Question # 1

Find the solution set of the following equation which lies in $[0,2\pi]$

(i) $\sin x = -\dfrac{\sqrt{3}}{2}$ (ii) $\operatorname{cosec}\theta = 2$ (iv) $\cot\theta = \dfrac{1}{\sqrt{3}}$

Solution

(i) Since $\sin x = -\dfrac{\sqrt{3}}{2}$ $\Rightarrow x = \sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$

$\Rightarrow x = \dfrac{5\pi}{3}, \dfrac{4\pi}{3}$ where $x \in [0, 2\pi]$

(ii) Since $\operatorname{cosec}\theta = 2$

$\Rightarrow \dfrac{1}{\sin\theta} = 2 \quad \Rightarrow \sin\theta = \dfrac{1}{2}$

$\Rightarrow \theta = \sin^{-1}\left(\dfrac{1}{2}\right) = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ where $\theta \in [0, 2\pi]$

(iii) *Do yourself*

(iv) Since $\cot\theta = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \dfrac{1}{\tan\theta} = \dfrac{1}{\sqrt{3}} \quad \Rightarrow \tan\theta = \sqrt{3}$

$\Rightarrow \theta = \tan^{-1}\left(\sqrt{3}\right)$

$\Rightarrow \theta = \dfrac{\pi}{3}, \dfrac{4\pi}{3}$ where $\theta \in [0, 2\pi]$

Question # 2

Solve the following trigonometric equations:

(i) $\tan^2\theta = \dfrac{1}{3}$ (ii) $\operatorname{cosec}^2\theta = \dfrac{4}{3}$ (iii) $\sec^2\theta = \dfrac{4}{3}$ (iv) $\cot^2\theta = \dfrac{1}{3}$

Solution

(i) Since $\tan^2\theta = \dfrac{1}{3} \quad \Rightarrow \tan\theta = \pm\dfrac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}}$ or $\tan\theta = -\dfrac{1}{\sqrt{3}}$

$\Rightarrow \theta = \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ or $\theta = \tan^{-1}\left(-\dfrac{1}{\sqrt{3}}\right)$

$\Rightarrow \theta = \dfrac{\pi}{6}$ or $\theta = \dfrac{5\pi}{6}$

Since period of $\tan\theta$ is π

Therefore general value of $\theta = \dfrac{\pi}{6} + n\pi\ ,\ \dfrac{5\pi}{6} + n\pi$

So Solution Set $= \left\{\dfrac{\pi}{6} + n\pi\right\} \cup \left\{\dfrac{5\pi}{6} + n\pi\right\}$ where $n \in \mathbb{Z}$

(ii) Since $\operatorname{cosec}^2\theta = \dfrac{4}{3}$

$\Rightarrow \operatorname{cosec}\theta = \pm\dfrac{2}{\sqrt{3}}$

$\Rightarrow \operatorname{cosec}\theta = \dfrac{2}{\sqrt{3}}$ or $\operatorname{cosec}\theta = -\dfrac{2}{\sqrt{3}}$

$\Rightarrow \sin\theta = \dfrac{\sqrt{3}}{2}$ or $\sin\theta = -\dfrac{\sqrt{3}}{2}$

$\Rightarrow \theta = \sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ or $\theta = \sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$

$\Rightarrow \theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$ or $\theta = \dfrac{4\pi}{3}, \dfrac{5\pi}{3}$

Since period of $\operatorname{cosec}\theta$ is 2π

Therefore general value of $\theta = \dfrac{\pi}{3} + 2n\pi\ ,\ \dfrac{2\pi}{3} + 2n\pi\ ,\ \dfrac{4\pi}{3} + 2n\pi\ ,\ \dfrac{5\pi}{3} + 2n\pi$

Solution set $= \left\{\dfrac{\pi}{3} + 2n\pi\right\} \cup \left\{\dfrac{2\pi}{3} + 2n\pi\right\} \cup \left\{\dfrac{4\pi}{3} + 2n\pi\right\} \cup \left\{\dfrac{5\pi}{3} + 2n\pi\right\}$ where $n \in \mathbb{Z}$.

(iii) Since $\sec^2\theta = \dfrac{4}{3}$

$\Rightarrow \sec\theta = \pm\dfrac{2}{\sqrt{3}}$

$\Rightarrow \sec\theta = \dfrac{2}{\sqrt{3}}$ or $\sec\theta = -\dfrac{2}{\sqrt{3}}$

$\Rightarrow \cos\theta = \dfrac{\sqrt{3}}{2}$ or $\cos\theta = -\dfrac{\sqrt{3}}{2}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ or $\theta = \cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$

$\Rightarrow \theta = \dfrac{\pi}{6}, \dfrac{11\pi}{6}$ or $\theta = \dfrac{5\pi}{6}, \dfrac{7\pi}{6}$

$\because$ period of $\sec\theta$ is 2π

$\therefore$ general values of $\theta = \dfrac{\pi}{6} + 2n\pi\ ,\ \dfrac{5\pi}{6} + 2n\pi\ ,\ \dfrac{7\pi}{6} + 2n\pi\ ,\ \dfrac{11\pi}{6} + 2n\pi$

S.Set $= \left\{\dfrac{\pi}{6} + 2n\pi\right\} \cup \left\{\dfrac{5\pi}{6} + 2n\pi\right\} \cup \left\{\dfrac{7\pi}{6} + 2n\pi\right\} \cup \left\{\dfrac{11\pi}{6} + 2n\pi\right\}$ where $n \in \mathbb{Z}$.

(iv)

Do yourself

Question # 3
Find the value of θ satisfying the following equation:
$3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$

Solution $3\tan^2\theta + 2\sqrt{3}\tan\theta + 1 = 0$

$$\Rightarrow \left(\sqrt{3}\tan\theta\right)^2 + 2\left(\sqrt{3}\tan\theta\right)(1) + (1)^2 = 0$$
$$\Rightarrow \left(\sqrt{3}\tan\theta + 1\right)^2 = 0$$
$$\Rightarrow \left(\sqrt{3}\tan\theta + 1\right) = 0$$
$$\Rightarrow \sqrt{3}\tan\theta = -1$$
$$\Rightarrow \tan\theta = -\frac{1}{\sqrt{3}}$$
$$\Rightarrow \theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$
$$\Rightarrow \theta = \frac{5\pi}{6}$$

$\because$ period of $\tan\theta$ is π

$\therefore$ general value of $\theta = \frac{5\pi}{6} + n\pi$, $n \in \mathbb{Z}$

Question # 4
Find the value of θ satisfying the following equation:
$\tan^2\theta - \sec\theta - 1 = 0$

Solution $\tan^2\theta - \sec\theta - 1 = 0$

$$\Rightarrow \left(\sec^2\theta - 1\right) - \sec\theta - 1 = 0$$
$$\Rightarrow \sec^2\theta - 1 - \sec\theta - 1 = 0$$
$$\Rightarrow \sec^2\theta - \sec\theta - 2 = 0$$
$$\Rightarrow \sec^2\theta - 2\sec\theta + \sec\theta - 2 = 0$$
$$\Rightarrow \sec\theta\left(\sec\theta - 2\right) + 1\left(\sec\theta - 2\right) = 0$$
$$\Rightarrow \left(\sec\theta + 1\right)\left(\sec\theta - 2\right) = 0$$
$$\Rightarrow \left(\sec\theta + 1\right) = 0 \quad \text{or} \quad \left(\sec\theta - 2\right) = 0$$
$$\Rightarrow \sec\theta = -1 \quad \text{or} \quad \sec\theta = +2$$
$$\Rightarrow \cos\theta = -1 \quad \text{or} \quad \cos\theta = \frac{1}{2}$$
$$\Rightarrow \theta = \cos^{-1}(-1) \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{1}{2}\right)$$

$\Rightarrow \theta = \dfrac{3\pi}{2}$ or $\theta = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

$\because$ period of $\cos\theta$ is 2π

$\therefore$ general value of θ = $\dfrac{3\pi}{2} + 2n\pi,\ \dfrac{\pi}{3} + 2n\pi,\ \dfrac{5\pi}{3} + 2n\pi$ where $n \in \mathbb{Z}$]

Question # 5

Find the value of θ satisfying the following equation:

$2\sin\theta + \cos^2\theta - 1 = 0$

Solution $2\sin\theta + \cos^2\theta - 1 = 0$

$\Rightarrow 2\sin\theta + 1 - \sin^2\theta - 1 = 0$

$\Rightarrow -\sin^2\theta + 2\sin\theta = 0$

$\Rightarrow -\sin\theta(\sin\theta - 2) = 0$

$\Rightarrow -\sin\theta = 0$ or $\sin\theta - 2 = 0$

$\Rightarrow \sin\theta = 0$ or $\sin\theta = 2$

$\Rightarrow \theta = \sin^{-1}(0)$ Which does not hold as $\sin\theta \in [-1,1]$

$\Rightarrow \theta = 0\ ,\ \pi$

$\because$ period of $\sin\theta$ is 2π

$\therefore$ general value of θ $= 0 + 2n\pi,\ \pi + 2n\pi$

$= 2n\pi,\ \pi + 2n\pi$ where $n \in \mathbb{Z}$

Question # 6

Find the value of θ satisfying the following equation:

$2\sin^2\theta - \sin\theta = 0$

Solution $2\sin^2\theta - \sin\theta = 0$

$\Rightarrow \sin\theta(2\sin\theta - 1) = 0 \Rightarrow \sin\theta = 0$ or $2\sin\theta - 1 = 0$

Now do yourself

Question # 7

Find the value of θ satisfying the following equation:

$3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

Solution $3\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta - 3\sin^2\theta = 0$

Dividing throughout by $\cos^2\theta$

$$\frac{3\cos^2\theta}{\cos^2\theta} - \frac{2\sqrt{3}\sin\theta\cos\theta}{\cos^2\theta} - \frac{3\sin^2\theta}{\cos^2\theta} = 0$$

$\Rightarrow 3 - 2\sqrt{3}\tan\theta - 3\tan^2\theta = 0$

$\Rightarrow -3\tan^2\theta - 2\sqrt{3}\tan\theta + 3 = 0$

$\Rightarrow 3\tan^2\theta + 2\sqrt{3}\tan\theta - 3 = 0$ ×ing by -1

$$\Rightarrow \tan\theta = \frac{-2\sqrt{3} \pm \sqrt{\left(2\sqrt{3}\right)^2 - 4(3)(-3)}}{2(3)}$$

$$\Rightarrow \tan\theta = \frac{-2\sqrt{3}\pm\sqrt{12+36}}{6} = \frac{-2\sqrt{3}\pm\sqrt{48}}{6}$$
$$= \frac{-2\sqrt{3}\pm\sqrt{16\times3}}{6} = \frac{-2\sqrt{3}\pm4\sqrt{3}}{6}$$
$$\Rightarrow \tan\theta = \frac{-2\sqrt{3}+4\sqrt{3}}{6} = \frac{2\sqrt{3}}{6} \quad \text{or} \quad \tan\theta = \frac{-2\sqrt{3}-4\sqrt{3}}{6} = -\frac{6\sqrt{3}}{6}$$
$$\Rightarrow \tan\theta = \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{\left(\sqrt{3}\right)^2} = \frac{1}{\sqrt{3}} \quad \text{or} \quad \Rightarrow \tan\theta = -\sqrt{3}$$
$$\Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \quad \text{or} \quad \theta = \tan^{-1}\left(-\sqrt{3}\right)$$
$$\Rightarrow \theta = \frac{\pi}{6} \quad \text{or} \quad \theta = \frac{11\pi}{6}$$

$\because$ period of $\tan\theta$ is π

$\therefore$ general value of $\theta = \frac{\pi}{6}+n\pi$, $\frac{11\pi}{6}+n\pi$ where $n\in\mathbb{Z}$.

Question # 8

Find the value of θ satisfying the following equation:

$4\sin^2\theta - 8\cos\theta + 1 = 0$

Solution $4\sin^2\theta - 8\cos\theta + 1 = 0$

$$\Rightarrow 4\left(1-\cos^2\theta\right) - 8\cos\theta + 1 = 0$$
$$\Rightarrow 4 - 4\cos^2\theta - 8\cos\theta + 1 = 0$$
$$\Rightarrow -4\cos^2\theta - 8\cos\theta + 5 = 0$$
$$\Rightarrow 4\cos^2\theta + 8\cos\theta - 5 = 0$$
$$\Rightarrow 4\cos^2\theta + 10\cos\theta - 2\cos\theta - 5 = 0$$
$$\Rightarrow 2\cos\theta\left(2\cos\theta+5\right) - 1\left(2\cos\theta+5\right) = 0$$
$$\Rightarrow \left(2\cos\theta+5\right)\left(2\cos\theta-1\right) = 0$$
$$\Rightarrow 2\cos\theta + 5 = 0 \quad \text{or} \quad 2\cos\theta - 1 = 0$$
$$\Rightarrow 2\cos\theta = -5 \quad \text{or} \quad 2\cos\theta = 1$$
$$\Rightarrow \cos\theta = \frac{-5}{2} \quad \text{or} \quad \cos\theta = \frac{1}{2}$$
$$\Rightarrow \theta = \cos^{-1}\left(\frac{-5}{2}\right) \quad \text{or} \quad \theta = \cos^{-1}\left(\frac{1}{2}\right)$$

Which is not possible as $\cos\theta\in[-1,1]$ or $\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$\because$ period of $\cos\theta$ is 2π

$\therefore$ general value of $\theta = \frac{\pi}{3}+2\pi n, \frac{5\pi}{3}+2n\pi$ where $n\in\mathbb{Z}$.

Question # 9

Find the solution set; $\sqrt{3}\tan x-\sec x-1=0$

Solution $\sqrt{3}\tan x-\sec x-1=0$ ……(i)

$$\Rightarrow \sqrt{3}\frac{\sin x}{\cos x}-\frac{1}{\cos x}-1=0$$

$$\Rightarrow \sqrt{3}\sin x-1-\cos x=0 \qquad \times\text{ing by } \cos\theta.$$

$$\Rightarrow \sqrt{3}\sin x-1=\cos x$$

On squaring both sides.

$$\left(\sqrt{3}\sin x-1\right)^2=(\cos x)^2$$

$$\Rightarrow 3\sin^2 x-2\sqrt{3}\sin x+1=\cos^2 x$$

$$\Rightarrow 3\sin^2 x-2\sqrt{3}\sin x+1=1-\sin^2 x$$

$$\Rightarrow 3\sin^2 x-2\sqrt{3}\sin x+1-1+\sin^2 x=0$$

$$\Rightarrow 4\sin^2 x-2\sqrt{3}\sin x=0$$

$$\Rightarrow 2\sin x\left(2\sin x-\sqrt{3}\right)=0$$

$$\Rightarrow 2\sin x=0 \quad \text{or} \quad 2\sin x=\sqrt{3}$$

$$\Rightarrow \sin x=0 \quad \text{or} \quad \sin x=\frac{\sqrt{3}}{2}$$

$$\Rightarrow x=\sin^{-1}(0) \quad \text{or} \quad x=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow x=0\ ,\ \pi \quad \text{or} \quad x=\frac{\pi}{3}\ ,\ \frac{2\pi}{3}$$

Now to check extraneous roots put $x=0$ in (i)

L.H.S $=\sqrt{3}\tan(0)-\sec(0)-1=0-1-1=-2\neq 0=$ R.H.S

Implies that $x=0$ is an extraneous root of given equation.

Now put $x=\pi$ in (i)

L.H.S $=\sqrt{3}\tan(\pi)-\sec(\pi)-1=0-(-1)-1=0=$ R.H.S

Implies that $x=\pi$ is a root of the equation.

Now put $x=\frac{\pi}{3}$ in (i)

L.H.S $=\sqrt{3}\tan\left(\frac{\pi}{3}\right)-\sec\left(\frac{\pi}{3}\right)-1=\sqrt{3}\left(\sqrt{3}\right)-2-1=1-2-1=0=$ R.H.S

Implies that $x=\frac{\pi}{3}$ is a root of given equation.. Since period of tan is π.

Now put $x=\frac{2\pi}{3}$ in (i)

L.H.S $=\sqrt{3}\tan\left(\frac{2\pi}{3}\right)-\sec\left(\frac{2\pi}{3}\right)-1$

$=\sqrt{3}\left(-\sqrt{3}\right)-(-2)-1=-3+2-1=-2=$ R.H.S

Implies $x=\dfrac{2\pi}{3}$ is an extraneous root of given equation.

$\because$ period of $\sin x$ is 2π

$\therefore$ general values of $x = \pi+2n\pi\ ,\ \dfrac{\pi}{3}+2n\pi$

Solution Set $=\{\pi+2n\pi\}\cup\left\{\dfrac{\pi}{3}+2n\pi\right\}$ where $n\in\mathbb{Z}$.

Question # 10

Find the solution set; $\cos 2x=\sin 3x$

Solution $\cos 2x=\sin 3x$

$\Rightarrow\ \cos^2 x-\sin^2 x=3\sin x-4\sin^3 x$ $\quad\because\ \cos 2x=\cos^2 x-\sin^2 x$

$\Rightarrow\ \cos^2 x-\sin^2 x-3\sin x+4\sin^3 x=0$ $\quad \sin 3x=3\sin x-4\sin^3 x$

$\Rightarrow\ \left(1-\sin^2 x\right)-\sin^2 x-3\sin x+4\sin^3 x=0$

$\Rightarrow\ 1-2\sin^2 x-3\sin x+4\sin^3 x=0$

$\Rightarrow\ 4\sin^3 x-2\sin^2 x-3\sin x+1=0$

Take $\sin x=1$ as a root then by synthetic division

1	4	-2	-3	1
	↓	4	2	1
	4	2	-1	0

$\Rightarrow\ (\sin x-1)\left(4\sin^2 x+2\sin x-1\right)=0$

$\Rightarrow\ \sin x-1=0$ or $4\sin^2 x+2\sin x-1=0$

$\Rightarrow\ \sin x=1$ or $\sin x=\dfrac{-2\pm\sqrt{(2)^2-4(4)(-1)}}{2(4)}$

$\Rightarrow\ x=\sin^{-1}(1)$ or $\sin x=\dfrac{-2\pm\sqrt{4+16}}{8}=\dfrac{-2\pm\sqrt{20}}{8}$

$\Rightarrow\ x=\dfrac{\pi}{2}$ or $\sin x=\dfrac{-2+\sqrt{20}}{8}$ or $\sin x=\dfrac{-2-\sqrt{20}}{8}$

$\sin x=0.309$ or $\sin x=-0.809$

$\Rightarrow\ x=\sin^{-1}(0.309)$ or $x=\sin^{-1}(-0.809)$

$\approx 18,\ 162$ or $\approx 234\ ,\ 306$

$\Rightarrow\ x=18\cdot\dfrac{\pi}{180}\ ,162\cdot\dfrac{\pi}{180}$ or $x=234\cdot\dfrac{\pi}{180}\ ,306\cdot\dfrac{\pi}{180}$

$=\dfrac{\pi}{10}\ ,\ \dfrac{9\pi}{10}$ or $=\dfrac{13\pi}{10}\ ,\ \dfrac{17\pi}{10}$

$\because$ period of $\sin x$ is 2π

$\therefore$ general value of $x = \frac{\pi}{10} + 2n\pi, \frac{9\pi}{10} + 2n\pi, \frac{13\pi}{10} + 2n\pi, \frac{17\pi}{10} + 2n\pi, \frac{\pi}{2} + 2n\pi$

S.Set $= \left\{\frac{\pi}{10} + 2n\pi\right\} \cup \left\{\frac{9\pi}{10} + 2n\pi\right\} \cup \left\{\frac{13\pi}{10} + 2n\pi\right\} \cup \left\{\frac{17\pi}{10} + 2n\pi\right\} \cup \left\{\frac{\pi}{2} + 2n\pi\right\}$

Question # 11

Find the solution set; $\sec 3\theta = \sec\theta$

Solution $\sec 3\theta = \sec\theta$

$\Rightarrow \frac{1}{\cos 3\theta} = \frac{1}{\cos\theta}$

$\Rightarrow \cos 3\theta = \cos\theta$

$\Rightarrow 4\cos^3\theta - 3\cos\theta = \cos\theta \qquad \because \cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\Rightarrow 4\cos^3\theta - 3\cos\theta - \cos\theta = 0$

$\Rightarrow 4\cos^3\theta - 4\cos\theta = 0$

$\Rightarrow 4\cos\theta\left(\cos^2\theta - 1\right) = 0$

$\Rightarrow 4\cos\theta = 0$ or $\cos^2\theta - 1 = 0$

$\Rightarrow \cos\theta = 0$ or $\cos^2\theta = 1$

$\Rightarrow \theta = \cos^{-1}(0)$ or $\cos\theta = \pm 1$

$\Rightarrow \theta = \cos^{-1}(1)$, $\theta = \cos^{-1}(-1)$

$\Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$ or $\theta = 0, \pi$

$\because$ period of $\cos\theta$ is 2π

$\therefore$ general values of $\theta = \frac{\pi}{2} + 2n\pi$, $\frac{3\pi}{2} + 2n\pi$, $0 + 2n\pi$, $\pi + 2n\pi$

$= \frac{\pi}{2} + 2n\pi$, $\frac{3\pi}{2} + 2n\pi$, $n\pi$

S. Set $= \left\{\frac{\pi}{2} + 2n\pi\right\} \cup \left\{\frac{3\pi}{2} + 2n\pi\right\} \cup \{n\pi\}$ where $n \in \mathbb{Z}$.

Question # 12

Find the solution set; $\tan 2\theta + \cot\theta = 0$

Solution $\tan 2\theta + \cot\theta = 0$

$\Rightarrow \tan 2\theta = -\cot\theta$

$\Rightarrow \frac{\sin 2\theta}{\cos 2\theta} = -\frac{\cos\theta}{\sin\theta}$

$\Rightarrow \frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = -\frac{\cos\theta}{\sin\theta}$

$\Rightarrow \left(2\sin\cos\theta\right)\left(\sin\theta\right) = \left(-\cos\theta\right)\left(\cos^2\theta - \sin^2\theta\right)$

$\Rightarrow 2\sin^2\theta\,\cos\theta = -\cos^3\theta + \sin^2\theta\,\cos\theta$

$\Rightarrow 2\sin^2\theta\cos\theta + \cos^3\theta - \sin^2\theta\,\cos\theta = 0$

$\Rightarrow \sin^2\theta\cos\theta + \cos^3\theta = 0$

$\Rightarrow\ \cos\theta\left(\sin^2\theta+\cos^2\theta\right)=0 \qquad \Rightarrow\ \cos\theta(1)=0$

$\Rightarrow\ \theta=\cos^{-1}(0)$

$=\dfrac{\pi}{2}\ ,\ \dfrac{3\pi}{2}$

$\because$ period of $\cos\theta$ is 2π

$\therefore$ general values of $\theta=\dfrac{\pi}{2}+2n\pi\ ,\ \dfrac{3\pi}{2}+2n\pi$

S. Set = $\left\{\dfrac{\pi}{2}+2n\pi\right\}\cup\left\{\dfrac{3\pi}{2}+2n\pi\right\}$ where $n\in\mathbb{Z}$.

Question # 13

Find the solution set; $\sin 2x+\sin x=0$

Solution $\sin 2x+\sin x=0$

$\Rightarrow\ 2\sin x\cos x+\sin x=0 \qquad \because\ \sin 2\theta=2\sin\theta\cos\theta$

$\Rightarrow\ \sin x\left(2\cos x+1\right)=0$

$\Rightarrow\ \sin x=0$ or $2\cos x+1=0$

$\Rightarrow\ x=\sin^{-1}(0)$ or $\cos x=-\dfrac{1}{2}$

$\Rightarrow\ x=\cos^{-1}\left(-\dfrac{1}{2}\right)$

$\Rightarrow\ x=0\ ,\ \pi$ or $x=\dfrac{2\pi}{3}\ ,\ \dfrac{4\pi}{3}$

$\because$ period of $\sin x$ and $\cos x$ is 2π

$\therefore$ general values of $x=0+2n\pi\ ,\ \pi+2n\pi\ ,\ \dfrac{2\pi}{3}+2n\pi\ ,\ \dfrac{4\pi}{3}+2n\pi$

$=n\pi\ ,\ \dfrac{2\pi}{3}+2n\pi\ ,\ \dfrac{4\pi}{3}+2n\pi$

So solution set $=\{n\pi\}\cup\left\{\dfrac{2\pi}{3}+2n\pi\right\}\cup\left\{\dfrac{4\pi}{3}+2n\pi\right\}$ where $n\in\mathbb{Z}$.

Question # 14

Find the solution set; $\sin 4x-\sin 2x=\cos 3x$

Solution $\sin 4x-\sin 2x=\cos 3x$

$\Rightarrow\ 2\cos\left(\dfrac{4x+2x}{2}\right)\sin\left(\dfrac{4x-2x}{2}\right)=\cos 3x$

$\Rightarrow\ 2\cos 3x\sin x-\cos 3x=0$

$\Rightarrow\ \cos 3x\left(2\sin x-1\right)=0$

$\Rightarrow\ \cos 3x=0$ or $2\sin x-1=0$

$\Rightarrow\ 3x=\cos^{-1}(0)$, $\sin x=\dfrac{1}{2}$

$\Rightarrow\ 3x=\dfrac{\pi}{2}\ ,\ \dfrac{3\pi}{2}$, $x=\sin^{-1}\left(\dfrac{1}{2}\right)$

$\Rightarrow\ x=\frac{\pi}{6}\ ,\ \frac{\pi}{2}\qquad ,\qquad x=\frac{\pi}{6}\ ,\ \frac{5\pi}{6}$

Since period of $\cos 3x$ is $\frac{2\pi}{3}$ and period of $\sin x$ is 2π

$\therefore$ general values of $x=\frac{\pi}{6}+\frac{2n\pi}{3}\ ,\ \frac{\pi}{2}+\frac{2n\pi}{3}\ ,\ \frac{\pi}{6}+2n\pi\ ,\ \frac{5\pi}{6}+2n\pi$

So solution set $=\left\{\frac{\pi}{6}+\frac{2n\pi}{3}\right\}\cup\left\{\frac{\pi}{2}+\frac{2n\pi}{3}\right\}\cup\left\{\frac{\pi}{6}+2n\pi\right\}\cup\left\{\frac{5\pi}{6}+2n\pi\right\}$ where $n\in\mathbb{Z}$.

Question # 15

Find the solution set; $\sin x+\cos 3x=\cos 5x$

Solution $\sin x+\cos 3x=\cos 5x$

$\Rightarrow\ \sin x=\cos 5x-\cos 3x$

$\Rightarrow\ \sin x=-2\sin\left(\frac{5x+3x}{2}\right)\sin\left(\frac{5x-3x}{2}\right)$

$\Rightarrow\ \sin x=-2\sin 4x\,\sin x$

$\Rightarrow\ \sin x+2\sin 4x\,\sin x=0$

$\Rightarrow\ \sin x\left(1+2\sin 4x\right)=0$

$\Rightarrow\ \sin x=0\qquad$ or $\qquad 1+2\sin 4x=0$

$\Rightarrow\ x=\sin^{-1}(0)\qquad$ or $\qquad \sin 4x=-\frac{1}{2}\ \Rightarrow\ 4x=\sin^{-1}\left(-\frac{1}{2}\right)$

$\Rightarrow\ x=0\ ,\ \pi\qquad$ or $\qquad 4x=\frac{7\pi}{6}\ ,\ \frac{11\pi}{6}\ \Rightarrow\ x=\frac{7\pi}{24}\ ,\ \frac{11\pi}{24}$

Since period of $\sin x$ is 2π and period of $\sin 4x$ is $\frac{2\pi}{4}=\frac{\pi}{2}$

$\therefore$ general values of $x=0+2n\pi\ ,\ \pi+2n\pi\ ,\ \frac{7\pi}{24}+\frac{n\pi}{2}\ ,\ \frac{11\pi}{24}+\frac{n\pi}{2}$

So solution set $=\{2n\pi\}\cup\{\pi+2n\pi\}\cup\left\{\frac{7\pi}{24}+\frac{n\pi}{2}\right\}\cup\left\{\frac{11\pi}{24}+\frac{n\pi}{2}\right\}$ where $n\in\mathbb{Z}$.

Question # 16

Find the solution set; $\sin 3x+\sin 2x+\sin x=0$

Solution $\sin 3x+\sin 2x+\sin x=0$

$\Rightarrow\ \left(\sin 3x+\sin x\right)+\sin 2x=0$

$\Rightarrow\ 2\sin\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)+\sin 2x=0$

$\Rightarrow\ 2\sin 2x\,\cos x+\sin 2x=0$

$\Rightarrow\ \sin 2x\left(2\cos x+1\right)=0$

$\Rightarrow\ \sin 2x=0\qquad$ or $\qquad 2\cos x+1=0$

$\Rightarrow\ 2x=\sin^{-1}(0)\qquad$ or $\qquad \cos x=-\frac{1}{2}$

$$\Rightarrow \ 2x = 0\,,\ \pi \qquad \text{or} \qquad x = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\Rightarrow \ x = 0\,,\ \frac{\pi}{2} \qquad \text{or} \qquad x = \frac{2\pi}{3}\,,\ \frac{4\pi}{3}$$

Since period of $\sin 2x$ is $\frac{2\pi}{2} = \pi$ and period of $\cos x$ is 2π

$\therefore$ general values of $x = 0 + n\pi\,,\ \frac{\pi}{2} + n\pi\,,\ \frac{2\pi}{3} + 2n\pi\,,\ \frac{4\pi}{3} + 2n\pi$

S. Set $= \{n\pi\} \cup \left\{\frac{\pi}{2} + n\pi\right\} \cup \left\{\frac{2\pi}{3} + 2n\pi\right\} \cup \left\{\frac{4\pi}{3} + 2n\pi\right\}$ where $n \in \mathbb{Z}$.

Question # 17

Find the solution set; $\sin 7x - \sin x = \sin 3x$

Solution

$$\sin 7x - \sin x = \sin 3x$$

$$\Rightarrow \ 2\cos\left(\frac{7x + x}{2}\right)\sin\left(\frac{7x - x}{2}\right) = \sin 3x$$

$$\Rightarrow \ 2\cos 4x \sin 3x - \sin 3x = 0$$

$$\Rightarrow \ \sin 3x(2\cos 4x - 1) = 0$$

$$\Rightarrow \ \sin 3x = 0 \qquad \text{or} \qquad 2\cos 4x - 1 = 0$$

$$\Rightarrow \ 3x = \sin^{-1}(0) \qquad \text{or} \qquad \cos 4x = \frac{1}{2}$$

$$\Rightarrow \ 3x = 0\,,\ \pi \qquad \text{or} \qquad 4x = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}\,,\ \frac{5\pi}{3}$$

$$\Rightarrow \ x = 0\,,\ \frac{\pi}{3} \qquad \text{or} \qquad x = \frac{\pi}{12}\,,\ \frac{5\pi}{12}$$

Since period of $\sin 3x$ is $\frac{2\pi}{3}$ and period of $\cos 4x$ is $\frac{2\pi}{4} = \frac{\pi}{2}$

$\therefore$ general values of $x = 0 + \frac{2n\pi}{3}\,,\ \frac{\pi}{3} + \frac{2n\pi}{3}\,,\ \frac{\pi}{12} + \frac{n\pi}{2}\,,\ \frac{5\pi}{12} + \frac{n\pi}{2}$

So S. set $= \left\{\frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{3} + \frac{2n\pi}{3}\right\} \cup \left\{\frac{\pi}{12} + \frac{n\pi}{2}\right\} \cup \left\{\frac{5\pi}{12} + \frac{n\pi}{2}\right\}$ where $n \in \mathbb{Z}$.

Question # 18

Find the solution set; $\sin x + \sin 3x + \sin 5x = 0$

Solution

$$\sin x + \sin 3x + \sin 5x = 0$$

$$\Rightarrow \ (\sin 5x + \sin x) + \sin 3x = 0$$

$$\Rightarrow \ 2\sin\left(\frac{5x + x}{2}\right)\cdot\cos\left(\frac{5x - x}{2}\right) + \sin 3x = 0$$

$$\Rightarrow \ 2\sin 3x \cos 2x + \sin 3x = 0$$

$$\Rightarrow \ \sin 3x(2\cos 2x + 1) = 0$$

$$\Rightarrow \ \sin 3x = 0 \qquad \text{or} \qquad 2\cos 2x + 1 = 0$$

$$\Rightarrow\ 3x=\sin^{-1}(0) \quad \text{or} \quad 2\cos 2x=-1 \ \Rightarrow\ 2x=\cos^{-1}\left(-\frac{1}{2}\right)$$

$$\Rightarrow\ 3x=0\,,\ \pi \qquad \text{or} \qquad 2x=\frac{2\pi}{3}\,,\ \frac{4\pi}{3}$$

$$\Rightarrow\ x=0\,,\ \frac{\pi}{3} \qquad \text{or} \qquad x=\frac{\pi}{3}\,,\ \frac{2\pi}{3}$$

Since period of $\sin 3x$ is $\frac{2\pi}{3}$ and period of $\cos 2x$ is $\frac{2\pi}{2}=\pi$

$\therefore$ general values of $x=0+\frac{2n\pi}{3}\,,\ \frac{\pi}{3}+\frac{2n\pi}{3}\,,\ \frac{\pi}{3}+n\pi\,,\ \frac{2\pi}{3}+n\pi$

S.Set = $\left\{\frac{2n\pi}{3}\right\}\cup\left\{\frac{\pi}{3}+\frac{2n\pi}{3}\right\}\cup\left\{\frac{\pi}{3}+n\pi\right\}\cup\left\{\frac{2\pi}{3}+n\pi\right\}$ where $n\in\mathbb{Z}$

Question # 19

Find the solution set; $\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta=0$

Solution $\sin\theta+\sin 3\theta+\sin 5\theta+\sin 7\theta=0$

$$\Rightarrow\ (\sin 7\theta+\sin\theta)+(\sin 5\theta+\sin 3\theta)=0$$

$$\Rightarrow\ 2\sin\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)+2\sin\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)=0$$

$$\Rightarrow\ 2\sin 4\theta\,\cos 3\theta+2\sin 4\theta\,\cos\theta=0$$

$$\Rightarrow\ 2\sin 4\theta\,(\cos 3\theta+\cos\theta)=0$$

$$\Rightarrow\ 2\sin 4\theta\left(2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right)=0$$

$$\Rightarrow\ 4\sin 4\theta\ \cos 2\theta\ \cos\theta=0$$

$$\Rightarrow\ \sin 4\theta=0 \quad \text{or} \quad \cos 2\theta=0 \quad \text{or} \quad \cos\theta=0$$

$$\Rightarrow\ 4\theta=\sin^{-1}(0)\ , \quad 2\theta=\cos^{-1}(0)\ , \quad \theta=\cos^{-1}(0)$$

$$\Rightarrow\ 4\theta=0\ ,\ \pi \quad \text{or} \quad 2\theta=\frac{\pi}{2}\,,\ \frac{3\pi}{2} \quad \text{or} \quad \theta=\frac{\pi}{2}\,,\ \frac{3\pi}{2}$$

$$\Rightarrow\ \theta=0\ ,\ \frac{\pi}{4} \quad \text{or} \quad \theta=\frac{\pi}{4}\,,\ \frac{3\pi}{4}$$

Since period of $\sin 4\theta$ is $\frac{2\pi}{4}=\frac{\pi}{2}$, $\cos 2\theta$ is $\frac{2\pi}{2}=\pi$ and $\cos\theta$ is 2π

$\therefore$ general values of $\theta=0+\frac{n\pi}{2}\,,\ \frac{\pi}{4}+\frac{n\pi}{2}\,,\ \frac{\pi}{4}+n\pi\,,\ \frac{3\pi}{4}+n\pi\,,\ \frac{\pi}{2}+2n\pi\,,\ \frac{3\pi}{2}+2n\pi$

S. Set $=\left\{\frac{n\pi}{2}\right\}\cup\left\{\frac{\pi}{4}+\frac{n\pi}{2}\right\}\cup\left\{\frac{\pi}{4}+n\pi\right\}\cup\left\{\frac{3\pi}{4}+n\pi\right\}\cup\left\{\frac{\pi}{2}+2n\pi\right\}\cup\left\{\frac{3\pi}{2}+2n\pi\right\}$.

Question # 20

Find the solution set; $\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta=0$

Solution $\cos\theta+\cos 3\theta+\cos 5\theta+\cos 7\theta=0$

$$\Rightarrow\ (\cos 7\theta+\cos\theta)+(\cos 5\theta+\cos 3\theta)=0$$

$$\Rightarrow\ 2\cos\left(\frac{7\theta+\theta}{2}\right)\cos\left(\frac{7\theta-\theta}{2}\right)+2\cos\left(\frac{5\theta+3\theta}{2}\right)\cos\left(\frac{5\theta-3\theta}{2}\right)=0$$

$$\Rightarrow\ 2\cos4\theta\ \cos3\theta+2\cos4\theta\ \cos\theta=0$$

$$\Rightarrow\ 2\cos4\theta\ (\cos3\theta+\cos\theta)=0$$

$$\Rightarrow\ 2\cos4\theta\left(2\cos\left(\frac{3\theta+\theta}{2}\right)\cos\left(\frac{3\theta-\theta}{2}\right)\right)=0$$

$$\Rightarrow\ 4\cos4\theta\ \cos2\theta\ \cos\theta=0$$

Now do yourself as above question.

If you found any error, submit at http://www.megalecture@gmail.com