Solution. $\varnothing$ no-element subsets

Conclusion. Above four examples show that, " every set has two improper subsets and remaining are proper subsets".

Example . Improper subsets = $\varnothing$, $\{x, y, z\}$

Proper subsets = $\{x\}, \{y\}, \{z\}, \{x, y\}, \{y, z\}, \{x, z\}$

Note that to find the number of subsets of a set having n elements, the formula is 2^n. For example, in

Example - 1. Number of subsets = $2^3 = 8$

Example - 2. Number of subsets = $2^2 = 4$

Example - 3. Number of subsets = $2^1 = 2$

Example - 4. Number of subsets = $2^0 = 1$

POWER SET. The set of all the subsets of a set is called its power set. It is written as P(S). e.g.

Power set of A = (x, y, z) above is

$$P(A) = \{\varnothing, \{x\}, \{y\}, \{z\}, \{x, y\}, \{y, z\}, \{x, z\}, \{x, y, z\}\}$$

EXERCISE 2.1

1. Write the following sets in set-builder notation :

Solution.

(i) $\{1, 2, 3, \ldots, 1000\} = \{x \mid x \in N \wedge x \le 1000\}$

(ii) $\{0, 1, 2, \ldots, 100\} = \{x \mid x \in W \wedge x \le 100\}$

(iii) $\{0, \pm 1, \pm 2, \ldots, \pm 1000\} = \{x \mid x \in Z \wedge -1000 \le x \le 1000\}$

(iv) $\{0, -1, -2, \ldots, -500\} = \{x \mid x \in Z \wedge -500 \le x \le 0\}$

(v) $\{100, 101, 102, \ldots, 400\} = \{x \mid x \in Z \wedge 100 \le x \le 400\}$

$= \{x \mid x \in N \wedge 100 \le x \le 400\}$

(vi) $\{-100, -101, -102, \ldots, -500\} = \{x \mid x \in Z \wedge -500 \le x \le -100\}$

(vii) {Peshawar, Lahore, Karachi, Quetta}

$= \{x \mid x$ is a capital of a province of Pakistan $\}$

(viii) **{January, June, July}**

$= \{x \mid x$ is a month of the Calender year beginning with letter J $\}$

(ix) **The set of all odd natural numbers**

$= \{x \mid x$ is an odd natural number$\}$

(x) **The set of all rational numbers.** $= \{x \mid x \in Q\}$

(xi) **The set of all real numbers between 1 and 2** $= \{x \mid x \in R \wedge 1 < x < 2\}$

(xii) **The set of all integers between –100 and 1000**

$= \{x \mid x \in Z \wedge -100 < x < 1000\}$

2. Write each of the following sets in descriptive and tabular form:

Solution.

(i) $\{x \mid x \in N \wedge x \leq 10\}$

Descriptive Form ↓	*Tabular Form* ↓
The set of first ten natural numbers	{1, 2, 3, .., 10}

(ii) $\{x \mid x \in N \wedge 4 < x < 12\}$

The set of natural numbers between 4 and 12 — {5, 6, 7, ..., 11}

(iii) $\{x \mid x \in Z \wedge -5 < x < 5\}$

The set of integers between –5 and 5 — {–4, –3, –2, ..., 4}

(iv) $\{x \mid x \in E \wedge 2 < x \leq 4\}$

The set of even integers between 2 and 5 — {4}

(v) $\{x \mid x \in P \wedge x < 12\}$

The set of prime numbers less than 12 — {2, 3, 5, 7, 11}

(vi) $\{x \mid x \in O \wedge 3 < x < 12\}$

The set of odd integers between 3 and 12 — {5, 7, 9, 11}

(vii) $\{x \mid x \in E \wedge 4 \leq x \leq 10\}$

The set of even integers between 2 and 12 — {4, 6, 8, 10}

(viii) $\{x \mid x \in E \wedge 4 < x < 6\}$

The set of even integers between 4 and 6 — { }

(ix) $\{x \mid x \in O \wedge 5 \leq x \leq 7\}$

The set of odd integers from 5 upto 7 — {5, 7}

(x) $\{x \mid x \in O \wedge 5 < x < 7\}$

The set of odd integers greater or equal 5 and less than 7 — { }

(xi) $\{x \mid x \in N \wedge x + 4 = 0\}$

The set of natural numbers x satisfying $x + 4 = 0$

Tabular form : as $x + 4 = 0$ $\Rightarrow$ $x = -4$ which $\notin$ N $\Rightarrow$ { }

(xii) $\{x \mid x \in Q \wedge x^2 = 2\}$

The set of rational numbers x satisfying $x^2 = 2$

Tabular form : as $x^2 = 2$ $\Rightarrow$ $x = \sqrt{2}$ which $\notin$ Q $\Rightarrow$ { }

(xiii) $\{x \mid x \in R \wedge x = x\}$

The set of real numbers x satisfying $x = x$

Tabular form : $x = x$ is satisfied by all reals. $\Rightarrow$ not possible

(xiv) $\{x \mid x \in Q \wedge x = -x\}$

The set of rational numbers x satisfying $x = -x$

Tabular form : $x = -x$ $\Rightarrow$ $x + x = 0$ $\Rightarrow$ $2x = 0$ $\Rightarrow$ $x = 0$ $\Rightarrow$ { 0 }

(xv) $\{x \mid x \in R \wedge x \neq x\}$

The set of real numbers x satisfying $x \neq x$

Tabular form : $x \neq x$ as there is no real number which is not equal to itself $\Rightarrow$ { }

(xvi) $\{x \mid x \in R \wedge x \notin Q\}$

The set of real numbers which are not rational. not possible

Tabular form : set of reals is the union of rational & irrational numbers, so irrational $\Rightarrow$ Q'

3. Which of the following sets are finite and which of these are infinite ?

Solution.

(i)	The set of students of your class.	[Finite]
(ii)	The set of all schools in Pakistan.	[Finite]
(iii)	The set of natural numbers between 3 and 10.	[Finite]
(iv)	The set of rational numbers between 3 and 10.	[Infinite]
(v)	The set of real numbers between 0 and 1.	[Infinite]
(vi)	The set of rationals between 0 and 1.	[Infinite]
(vii)	The set of whole between 0 and 1.	[Finite]
(viii)	The set of all leaves of trees in Pakistan.	[Finite]
(ix)	P (N)	[Infinite]
(x)	P { a, b, c }	[Finite]

(xi) $\{1, 2, 3, 4, ...\}$ [Infinite]

(xii) $\{1, 2, 3, ..., 100000000\}$ [Finite]

(xiii) $\{x \mid x \in R \wedge x \neq x\}$ [Finite]

(xiv) $\{x \mid x \in R \wedge x^2 = -16\}$ [Finite]

(xv) $\{x \mid x \in Q \wedge x^2 = 5\}$ [Finite]

(xvi) $\{x \mid x \in Q \wedge 0 \leq x \leq 1\}$ [Infinite]

4. Write two proper subsets of each of the following sets:

(i) $\{a, b, c\}$ (ii) $\{0, 1\}$ (iii) N (iv) Z (v) Q

(vi) R (vii) W (viii) $\{x \mid x \in Q \wedge 0 < x \leq 2\}$

Solution.

(i) Two proper subsets of $\{a, b, c\}$ are : $\{a\}, \{a, b\}$

(ii) Two proper subsets of $\{0, 1\}$ are : $\{0\}, \{1\}$

(iii) Two proper subsets of N are : $\{1\}, \{1, 2\}$

(iv) Two proper subsets of Z are : $\{1\}, \{1, 2\}$

(v) Two proper subsets of Q are : $\{1\}, \{1, 2\}$

(vi) Two proper subsets of R are : $\{1\}, \{1, 2\}$

(vii) Two proper subsets of W are : $\{1\}, \{1, 2\}$

(viii) Two proper subsets of $\{x \mid x \in Q \wedge 0 < x \leq 2\}$ are : $\{1\}, \{1, 2\}$

5. Is there any set which has no proper subset? If so, name the set.

Solution. Yes, empty set or $\{\ \}$ or $\varnothing$ is the set which has no proper subset.

6. What is the difference between $\{a, b\}$ and $\{\{a, b\}\}$?

Solution.

$\{a, b\}$ is a set which contains two elements a and b.

and $\{\{a, b\}\}$ is a set which contains only one element $\{a, b\}$.

7. Which of the following sentences are true & which of them are false?

(i) $\{1, 2\} = \{2, 1\}$ (ii) $\varnothing \subseteq \{\{a\}\}$ (iii) $\{a\} \subseteq \{\{a\}\}$

(iv) $\{a\} \in \{\{a\}\}$ (v) $a \in \{\{a\}\}$ (vi) $\varnothing \in \{\{a\}\}$

Solution.

(i) $\{1, 2\} = \{2, 1\}$ [True]

(ii) $\varnothing \subseteq \{\{a\}\}$ [True]

(iii) $\{a\} \subseteq \{\{a\}\}$ [False]

(iv) $\{a\} \in \{\{a\}\}$ [True]

(v) $a \in \{\{a\}\}$ [False]

(vi) $\varnothing \in \{\{a\}\}$ [False]

8. What is the number of elements of the power set of each of the following sets ?

(i) $\{\ \}$ (ii) $\{0, 1\}$ (iii) $\{1, 2, 3, 4, 5, 6, 7\}$

(iv) $\{0, 1, 2, 3, 4, 5, 6, 7\}$ (v) $\{a, \{b, c\}\}$ (vi) $\{\{a, b\}, \{b, c\}, \{d, e\}\}$

Solution.

Note that formula to find the number of elements in power set is 2^n.

Number of elements in the

(i)	Power set of $\{\ \}$	is $2^0 = 1$
(ii)	Power set of $\{0, 1\}$	is $2^2 = 4$
(iii)	Power set of $\{1, 2, 3, 4, 5, 6, 7\}$	is $2^7 = 128$
(iv)	Power set of $\{0, 1, 2, 3, 4, 5, 6, 7\}$	is $2^8 = 256$
(v)	Power set of $\{a, \{b, c\}\}$	is $2^2 = 4$
(v)	Power set of $\{\{a, b\}, \{b, c\}, \{d, e\}\}$	is $2^3 = 8$

9. Write down the power set of each of the following sets :

(i) $\{9, 11\}$ (ii) $\{+, -, \times, \div\}$ (iii) $\{\varnothing\}$ (iv) $\{a, \{b, c\}\}$

Solution.

(i) Power set of $\{9, 11\}$ is : $\{\varnothing, \{9\}, \{11\}, \{9, 11\}\}$

(ii) Power set of $\{+, -, \times, \div\}$ is $\{\varnothing, \{+\}, \{-\}, \{\times\}, \{\div\}, \{+,-\}, \{+, \times\}, \{+, \div\},$
$\{-, \times\}, \{-, \div\}, \{\times, \div\}, \{+, -, \times\}, \{+, -, \div\}, \{+, \times, \div\}, \{-, \times, \div\}, \{+, -, \times, \div\}\}$

(iii) Power set of $\{\varnothing\}$ is : $\{\varnothing, \{\varnothing\}\}$

(iv) Power set of $\{a, \{b, c\}\}$ is : $\{\varnothing, \{a\}, \{\{b, c\}\}, \{a, \{b, c\}\}\}$

10. Which of the pairs of sets are equivalent ? Which of them are also equal ?

Solution.

(i) $\{a, b, c\}$, $\{1, 2, 3\}$

are equivalent sets (since, each has three elements)

(ii) **The set of first 10 whole numbers ; $\{0, 1, 2, ..., 9\}$**

are equal sets (since, each has same ten elements)

(iii) **The set of angles of a quadrilateral *ABCD*;**
set of the sides of the same quadrilateral.
are equivalent sets (since, each has four elements)

(iv) **Set of the sides of a hexagon *ABCDEF*;**
set of the angles of the same hexagon.
are equivalent sets (since, each has six elements)

(v) $\{1, 2, 3, 4, ...\}$; $\{2, 4, 6, 8, ...\}$
are equivalent sets (since, 1–1 correspondence can be established)

(vi) $\{1, 2, 3, 4, ...\}$; $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...\right\}$
are equivalent sets (since, 1–1 correspondence can be established)

(vii) $\{5, 10, 15, 20, ..., 55555\}$; $\{5, 10, 15, 20, ...\}$
are not equivalent sets
(since, first set has finite and second infinite number of elements).

§ 2.2 OPERATIONS ON SETS

UNIVERSAL SET. The set of all objects under consideration is called the universal Set. It is usually denoted by U. Any universal set can be restricted to a lower set according to the situation. e.g.,

U = set of all natural numbers

$U = \{1, 2, 3, ..., 100\}$

$U = \{a, b, c, ..., x, y, z\}$, etc.

OPERATIONS ON SETS.

Union of Two Sets. The union of sets A and B, denoted by $A \cup B$, is a set whose elements are the elements of A or the elements of B. In set builder form,

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\} \quad \text{OR}$$

$$A \cup B = \{x \mid x \in A \vee x \in B\}.$$

For example, if $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6\}$, then

$$A \cup B = \{1, 2, 3, 4, 5, 6\}.$$