

$$y(y+10) = 0$$

$$y = 0, y + 10 = 0 \Rightarrow y = -10$$

if $y=0$ Then

$$x^2 + 5x = 0$$

$$x(x+5) = 0$$

$$x=0, x+5=0$$

$$x = -5$$

if $y=-10$ Then

$$x^2 + 5x = -10$$

$$x^2 + 5x + 10 = 0$$

$$\text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25-40}}{2} \Rightarrow x = \frac{-5 \pm \sqrt{-15}}{2}$$

$$\left\{ 0, -5, \frac{-5 \pm \sqrt{-15}}{2} \right\}$$

Q.7 $(x-1)(x+5)(x+8)(x+2) = 880$

$$(x-1)(x+5)(x+8)(x+2) = 880$$

Re-arranging it

$$(x-1)(x+8) \cdot (x+2)(x+5) = 880$$

$$(x^2 + 8x - x - 8) \cdot (x^2 + 5x + 2x + 10) = 880$$

$$(x^2 + 7x - 8) \cdot (x^2 + 7x + 10) = 880$$

put $x^2 + 7x = y$ Then

$$(y-8)(y+10) = 880$$

$$y^2 + 10y - 8y - 80 = 880$$

$$y^2 + 2y - 80 - 880 = 0$$

$$y^2 + 2y - 960 = 0$$

$$y^2 + 32y - 30y - 960 = 0$$

$$y(y+32) - 30(y+32) = 0$$

$$(y-30)(y+32) = 0$$

$$y-30=0, y+32=0$$

$$y = 30, y = -32$$

if $y=30$ Then

$$x^2 + 7x = 30$$

$$x^2 + 7x - 30 = 0$$

$$x^2 - 3x + 10x - 30 = 0,$$

$$x(x-3) + 10(x-3) = 0,$$

if $y=-32$ Then

$$x^2 + 7x = -32$$

$$x^2 + 7x + 32 = 0$$

Using

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$(x-3)(x+10)=0 \quad x = \frac{-7 \pm \sqrt{(7)^2 - 4(1)(32)}}{2(1)}$$

$$x-3=0, x+10=0$$

$$x=3, x=-10 \quad x = \frac{-7 \pm \sqrt{49-128}}{2}$$

$$x = \frac{-7 \pm \sqrt{-79}}{2}$$

$$\left\{ 3, -10, \frac{-7 \pm \sqrt{-79}}{2} \right\}$$

$$\mathbf{Q.8} (x-5)(x-7)(x+6)(x+4)-504=0$$

$$(x-5)(x-7)(x+6)(x+4)=504$$

Re-arranging it

$$(x-5)(x+4) \cdot (x-7)(x+6)=504$$

$$(x^2+4x-5x-20) \cdot (x^2+6x-7x-42)=504$$

$$(x^2-x-20) \cdot (x^2-x-42)=504$$

put $x^2-x=y$ Then

$$(y-20)(y-42)=504$$

$$y^2-42y-20y+840-504=0$$

$$y^2-62y+336=0$$

$$y^2-6y-56y+336=0$$

$$y(y-6)-56(y-6)=0$$

$$(y-6)(y-56)=0$$

$$y-6=0, y-56=0$$

$$y=6, y=56$$

If $y=6$ Then, If $y=56$ Then

$$x^2-x=6$$

$$x^2-x-6=0$$

$$x^2+2x-3x-6=0$$

$$x(x+2)-3(x+2)=0$$

$$(x-3)(x+2)=0$$

$$x-3=0, x+2=0$$

$$x=3, x=-2$$

$$\{ 3, -2, 8, -7 \}$$

$$x^2-x=56$$

$$x^2-x-56=0$$

$$x^2+7x-8x-56=0$$

$$x(x+7)-8(x+7)=0$$

$$(x-8)(x+7)=0$$

$$x-8=0, x+7=0$$

$$x=8, x=-7$$

$$\mathbf{Q.9} (x-1)(x-2)(x-8)(x+5)+360=0$$

$$(x-1)(x-2)(x-8)(x+5)=-360$$

Re-arranging it

$$(x-1)(x-2) \cdot (x-8)(x+5)=-360$$

$$(x^2-2x-x+2) \cdot (x^2+5x-8x-40)=-360$$

$$(x^2-3x+2) \cdot (x^2-3x-40)=-360$$

put $x^2-3x=y$ Then

$$(y+2)(y-40)=-360$$

$$y^2-40y+2y-80+360=0$$

$$y^2-38y+280=0$$

$$y^2-10y-28y+280=0$$

$$y(y-10)-28(y-10)=0$$

$$(y-10)(y-28)=0$$

$$y-10=0, y-28=0$$

$$y=10, y=28$$

If $y=10$ Then, If $y=28$ Then

$$x^2-3x=10$$

$$x^2-3x-10=0$$

$$x^2+2x-5x-10=0$$

$$x(x+2)-5(x+2)=0$$

$$(x-5)(x+2)=0$$

$$x-5=0, x+2=0$$

$$x=5, x=-2$$

$$x^2-3x=28$$

$$x^2-3x-28=0$$

$$x^2+4x-7x-28=0$$

$$x(x+4)-7(x+4)=0$$

$$(x-7)(x+4)=0$$

$$x-7=0, x+4=0$$

$$x=7, x=-4$$

$$\{ 5, -2, 7, -4 \}$$

$$\mathbf{Q.10} (x+1)(2x+3)(x+5)(x+3)=945$$

Re-arranging it

$$(x+1)(x+3) \cdot (2x+3)(2x+5)=945$$

$$(x^2+3x+x+3) \cdot (4x^2+10x+6x+15)=945$$

$$(x^2+4x+3) \cdot (4x^2+16x+15)=945$$

$$(x^2+4x+3) \cdot (4(x^2+4x)+15)=945$$

15

put $x^2+4x=y$ Then

$$(y+3)(4y+15) = 945$$

$$4y^2+15y+12y+45-945=0$$

$$4y^2+27y-900=0$$

$$4y^2-48y+75y-900=0$$

$$4y(y-12)+75(y-12)=0$$

$$(y-12)(4y+75)=0$$

$$y-12=0, 4y+75=0$$

$$y=12, y=-\frac{75}{4}$$

If $y=12$ then If $y=-\frac{75}{4}$ then

$$x^2+4x=12, x^2+4x=-\frac{75}{4}$$

$$x^2+4x-12=0, 4x^2+16x=-75$$

$$x^2+4x-12=0, 4x^2+16x+75=0$$

$$x^2-2x+6x-12=0, \text{ using } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x(x-2)+6(x-2)=0$$

$$(x-2)(x+6)=0$$

$$x-2=0, x+6=0$$

$$x=2, x=-6$$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(4)(75)}}{2(4)}$$

$$x = \frac{-16 \pm \sqrt{256-1200}}{8}$$

$$x = \frac{-16 \pm \sqrt{-944}}{8} \Rightarrow x = \frac{-16 \pm \sqrt{16 \times 59}}{8}$$

$$x = \frac{-16 \pm 4\sqrt{59}}{8} \Rightarrow x = \frac{4(-4 \pm \sqrt{59})}{8}$$

$$x = \frac{-4 \pm \sqrt{59}}{2} \Rightarrow x = \frac{-4 \pm \sqrt{59} \sqrt{-1}}{2}$$

$$x = \frac{-4 \pm \sqrt{59}i}{2} \because \sqrt{-1} = i \text{ (iota)}$$

$$\left\{ -6, 2, \frac{-4 \pm \sqrt{59}i}{2} \right\}$$

Q.11 $(2x-7)(x^2-9)(2x+5)-91=0$

$$(2x-7)(x-3)(x+3) \cdot (2x+5) = 91$$

Re-arranging it

$$(2x-7)(x+3) \cdot (x-3)(2x+5) = 91$$

$$(2x^2+6x-7x-21) \cdot (2x^2+5x-6x-15) = 91$$

$$(2x^2-x-21) \cdot (2x^2-x-15) = 91$$

put $2x^2-x=y$ then

$$(y-21)(y-15) = 91$$

$$y^2-15y-21y+315-91=0$$

$$y^2-36y+224=0$$

$$y^2-8y-28y+224=0$$

$$y(y-8)-28(y-8)=0$$

$$(y-8)(y-28)=0$$

$$y-8=0, y-28=0$$

$$y=8, y=28$$

If $y=8$ then, If $y=28$ then

$$2x^2-x=8$$

$$2x^2-x=28$$

$$2x^2-x-8=0$$

$$2x^2-x-28=0$$

$$\text{using } x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$2x^2-8x+7x-28=0$$

$$2x(x-4)+7(x-4)=0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-8)}}{2(2)}$$

$$(x-4)(2x+7)=0$$

$$x-4=0, 2x+7=0$$

$$x=4, x=-\frac{7}{2}$$

$$x = \frac{1 \pm \sqrt{1+64}}{4}$$

$$x = \frac{1 \pm \sqrt{65}}{4} \left\{ 4, -\frac{7}{2}, \frac{1 \pm \sqrt{65}}{4} \right\}$$

Q.12 $(x^2+6x+8)(x^2+14x+48)=105$

Factorizing it

$$(x^2+2x+4x+8) \cdot (x^2+6x+8x+48)=105$$

$$[x(x+2)+4(x+2)][x(x+6)+8(x+6)]=105$$

$$(x+2)(x+4) \cdot (x+6)(x+8)=105$$

Re-arranging it

$$(x+2)(x+8) \cdot (x+4)(x+6)=105$$

$$(x^2+8x+2x+16) \cdot (x^2+6x+4x+24)=105$$

$$(x^2+10x+16) \cdot (x^2+10x+24)=105$$

put $x^2 + 10x = y$ Then

$$(y+16)(y+24) = 105$$

$$y^2 + 24y + 16y + 384 - 105 = 0$$

$$y^2 + 40y + 279 = 0$$

$$y^2 + 9y + 31y + 279 = 0$$

$$y(y+9) + 31(y+9) = 0$$

$$(y+9)(y+31) = 0$$

$$y+9=0, y+31=0$$

$$y = -9, y = -31$$

if $y = -9$ then, if $y = -31$ then

$$x^2 + 10x = -9$$

$$x^2 + 10x = -31$$

$$x^2 + 10x + 9 = 0$$

$$x^2 + 10x + 31 = 0$$

$$x^2 + x + 9x + 9 = 0$$

$$x(x+1) + 9(x+1) = 0$$

$$(x+1)(x+9) = 0$$

$$x+1=0, x+9=0$$

$$x = -1, x = -9$$

$$\text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(1)(31)}}{2(1)}$$

$$x = \frac{-10 \pm \sqrt{100 - 124}}{2}$$

$$x = \frac{-10 \pm \sqrt{-24}}{2}$$

$$\Rightarrow x = \frac{-10 \pm \sqrt{4(-6)}}{2}$$

$$x = \frac{-10 \pm 2\sqrt{-6}}{2}$$

$$\Rightarrow x = \frac{2(-5 \pm \sqrt{-6})}{2}$$

$$x = -5 \pm \sqrt{-6} \Rightarrow x = -5 \pm \sqrt{6}\sqrt{-1}$$

$$x = -5 \pm \sqrt{6}i \quad \because \sqrt{-1} = i$$

$$\{-1, -9, -5 \pm \sqrt{6}i\}$$

$$\text{OR } \{-1, -9, -5 \pm \sqrt{6}i\}$$

$$\mathbf{Q.13} (x^2 + 6x - 27)(x^2 - 2x - 35) = 385$$

Factorizing it

$$(x^2 - 3x + 9x - 27) \cdot (x^2 + 5x - 7x - 35) = 385$$

$$[x(x-3) + 9(x-3)] [x(x+5) - 7(x+5)] = 385$$

$$(x-3)(x+9) \cdot (x-7)(x+5) = 385$$

Re-arranging it

$$(x-3)(x+5) \cdot (x-7)(x+9) = 385$$

$$(x^2 + 5x - 3x - 15) \cdot (x^2 + 9x - 7x - 63) = 385$$

$$(x^2 + 2x - 15) \cdot (x^2 + 2x - 63) = 385$$

put $x^2 + 2x = y$ Then

$$(y-15)(y-63) = 385$$

$$y^2 - 63y - 15y + 945 - 385 = 0$$

$$y^2 - 78y + 560 = 0$$

$$y^2 - 8y - 70y + 560 = 0$$

$$y(y-8) - 70(y-8) = 0$$

$$(y-8)(y-70) = 0$$

$$y-8=0, y-70=0$$

$$y = 8, y = 70$$

if $y = 8$ then

if $y = 70$ then

$$x^2 + 2x = 8$$

$$x^2 + 2x = 70$$

$$x^2 + 2x - 8 = 0$$

$$x^2 + 2x - 70 = 0$$

$$x^2 - 2x + 4x - 8 = 0$$

$$\text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x(x-2) + 4(x-2) = 0$$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-70)}}{2(1)}$$

$$(x-2)(x+4) = 0$$

$$x-2=0, x+4=0$$

$$x = 2, x = -4$$

$$x = \frac{-2 \pm \sqrt{4 + 280}}{2}$$

$$x = \frac{-2 \pm \sqrt{284}}{2}$$

$$\Rightarrow x = \frac{-2 \pm \sqrt{4 \times 71}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{71}}{2} \Rightarrow x = \frac{2(-1 \pm \sqrt{71})}{2}$$

$$x = -1 \pm \sqrt{71} \quad \{2, -4, -1 \pm \sqrt{71}\}$$

$$\mathbf{Q.14} 4 \cdot 2^{2x+1} - 9 \cdot 2^x + 1 = 0$$

$$4 \cdot 2^{2x} \cdot 2^1 - 9 \cdot 2^x + 1 = 0$$

$$4 \cdot (2^x)^2 \cdot 2 - 9 \cdot 2^x + 1 = 0$$

$$8 \cdot (2^x)^2 - 9 \cdot 2^x + 1 = 0$$

put $2^x = y$ Then

$$8y^2 - 9y + 1 = 0$$

$$8y^2 - 8y - y + 1 = 0$$

$$8y(y-1) - 1(y-1) = 0$$

$$(y-1)(8y-1) = 0$$

$$y-1=0, \quad 8y-1=0$$

$$y=1, \quad y=\frac{1}{8}$$

if $y=1$ Then, if $y=\frac{1}{8}$ Then

$$2^x=1, \quad 2^x=\frac{1}{8}$$

$$2^x=2^0, \quad 2^x=\frac{1}{2^3}$$

$$2^x=2^0, \quad 2^x=2^{-3}$$

$$\Rightarrow x=0, \quad x=-3$$

$\{0, -3\}$

Q.15 $2^x + 2^{-x+6} - 20 = 0$

$$2^x + 2^{-x} \cdot 2^6 - 20 = 0$$

$$2^x + 2^{-x} \cdot 64 - 20 = 0$$

$$2^x + \frac{64}{2^x} - 20 = 0$$

put $2^x = y$ Then

$$y + \frac{64}{y} - 20 = 0$$

$$y^2 + 64 - 20y = 0$$

$$y^2 - 20y + 64 = 0$$

$$y^2 - 4y - 16y + 64 = 0$$

$$y(y-4) - 16(y-4) = 0$$

$$(y-4)(y-16) = 0$$

$$y-4=0, \quad y-16=0$$

$$y=4, \quad y=16$$

if $y=4$ Then if $y=16$ Then

$$2^x=4, \quad 2^x=16$$

$$2^x=2^2$$

$$\Rightarrow x=2$$

$$2^x=2^4$$

$$x=4$$

$\{2, 4\}$

Q.16 $4^x - 3 \cdot 2^{x+3} + 128 = 0$

$$(2^2)^x - 3 \cdot 2^x \cdot 2^3 + 128 = 0$$

$$(2^x)^2 - 3 \cdot 2^x \cdot 8 + 128 = 0$$

$$(2^x)^2 - 24 \cdot 2^x + 128 = 0$$

put $2^x = y$ Then

$$y^2 - 24y + 128 = 0$$

$$y^2 - 8y - 16y + 128 = 0$$

$$y(y-8) - 16(y-8) = 0$$

$$(y-8)(y-16) = 0$$

$$y-8=0, \quad y-16=0$$

$$y=8, \quad y=16$$

if $y=8$ Then, if $y=16$ Then

$$2^x=8,$$

$$2^x=16$$

$$2^x=2^3,$$

$$2^x=2^4$$

$$\Rightarrow x=3,$$

$$x=4$$

$\{3, 4\}$

Q.17 $3^{2x-1} - 12 \cdot 3^x + 81 = 0$

$$3^{2x} \cdot 3^{-1} - 12 \cdot 3^x + 81 = 0$$

$$(3^x)^2 \cdot \frac{1}{3} - 12 \cdot 3^x + 81 = 0$$

Multiplying by 3

$$(3^x)^2 - 36 \cdot 3^x + 243 = 0$$

put $3^x = y$ Then

$$y^2 - 36y + 243 = 0$$

$$y^2 - 9y - 27y + 243 = 0$$

$$y(y-9) - 27(y-9) = 0$$

$$(y-9)(y-27) = 0$$

$$\begin{aligned}
 y-9 &= 0, & y-27 &= 0 \\
 y &= 9, & y &= 27 \\
 \text{If } y=9 \text{ Then} & , & \text{If } y=27 \text{ Then} & \\
 3^x &= 9, & 3^x &= 27 \\
 3^x &= 3^2, & 3^x &= 3^3 \\
 \Rightarrow x &= 2, & x &= 3 \\
 & & \{2, 3\} &
 \end{aligned}$$

Q.18 $(x + \frac{1}{x})^2 - 3(x + \frac{1}{x}) - 4 = 0$

put $x + \frac{1}{x} = y$ then

$$\begin{aligned}
 y^2 - 3y - 4 &= 0 \\
 y^2 + y - 4y - 4 &= 0 \\
 y(y+1) - 4(y+1) &= 0 \\
 (y+1)(y-4) &= 0 \\
 y+1 &= 0, & y-4 &= 0 \\
 y &= -1, & y &= 4
 \end{aligned}$$

$$\begin{aligned}
 \text{If } y=-1 \text{ Then} & , & \text{If } y=4 \text{ Then} & \\
 x + \frac{1}{x} &= -1, & x + \frac{1}{x} &= 4 \\
 x^2 + 1 &= -x, & x^2 + 1 &= 4x \\
 x^2 + x + 1 &= 0, & x^2 - 4x + 1 &= 0
 \end{aligned}$$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\begin{aligned}
 x &= \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)} & x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)} \\
 x &= \frac{-1 \pm \sqrt{1-4}}{2} & x &= \frac{4 \pm \sqrt{16-4}}{2} \\
 x &= \frac{-1 \pm \sqrt{-3}}{2} & x &= \frac{4 \pm \sqrt{12}}{2} \\
 x &= \frac{-1 \pm \sqrt{3}i}{2} & x &= \frac{4 \pm \sqrt{4 \times 3}}{2} \\
 x &= \frac{-1 \pm \sqrt{3}i}{2} & x &= \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3} \\
 & & \{ \frac{-1 \pm \sqrt{3}i}{2}, 2 \pm \sqrt{3} \} &
 \end{aligned}$$

Q.19 $x^2 + x - 4 + \frac{1}{x} + \frac{1}{x^2} = 0$

$$x^2 + \frac{1}{x^2} + x + \frac{1}{x} - 4 = 0$$

put $x + \frac{1}{x} = y$ then

$$(x + \frac{1}{x})^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2 \Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$$

Given equation takes form

$$y^2 - 2 + y - 4 = 0$$

$$y^2 + y - 6 = 0$$

$$y^2 - 2y + 3y - 6 = 0$$

$$y(y-2) + 3(y-2) = 0$$

$$(y-2)(y+3) = 0$$

$$y-2 = 0, & y+3 = 0$$

$$y = 2, & y = -3$$

If $y = 2$ then, If $y = -3$ then

$$x + \frac{1}{x} = 2, & x + \frac{1}{x} = -3$$

$$x^2 + 1 = 2x, & x^2 + 1 = -3x$$

$$x^2 - 2x + 1 = 0, & x^2 + 3x + 1 = 0$$

$$(x-1)^2 = 0 \quad \text{Using } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x-1 = 0$$

$$\Rightarrow x = 1 \quad x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9-4}}{2} \Rightarrow x = \frac{-3 \pm \sqrt{5}}{2}$$

$$\{1, \frac{-3 \pm \sqrt{5}}{2}\}$$

Q.20 $(x - \frac{1}{x})^2 + 3(x + \frac{1}{x}) = 0$

$$x^2 + \frac{1}{x^2} - 2 + 3(x + \frac{1}{x}) = 0$$

put $x + \frac{1}{x} = y$

$$\Rightarrow (x + \frac{1}{x})^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2 \Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$$

Then given equation takes form

$$y^2 - 2 - 2 + 3y = 0$$

$$y^2 + 3y - 4 = 0$$

$$y^2 - y + 4y - 4 = 0$$

$$y(y-1) + 4(y-1) = 0$$

$$(y-1)(y+4) = 0$$

$$y-1=0, y+4=0$$

$$y=1, y=-4$$

If $y=1$ then, If $y=-4$ then

$$x + \frac{1}{x} = 1, x + \frac{1}{x} = -4$$

$$x^2 + 1 = x, x^2 + 1 = -4x$$

$$x^2 - x + 1 = 0, x^2 + 4x + 1 = 0$$

Using $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{1-4}}{2}$$

$$x = \frac{1 \pm \sqrt{-3}}{2}$$

$$x = \frac{1 \pm \sqrt{3}i}{2}$$

$$x = \frac{1 \pm \sqrt{3}i}{2}$$

$$x = \frac{-4 \pm \sqrt{(4)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16-4}}{2}$$

$$x = \frac{-4 \pm \sqrt{12}}{2}$$

$$x = \frac{-4 \pm \sqrt{4 \times 3}}{2}$$

$$x = \frac{-4 \pm 2\sqrt{3}}{2}$$

$$x = 2(-2 \pm \sqrt{3})$$

$$x = -2 \pm \sqrt{3}$$

OR. $\left\{ \frac{1 \pm \sqrt{3}}{2}, -2 \pm \sqrt{3} \right\}$

$\left\{ \frac{1 \pm \sqrt{3}i}{2}, -2 \pm \sqrt{3} \right\}$

Q.21 $2x^4 - 3x^3 - x^2 - 3x + 2 = 0$

Dividing by x^2

$$\frac{2x^4}{x^2} - \frac{3x^3}{x^2} - \frac{x^2}{x^2} - \frac{3x}{x^2} + \frac{2}{x^2} = \frac{0}{x^2}$$

$$2x^2 - 3x - 1 - \frac{3}{x} + \frac{2}{x^2} = 0$$

$$2x^2 + \frac{2}{x^2} - 3x - \frac{3}{x} - 1 = 0$$

$$2\left(x^2 + \frac{1}{x^2}\right) - 3\left(x + \frac{1}{x}\right) - 1 = 0$$

put $x + \frac{1}{x} = y$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2 \Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$$

The given equation takes form

$$2(y^2 - 2) - 3y - 1 = 0$$

$$2y^2 - 4 - 3y - 1 = 0$$

$$2y^2 - 3y - 5 = 0$$

$$2y^2 + 2y - 5y - 5 = 0$$

$$2y(y+1) - 5(y+1) = 0$$

$$(y+1)(2y-5) = 0$$

$$y+1=0, 2y-5=0 \Rightarrow y = -1, y = \frac{5}{2}$$

$$y = -1, y = \frac{5}{2}$$

If $y = -1$ then

$$x + \frac{1}{x} = -1$$

$$x^2 + 1 = -x$$

$$x^2 + x + 1 = 0$$

Using

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$x = \frac{-1 \pm \sqrt{-3}}{2}$$

$$x = \frac{-1 \pm \sqrt{3}i}{2}$$

If $y = \frac{5}{2}$ then

$$x + \frac{1}{x} = \frac{5}{2}$$

$$2x^2 + 2 = 5x$$

$$2x^2 - 5x + 2 = 0$$

$$2x^2 - x - 4x + 2 = 0$$

$$x(2x-1) - 2(2x-1) = 0$$

$$(x-2)(2x-1) = 0$$

$$x-2=0, 2x-1=0$$

$$x=2, x=\frac{1}{2}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{3}i}{2}$$

$$\left\{ 2, \frac{1}{2}, \frac{-1 \pm \sqrt{3}i}{2} \right\} \text{ OR } \left\{ 2, \frac{1}{2}, \frac{-1 \pm \sqrt{3}i}{2} \right\}$$

MEGA LECTURE

Q.22 $2x^4 + 3x^3 - 4x^2 - 3x + 2 = 0$

Dividing by x^2

$$\frac{2x^4}{x^2} + \frac{3x^3}{x^2} - \frac{4x^2}{x^2} - \frac{3x}{x^2} + \frac{2}{x^2} = \frac{0}{x^2}$$

$$2x^2 + 3x - 4 - \frac{3}{x} + \frac{2}{x^2} = 0$$

$$2x^2 + \frac{2}{x^2} + 3x - \frac{3}{x} - 4 = 0$$

$$2(x^2 + \frac{1}{x^2}) + 3(x - \frac{1}{x}) - 4 = 0$$

put $x - \frac{1}{x} = y$

$$\Rightarrow (x - \frac{1}{x})^2 = y^2 \Rightarrow x^2 + \frac{1}{x^2} - 2 = y^2$$

$$x^2 + \frac{1}{x^2} = y^2 + 2$$

Given equation takes form

$$2(y^2 + 2) + 3y - 4 = 0$$

$$2y^2 + 4 + 3y - 4 = 0$$

$$2y^2 + 3y = 0$$

$$y(2y + 3) = 0$$

$$y = 0, \quad 2y + 3 = 0$$

$$y = 0, \quad y = -\frac{3}{2}$$

if $y = 0$ then

$$x - \frac{1}{x} = 0$$

$$x^2 - 1 = 0$$

$$(x-1)(x+1) = 0$$

$$x-1=0, x+1=0$$

$$x=1, x=-1$$

if $y = -\frac{3}{2}$ then

$$x - \frac{1}{x} = -\frac{3}{2}$$

$$2x^2 - 2 = -3x$$

$$2x^2 + 3x - 2 = 0$$

$$2x^2 - x + 4x - 2 = 0$$

$$x(2x-1) + 2(2x-1) = 0$$

$$(2x-1)(x+2) = 0$$

$$2x-1=0, x+2=0$$

$$x = \frac{1}{2}, x = -2$$

$$\left\{ 1, -1, \frac{1}{2}, -2 \right\}$$

Q.23 $6x^4 - 35x^3 + 62x^2 - 35x + 6 = 0$

Dividing by x^2

$$\frac{6x^4}{x^2} - \frac{35x^3}{x^2} + \frac{62x^2}{x^2} - \frac{35x}{x^2} + \frac{6}{x^2} = \frac{0}{x^2}$$

$$6x^2 - 35x + 62 - \frac{35}{x} + \frac{6}{x^2} = 0$$

$$6x^2 + \frac{6}{x^2} - 35x - \frac{35}{x} + 62 = 0$$

$$6(x^2 + \frac{1}{x^2}) - 35(x + \frac{1}{x}) + 62 = 0$$

put $x + \frac{1}{x} = y$

$$\Rightarrow (x + \frac{1}{x})^2 = y^2$$

$$x^2 + \frac{1}{x^2} + 2 = y^2 \Rightarrow x^2 + \frac{1}{x^2} = y^2 - 2$$

Given equation takes form

$$6(y^2 - 2) - 35y + 62 = 0$$

$$6y^2 - 12 - 35y + 62 = 0$$

$$6y^2 - 35y + 50 = 0$$

$$6y^2 - 15y - 20y + 50 = 0$$

$$3y(2y-5) - 10(2y-5) = 0$$

$$(2y-5)(3y-10) = 0$$

$$2y-5=0, 3y-10=0$$

$$y = \frac{5}{2}, y = \frac{10}{3}$$

if $y = \frac{5}{2}$ then, if $y = \frac{10}{3}$ then

$$x + \frac{1}{x} = \frac{5}{2}, x + \frac{1}{x} = \frac{10}{3}$$

$$2x^2 + 2 = 5x$$

$$2x^2 - 5x + 2 = 0$$

$$2x^2 - x - 4x + 2 = 0$$

$$x(2x-1) - 2(2x-1) = 0$$

$$(2x-1)(x-2) = 0$$

$$2x-1=0, x-2=0$$

$$x = \frac{1}{2}, x = 2 \left\{ \frac{1}{2}, 2, \frac{1}{3}, 3 \right\}$$

$$3x^2 + 3 = 10x$$

$$3x^2 - 10x + 3 = 0$$

$$3x^2 - x - 9x + 3 = 0$$

$$x(3x-1) - 3(3x-1) = 0$$

$$(3x-1)(x-3) = 0$$

$$3x-1=0, x-3=0$$

$$x = \frac{1}{3}, x = 3$$

Q.24 $x^4 - 6x^2 + 10 - \frac{6}{x^2} + \frac{1}{x^4} = 0$

$$x^4 + \frac{1}{x^4} - 6x^2 - \frac{6}{x^2} + 10 = 0$$

$$x^4 + \frac{1}{x^4} - 6\left(x^2 + \frac{1}{x^2}\right) + 10 = 0$$

put $x^2 + \frac{1}{x^2} = y$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = y^2$$

$$x^4 + \frac{1}{x^4} + 2 = y^2 \Rightarrow x^4 + \frac{1}{x^4} = y^2 - 2$$

Given equation takes form.

$$y^2 - 2 - 6y + 10 = 0$$

$$y^2 - 6y + 8 = 0$$

$$y^2 - 2y - 4y + 8 = 0$$

$$y(y-2) - 4(y-2) = 0$$

$$(y-2)(y-4) = 0$$

$$y-2=0, \quad y-4=0$$

$$y=2, \quad y=4$$

if $y=2$ then

$$x^2 + \frac{1}{x^2} = 2$$

$$x^4 + 1 = 2x^2$$

$$x^4 - 2x^2 + 1 = 0$$

$$(x^2 - 1)^2 = 0$$

$$x^2 - 1 = 0$$

$$(x-1)(x+1) = 0$$

$$x-1=0, \quad x+1=0$$

$$x=1, \quad x=-1$$

$$z = \frac{4 \pm \sqrt{16-4}}{2} \Rightarrow z = \frac{4 \pm \sqrt{12}}{2}$$

$$z = \frac{4 \pm \sqrt{4 \times 3}}{2} \Rightarrow z = \frac{4 \pm 2\sqrt{3}}{2} = 2(2 \pm \sqrt{3})$$

$$z = 2 \pm \sqrt{3} \quad \text{So } x^2 = 2 \pm \sqrt{3}$$

$$\Rightarrow x = \pm \sqrt{2 \pm \sqrt{3}}$$

$$\{1, -1, \pm \sqrt{2 \pm \sqrt{3}}\}$$

Radical

Another word used for root, The sign $\sqrt{\quad}$ is called radical sign. A number placed to the left of the sign shows the type of root eg. $\sqrt[2]{\quad}$, $\sqrt[3]{\quad}$, $\sqrt[4]{\quad}$, ..., $\sqrt[n]{\quad}$ denote square root, Cube roots, Fourth root, ..., n th roots respectively. Here numbers 2, 3, 4 and n are called index. If there is no number to the left of the sign $\sqrt{\quad}$, then root is called square root.

Radical Equations The equations involving radical expressions of the variable are called radical equations.

For example; $\sqrt{2x+8} + \sqrt{x+5} = 7$

$$3x^2 + 2x - \sqrt{3x^2 + 2x - 1} - 7 = 0$$

$$\sqrt{x^2 + 2x - 3} + \sqrt{x^2 + 7x - 8} = \sqrt{5x^2 + 15x - 20}$$

are radical equations.

Extraneous Root

A number obtained in the process of solving an equation, which is actually not a root of the given equation. In other words, we may say that, a root that does not satisfy given equation is called an extraneous root.

* There are some types of Radical Equations

Type No. 1

The equation of the form

$$l(ax^2 + bx) + m\sqrt{ax^2 + bx + c} = 0$$

Questions of this Type are

Example 1, Q. No. 1, 2, 10