

Convert the following theorems to logical form and prove them by constructing truth table:-

Question # 1

$$(A \cap B)' = A' \cup B'$$

Solution

The corresponding formula of logic is

$$\sim(p \wedge q) = \sim p \vee \sim q$$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \vee \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

The last two columns of the above table shows that $\sim(p \wedge q) = \sim p \vee \sim q$ and hence $(A \cap B)' = A' \cup B'$.

Question # 2

$$(A \cup B) \cup C = A \cup (B \cup C)$$

Solution

The corresponding formula of logic is

$$(p \vee q) \vee r = p \vee (q \vee r)$$

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

The last two columns of the above table shows that $(p \vee q) \vee r = p \vee (q \vee r)$ and hence $(A \cup B) \cup C = A \cup (B \cup C)$.

Question # 3

$$(A \cap B) \cap C = A \cap (B \cap C)$$

Solution

The corresponding formula of logic is

$$(p \wedge q) \wedge r = p \wedge (q \wedge r)$$

p	q	r	$p \wedge q$	$q \wedge r$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

The last two columns of the above table shows that $(p \wedge q) \wedge r = p \wedge (q \wedge r)$ and hence $(A \cap B) \cap C = A \cap (B \cap C)$.

Question # 4

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Solution

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

The corresponding formula of logic is

$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$$

p	q	r	$q \wedge r$	$p \vee q$	$p \vee r$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

The last two columns of the above table shows that $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$ and hence $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

If you found any error, please report us at www.megalecture.com

Book:

Exercise 2.4

Text Book of Algebra and Trigonometry Class XI

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