IMPORTANT FORMULAS

The Law of Cosine

- $a^2 = b^2 + c^2 - 2bc\cos\alpha$
- $b^2 = c^2 + a^2 - 2ca\cos\beta$
- $c^2 = a^2 + b^2 - 2ab\cos\gamma$
- $\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$
- $\cos\beta = \dfrac{c^2 + a^2 - b^2}{2ca}$
- $\cos\gamma = \dfrac{a^2 + b^2 - c^2}{2ab}$

The Law of Sine

- $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

The Law of Tangent

- $\dfrac{a-b}{a+b} = \dfrac{\tan\dfrac{\alpha-\beta}{2}}{\tan\dfrac{\alpha+\beta}{2}}$
- $\dfrac{b-c}{b+c} = \dfrac{\tan\dfrac{\beta-\gamma}{2}}{\tan\dfrac{\beta+\gamma}{2}}$
- $\dfrac{c-a}{c+a} = \dfrac{\tan\dfrac{\gamma-\alpha}{2}}{\tan\dfrac{\gamma+\alpha}{2}}$

Half Angle Formulas

- $\sin\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{bc}}$
- $\sin\dfrac{\beta}{2} = \sqrt{\dfrac{(s-c)(s-a)}{ca}}$
- $\sin\dfrac{\gamma}{2} = \sqrt{\dfrac{(s-a)(s-b)}{ab}}$
- $\cos\dfrac{\alpha}{2} = \sqrt{\dfrac{s(s-a)}{bc}}$
- $\cos\dfrac{\beta}{2} = \sqrt{\dfrac{s(s-b)}{ac}}$
- $\cos\dfrac{\gamma}{2} = \sqrt{\dfrac{s(s-c)}{ab}}$
- $\tan\dfrac{\alpha}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$
- $\tan\dfrac{\beta}{2} = \sqrt{\dfrac{(s-c)(s-a)}{s(s-b)}}$
- $\tan\dfrac{\gamma}{2} = \sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}}$

where $s = \dfrac{a+b+c}{2} \Rightarrow 2s = a+b+c$

Area of the Triangle $(=\Delta)$

- $\Delta = \dfrac{1}{2}bc\sin\alpha = \dfrac{1}{2}ca\sin\beta = \dfrac{1}{2}ab\sin\gamma$
- $\Delta = \dfrac{a^2\sin\beta\sin\gamma}{2\sin\alpha} = \dfrac{b^2\sin\gamma\sin\alpha}{2\sin\beta} = \dfrac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$
- $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ *(Hero's Formula)*

Circum Radius $(=R)$

- $R = \dfrac{a}{2\sin\alpha} = \dfrac{b}{2\sin\beta} = \dfrac{c}{2\sin\gamma}$
- $R = \dfrac{abc}{4\Delta}$

In-Radius $(=r)$

- $r = \dfrac{\Delta}{s}$

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Escribed Circle

- $r_1 = \dfrac{\Delta}{s-a}$
- $r_2 = \dfrac{\Delta}{s-b}$
- $r_3 = \dfrac{\Delta}{s-c}$

Question # 1

Show that

(i) $r = 4R\sin\dfrac{\alpha}{2}\sin\dfrac{\beta}{2}\sin\dfrac{\gamma}{2}$ (ii) $s = 4R\cos\dfrac{\alpha}{2}\cos\dfrac{\beta}{2}\cos\dfrac{\gamma}{2}$

Solution

(i)

$$\text{R.H.S} = 4R\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$$

$$= 4R\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{(s-a)(s-c)}{ac}}\sqrt{\frac{(s-a)(s-b)}{ab}}$$

$$= 4R\sqrt{\frac{(s-b)(s-c)(s-a)(s-c)(s-a)(s-b)}{(bc)(ac)(ab)}}$$

$$= 4R\sqrt{\frac{(s-a)^2(s-b)^2(s-c)^2}{a^2b^2c^2}}$$

$$= 4R\frac{(s-a)(s-b)(s-c)}{abc}$$

$$= 4\left(\frac{abc}{4\Delta}\right)\frac{(s-a)(s-b)(s-c)}{abc} \qquad \because R = \frac{abc}{4\Delta}$$

$$= \frac{(s-a)(s-b)(s-c)}{\Delta} \qquad = \frac{s(s-a)(s-b)(s-c)}{s\Delta}$$

$$= \frac{\Delta^2}{s\Delta} \qquad \because \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \frac{\Delta}{s} = r = \text{L.H.S}$$

(ii)

$$\text{R.H.S} = 4R\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$$

$$= 4R\sqrt{\frac{s(s-a)}{bc}}\sqrt{\frac{s(s-b)}{ac}}\sqrt{\frac{s(s-c)}{ab}}$$

$$= 4R\sqrt{\frac{s^2\cdot s(s-a)(s-b)(s-c)}{(bc)(ac)(ab)}} \qquad = 4R\sqrt{\frac{s^2\Delta^2}{a^2b^2c^2}}$$

$$= 4R\frac{s\Delta}{abc} \qquad = 4\left(\frac{abc}{4\Delta}\right)s\frac{\Delta}{abc} \qquad \because R = \frac{abc}{4\Delta}$$

$$= s \quad = \text{L.H.S}$$

Question # 2

Show that:

$$r = a\sin\frac{\beta}{2}\sin\frac{\gamma}{2}\sec\frac{\alpha}{2} = b\sin\frac{\gamma}{2}\sin\frac{\alpha}{2}\sec\frac{\beta}{2} = c\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sec\frac{\gamma}{2}$$

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Solution

We take

$$a\sin\frac{\beta}{2}\sin\frac{\gamma}{2}\sec\frac{\alpha}{2}=a\sin\frac{\beta}{2}\sin\frac{\gamma}{2}\frac{1}{\cos\frac{\alpha}{2}}$$

$$=a\sqrt{\frac{(s-a)(s-c)}{ac}}\sqrt{\frac{(s-a)(s-b)}{ab}}\frac{1}{\sqrt{\frac{s(s-a)}{bc}}}$$

$$=a\sqrt{\frac{(s-a)(s-c)}{ac}}\sqrt{\frac{(s-a)(s-b)}{ab}}\sqrt{\frac{bc}{s(s-a)}}$$

$$=a\sqrt{\frac{(s-a)(s-c)(s-a)(s-b)(bc)}{(ac)(ab)s(s-a)}}$$

$$=a\sqrt{\frac{(s-a)(s-b)(s-c)}{a^2s}}\qquad=a\sqrt{\frac{s(s-a)(s-b)(s-c)}{a^2s^2}}$$

$$=a\frac{\sqrt{s(s-a)(s-b)(s-c)}}{as}\qquad=\frac{\Delta}{s}\qquad=r$$

$$\Rightarrow\quad a\sin\frac{\beta}{2}\sin\frac{\gamma}{2}\sec\frac{\alpha}{2}=r\quad\ldots\ldots\ldots\ldots\ldots(i)$$

Similarly prove yourself

$$b\sin\frac{\gamma}{2}\sin\frac{\alpha}{2}\sec\frac{\beta}{2}=r\quad\ldots\ldots\ldots\ldots(ii)$$

$$c\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sec\frac{\gamma}{2}=r\quad\ldots\ldots\ldots\ldots(iii)$$

From (i), (ii) and (iii)

$$r=a\sin\frac{\beta}{2}\sin\frac{\gamma}{2}\sec\frac{\alpha}{2}=b\sin\frac{\gamma}{2}\sin\frac{\alpha}{2}\sec\frac{\beta}{2}=c\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sec\frac{\gamma}{2}$$

Question # 3

Show that:

(i) $r_1=4R\sin\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$ (ii) $r_2=4R\cos\frac{\alpha}{2}\sin\frac{\beta}{2}\cos\frac{\gamma}{2}$

(iii) $r_3=4R\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\sin\frac{\gamma}{2}$

Solution

$$\text{R.H.S}=4R\sin\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$$

$$=4R\sqrt{\frac{(s-b)(s-c)}{bc}}\sqrt{\frac{s(s-b)}{ac}}\sqrt{\frac{s(s-c)}{ab}}$$

$$=4R\sqrt{\frac{(s-b)(s-c)s(s-b)s(s-c)}{(bc)(ac)(ab)}}\qquad=4R\sqrt{\frac{s^2(s-b)^2(s-c)^2}{a^2b^2c^2}}$$

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$$= 4R\frac{s(s-b)(s-c)}{abc} \quad = 4\frac{abc}{4\Delta}\frac{s(s-b)(s-c)}{abc}\cdot\frac{(s-a)}{(s-a)} \qquad \because\ R = \frac{abc}{4\Delta}$$

$$= \frac{s(s-a)(s-b)(s-c)}{\Delta(s-a)}$$

$$= \frac{\Delta^2}{\Delta(s-a)} \quad = \frac{\Delta}{(s-a)} \quad = r_1 \quad = \text{R.H.S}$$

(ii) & (iii) *Do yourself*

Question # 4
Show that:

(i) $r_1 = s\tan\frac{\alpha}{2}$ (ii) $r_2 = s\tan\frac{\beta}{2}$ (iii) $r_3 = s\tan\frac{\gamma}{2}$

Solution

$$\text{R.H.S} = s\tan\frac{\alpha}{2}$$

$$= s\sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \quad = s\sqrt{\frac{(s-b)(s-c)}{s(s-a)}\cdot\frac{s(s-a)}{s(s-a)}}$$

$$= s\sqrt{\frac{s(s-a)(s-b)(s-c)}{s^2(s-a)^2}}$$

$$= s\sqrt{\frac{\Delta^2}{s^2(s-a)^2}} = s\,\frac{\Delta}{s(s-a)} \quad = \frac{\Delta}{(s-a)} \quad = r_1 \ = \ \text{L.H.S}$$

(ii) & (iii) *Do yourself*

Question # 5
Prove that:

(i) $r_1 r_2 + r_2 r_3 + r_3 r_1 = s^2$ (ii) $r\, r_1 r_2 r_3 = \Delta^2$

(iii) $r_1 + r_2 + r_3 - r = 4R$ (iv) $r_1 r_2 r_3 = r s^2$

Solution

(i) $\text{L.H.S} = r_1 r_2 + r_2 r_3 + r_3 r_1$

$$= \left(\frac{\Delta}{s-a}\right)\left(\frac{\Delta}{s-b}\right) + \left(\frac{\Delta}{s-b}\right)\left(\frac{\Delta}{s-c}\right) + \left(\frac{\Delta}{s-c}\right)\left(\frac{\Delta}{s-a}\right)$$

$$= \frac{\Delta^2}{(s-a)(s-b)} + \frac{\Delta^2}{(s-b)(s-c)} + \frac{\Delta^2}{(s-c)(s-a)}$$

$$= \Delta^2\left(\frac{1}{(s-a)(s-b)} + \frac{1}{(s-b)(s-c)} + \frac{1}{(s-c)(s-a)}\right)$$

$$= \Delta^2\left(\frac{s-c+s-a+s-b}{(s-a)(s-b)(s-c)}\right) \quad = \Delta^2\left(\frac{3s-(a+b+c)}{(s-a)(s-b)(s-c)}\right)$$

$$= \Delta^2\left(\frac{3s-2s}{(s-a)(s-b)(s-c)}\right) \qquad \because\ s = \frac{a+b+c}{2}$$

$$= \Delta^2\left(\frac{s}{(s-a)(s-b)(s-c)}\cdot\frac{s}{s}\right)$$

$$= \Delta^2\left(\frac{s^2}{s(s-a)(s-b)(s-c)}\right) \quad = \Delta^2\left(\frac{s^2}{\Delta^2}\right) \quad = s^2 \quad = \text{R.H.S}$$

(ii) L.H.S $= r\, r_1\, r_2\, r_3$

$$= \left(\frac{\Delta}{s}\right)\left(\frac{\Delta}{s-a}\right)\left(\frac{\Delta}{s-b}\right)\left(\frac{\Delta}{s-c}\right)$$

$$= \frac{\Delta^4}{s(s-a)(s-b)(s-c)} \quad = \frac{\Delta^4}{\Delta^2} \quad = \Delta^2 \quad = \text{R.H.S}$$

(iii) L.H.S $= r_1 + r_2 + r_3 - r$

$$= \frac{\Delta}{s-a} + \frac{\Delta}{s-b} + \frac{\Delta}{s-c} - \frac{\Delta}{s} \quad = \Delta\left(\frac{1}{s-a} + \frac{1}{s-b} + \frac{1}{s-c} - \frac{1}{s}\right)$$

$$= \Delta\left(\frac{(s-b)+(s-a)}{(s-a)(s-b)} + + \frac{s-(s-c)}{s(s-c)}\right) \quad = \Delta\left(\frac{2s-b-a}{(s-a)(s-b)} + \frac{s-s+c}{s(s-c)}\right)$$

$$= \Delta\left(\frac{a+b+c-b-a}{(s-a)(s-b)} + \frac{c}{s(s-c)}\right) \qquad \because 2s = a+b+c$$

$$= \Delta\left(\frac{c}{(s-a)(s-b)} + \frac{c}{s(s-c)}\right) \quad = c\Delta\left(\frac{1}{(s-a)(s-b)} + \frac{1}{s(s-c)}\right)$$

$$= c\Delta\left(\frac{s(s-c)-(s-a)(s-b)}{s(s-a)(s-b)(s-c)}\right) \quad = c\Delta\left(\frac{s^2 - sc + s^2 - as - bs + ab}{\Delta^2}\right)$$

$$= c\left(\frac{2s^2 - s(a+b+c) + ab}{\Delta}\right) \quad = c\left(\frac{2s^2 - s(2s) + ab}{\Delta}\right)$$

$$= c\left(\frac{2s^2 - 2s^2 + ab}{\Delta}\right) \quad = \frac{abc}{\Delta} \quad = 4\cdot\frac{abc}{4\Delta} \quad = 4R \quad = \text{R.H.S}$$

(iv) L.H.S $= \left(\frac{\Delta}{s-a}\right)\left(\frac{\Delta}{s-b}\right)\left(\frac{\Delta}{s-c}\right)$

$$= \frac{\Delta^3}{(s-a)(s-b)(s-c)} \quad = \frac{s\Delta^3}{s(s-a)(s-b)(s-c)}$$

$$= \frac{s\Delta^3}{\Delta^2} \quad = s\Delta \quad = s^2\frac{\Delta}{s} \quad = s^2\, r \quad = r\, s^2 \quad = \text{R.H.S}$$

Question # 6

Find R, r, r_1, r_2 and r_3, if measures of the sides of triangle ABC are

(i) $a = 13$, $b = 14$, $c = 15$ (ii) $a = 34,$ $b = 20,$, $c = 42$

Solution

(i) $a = 13$, $b = 14$, $c = 15$

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$$s=\frac{a+b+c}{2}=\frac{13+14+15}{2}=21$$
$$s-a=21-13=8$$
$$s-b=21-14=7$$
$$s-c=21-15=6$$

So $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$

$$=\sqrt{21(8)(7)(6)}=\sqrt{7056}=84$$

Now

$$R=\frac{abc}{4\Delta}=\frac{(13)(14)(15)}{4(84)}=8.125$$
$$r=\frac{\Delta}{s}=\frac{84}{21}=4$$
$$r_1=\frac{\Delta}{s-a}=\frac{84}{8}=10.5$$
$$r_2=\frac{\Delta}{s-b}=\frac{84}{7}=12$$
$$r_3=\frac{\Delta}{s-c}=\frac{84}{6}=14$$

(ii) *Do yourself*

Question # 7

Prove that in an equilateral triangle,

(i) $r:R:r_1=1:2:3$ (ii) $r:R:r_1:r_2:r_3=1:2:3:3:3$

Solution

(i) *Do yourself*

(ii) In equilateral triangle all the sides are equal so $a=b=c$

$$s=\frac{a+b+c}{2}=\frac{a+a+a}{2}=\frac{3a}{2}$$
$$s-a=\frac{3a}{2}-a=\left(\frac{3}{2}-1\right)a=\frac{1}{2}a$$

Now

$$\Delta=\sqrt{s(s-a)(s-b)(s-c)}$$
$$=\sqrt{s(s-a)(s-a)(s-a)}=\sqrt{s(s-a)^3}=\sqrt{\frac{3a}{2}\left(\frac{1}{2}a\right)^3}$$
$$=\sqrt{\frac{3a}{2}\left(\frac{a^3}{8}\right)}=\sqrt{\frac{3a^4}{16}}=\frac{\sqrt{3}\,a^2}{4}$$

Now $$r=\frac{\Delta}{s}=\frac{\sqrt{3}\,a^2/4}{3a/2}=\frac{\sqrt{3}\,a^2}{4}\cdot\frac{2}{3a}=\frac{\sqrt{3}\,a}{6}$$

$$R=\frac{abc}{4\Delta}=\frac{a\cdot a\cdot a}{4\sqrt{3}a^2/4}=\frac{a}{\sqrt{3}}=\frac{a}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}\,a}{3}$$

$$r_1=\frac{\Delta}{s-a}=\frac{\sqrt{3}a^2/4}{\frac{1}{2}a}=\frac{\sqrt{3}\,a^2}{4}\cdot\frac{2}{a}=\frac{\sqrt{3}\,a}{2}$$

$$r_2=\frac{\Delta}{s-b}=\frac{\Delta}{s-a}=\frac{\sqrt{3}\,a}{2}$$

$$r_3=\frac{\Delta}{s-c}=\frac{\Delta}{s-a}=\frac{\sqrt{3}\,a}{2}$$

Now

$$r:R:r_1:r_2:r_3=\frac{\sqrt{3}\,a}{6}:\frac{\sqrt{3}\,a}{3}:\frac{\sqrt{3}\,a}{2}:\frac{\sqrt{3}\,a}{2}:\frac{\sqrt{3}\,a}{2}$$

$$=\frac{1}{6}:\frac{1}{3}:\frac{1}{2}:\frac{1}{2}:\frac{1}{2} \qquad \div\text{ing by } \sqrt{3}\,a$$

$$=1:2:3:3:3 \qquad \times\text{ing by } 6$$

Question # 8

Prove that:

(i) $\Delta=r^2\cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$ (ii) $r=s\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

(iii) $\Delta=4Rr\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$

Solution

(i) R.H.S $=r^2\cot\frac{\alpha}{2}\cot\frac{\beta}{2}\cot\frac{\gamma}{2}$

$$=r^2\,\frac{1}{\tan\frac{\alpha}{2}}\cdot\frac{1}{\tan\frac{\beta}{2}}\cdot\frac{1}{\tan\frac{\gamma}{2}}$$

$$=r^2\,\frac{1}{\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}}\cdot\frac{1}{\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}}\cdot\frac{1}{\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}}$$

$$=r^2\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}\cdot\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}\cdot\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$$

$$=r^2\sqrt{\frac{s^3(s-a)(s-b)(s-c)}{(s-a)^2(s-b)^2(s-c)^2}}=r^2\sqrt{\frac{s^3}{(s-a)(s-b)(s-c)}}$$

$$=r^2\sqrt{\frac{s^3}{(s-a)(s-b)(s-c)}\cdot\frac{s}{s}}=r^2\sqrt{\frac{s^4}{s(s-a)(s-b)(s-c)}}$$

$$=r^2\sqrt{\frac{s^4}{\Delta^2}}=r^2\,\frac{s^2}{\Delta}=\left(\frac{\Delta}{s}\right)^2\frac{s^2}{\Delta} \qquad \because\ r=\frac{\Delta}{s}$$

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$$= \frac{\Delta^2}{s^2}\frac{s^2}{\Delta} = \Delta = \text{L.H.S}$$

(ii) *Do yourself*

(iii) $$\text{R.H.S} = 4Rr\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$$

$$= 4Rr\sqrt{\frac{s(s-a)}{bc}}\cdot\sqrt{\frac{s(s-b)}{ca}}\cdot\sqrt{\frac{s(s-c)}{ab}}$$

$$= 4Rr\sqrt{\frac{s(s-a)\cdot s(s-b)\cdot s(s-c)}{(bc)(ac)(ab)}}$$

$$= 4Rr\sqrt{\frac{s^2\cdot s(s-a)(s-b)(s-c)}{a^2b^2c^2}} = 4Rr\sqrt{\frac{s^2\cdot\Delta^2}{a^2b^2c^2}}$$

$$= 4Rr\frac{s\Delta}{abc} = 4\left(\frac{abc}{4\Delta}\right)\left(\frac{\Delta}{s}\right)\frac{s\Delta}{abc} = \Delta = \text{L.H.S}$$

Question # 9

Show that

(i) $\dfrac{1}{2rR} = \dfrac{1}{ab} + \dfrac{1}{bc} + \dfrac{1}{ca}$ (ii) $\dfrac{1}{r} = \dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}$

Solution

(i) $$\text{L.H.S} = \frac{1}{2rR}$$

$$= \frac{1}{2\left(\frac{\Delta}{s}\right)\left(\frac{abc}{4\Delta}\right)} = \frac{4s\Delta}{2\Delta abc} = \frac{2s}{abc} = \frac{a+b+c}{abc} \qquad \because 2s = a+b+c$$

$$= \frac{a}{abc} + \frac{b}{abc} + \frac{c}{abc}$$

$$= \frac{1}{bc} + \frac{1}{ac} + \frac{1}{ab} = \frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = \text{R.H.S}$$

(ii) $$\text{R.H.S} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$$

$$= \frac{1}{\frac{\Delta}{s-a}} + \frac{1}{\frac{\Delta}{s-b}} + \frac{1}{\frac{\Delta}{s-c}} = \frac{s-a}{\Delta} + \frac{s-b}{\Delta} + \frac{s-c}{\Delta}$$

$$= \frac{s-a+s-b+s-c}{\Delta} = \frac{3s-(a+b+c)}{\Delta}$$

$$= \frac{3s-2s}{\Delta} \qquad \because 2s = a+b+c$$

$$= \frac{s}{\Delta} = \frac{1}{\Delta/s} = \frac{1}{r} = \text{L.H.S}$$

Question # 10

We take $\dfrac{a\sin\frac{\beta}{2}\sin\frac{\gamma}{2}}{\cos\frac{\alpha}{2}} = a\sin\frac{\beta}{2}\sin\frac{\gamma}{2}\,\dfrac{1}{\cos\frac{\alpha}{2}}$

Solution *Now see Question # 2*

Question # 11

Prove that: $abc(\sin\alpha+\sin\beta+\sin\gamma)=4\Delta s$

Solution

L.H.S $= abc(\sin\alpha+\sin\beta+\sin\gamma)$

Since $\Delta=\frac{1}{2}ab\sin\gamma=\frac{1}{2}bc\sin\alpha=\frac{1}{2}ca\sin\beta$

$\therefore\ \sin\gamma=\dfrac{2\Delta}{ab}\ ,\quad \sin\alpha=\dfrac{2\Delta}{bc}\ ,\quad \sin\beta=\dfrac{2\Delta}{ca}$

Thus L.H.S $= abc\left(\dfrac{2\Delta}{bc}+\dfrac{2\Delta}{ac}+\dfrac{2\Delta}{ab}\right)$

$= abc\left(\dfrac{2\Delta a+2\Delta b+2\Delta c}{abc}\right) = 2\Delta a+2\Delta b+2\Delta c$

$=2\Delta(a+b+c) = 2\Delta(2s) \qquad \because\ 2s=a+b+c$

$=4\Delta s$ = R.H.S

Question # 12

(i) $(r_1+r_2)\tan\dfrac{\gamma}{2}=c$ (ii) $(r_3-r)\cot\dfrac{\gamma}{2}=c$

Solution

(i) L.H.S $=(r_1+r_2)\tan\dfrac{\gamma}{2}$

$=\left(\dfrac{\Delta}{s-a}+\dfrac{\Delta}{s-b}\right)\sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}}$

$=\left(\dfrac{\Delta(s-b)+\Delta(s-a)}{(s-a)(s-b)}\right)\sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}\cdot\dfrac{s(s-c)}{s(s-c)}}$

$=\Delta\left(\dfrac{s-b+s-a}{(s-a)(s-b)}\right)\sqrt{\dfrac{s(s-a)(s-b)(s-c)}{s^2(s-c)^2}}$

$=\Delta\left(\dfrac{2s-a-b}{(s-a)(s-b)}\right)\sqrt{\dfrac{\Delta^2}{s^2(s-c)^2}}$

$=\Delta\left(\dfrac{a+b+c-a-b}{(s-a)(s-b)}\right)\dfrac{\Delta}{s(s-c)} \qquad \because\ 2s=a+b+c$

$=\dfrac{\Delta^2 c}{s(s-a)(s-b)(s-c)} = \dfrac{\Delta^2 c}{\Delta^2} = c$ = R.H.S

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(ii) $\text{L.H.S} = (r_3 - r)\cot\dfrac{\gamma}{2}$

$$= \left(\frac{\Delta}{s-c} - \frac{\Delta}{s}\right)\frac{1}{\tan\frac{\gamma}{2}} \quad = \Delta\left(\frac{1}{s-c} - \frac{1}{s}\right)\frac{1}{\sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}}}$$

$$= \Delta\left(\frac{s-(s-c)}{s(s-c)}\right)\sqrt{\frac{s(s-c)}{(s-a)(s-b)}} \quad = \Delta\left(\frac{c}{s(s-c)}\right)\sqrt{\frac{s(s-c)}{(s-a)(s-b)}\cdot\frac{s(s-c)}{s(s-c)}}$$

$$= \Delta\left(\frac{c}{s(s-c)}\right)\sqrt{\frac{s^2(s-c)^2}{s(s-a)(s-b)(s-c)}} \quad = \Delta\left(\frac{c}{s(s-c)}\right)\sqrt{\frac{s^2(s-c)^2}{\Delta^2}}$$

$$= \Delta\left(\frac{c}{s(s-c)}\right)\frac{s(s-c)}{\Delta} \quad = c \quad = \text{R.H.S}$$

If you found any error, submit at
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