Question # 1
Solve the following triangle ABC in which:

$b=95,\quad c=34,\quad \alpha=52^\circ.$

Solution

$b=95,\quad c=34,\quad \alpha=52^\circ.$

By law of cosine

$$a^2=b^2+c^2-2bc\cos\alpha$$
$$=(95)^2+(34)^2-2(95)(34)\cos52^\circ$$
$$=9025+1156-6460(0.6157)\quad =6203.578$$
$$\Rightarrow\ a=\sqrt{6203.578}\quad\Rightarrow\ \boxed{a=78.76}$$

Again by law of cosine

$$\cos\beta=\frac{c^2+a^2-b^2}{2ca}=\frac{(34)^2+(78.76)^2-(95)^2}{2(34)(78.76)}$$
$$=\frac{1156+6203.138-9025}{5355.68}=-\frac{1665.862}{5355.68}=-0.311$$
$$\beta=\cos^{-1}(-0.311)\quad\Rightarrow\ \boxed{\beta=108^\circ7'20''}$$

Since in any triangle

$$\alpha+\beta+\gamma=180$$
$$\Rightarrow\ \gamma=180-\alpha-\beta\ =180^\circ-52^\circ-108^\circ7'20''\quad\Rightarrow\ \boxed{\gamma=19^\circ52'40''}$$

Question # 2
Solve the following triangle ABC in which:

$b=12.5\ ,\quad c=23\ ,\quad \alpha=38^\circ20'$

Solution

$b=12.5\ ,\quad c=23\ ,\quad \alpha=38^\circ20'$

By law of cosine

$$a^2=b^2+c^2-2bc\cos\alpha$$
$$=(12.5)^2+(23)^2-2(12.5)(23)\cos38^\circ20'$$
$$=156.25+529-575(0.7844)\quad =234.21$$
$$\Rightarrow\ a=\sqrt{234.21}\quad\Rightarrow\ \boxed{a=15.304}$$

Again by law of cosine

$$\cos\beta=\frac{c^2+a^2-b^2}{2ca}=\frac{(23)^2+(15.304)^2-(12.5)^2}{2(23)(15.304)}$$
$$=\frac{529+234.21-156.25}{703.984}=\frac{606.96}{703.984}=0.8622$$
$$\beta=\cos^{-1}(0.8622)\quad\Rightarrow\ \boxed{\beta=30^\circ26'}$$

Since in any triangle

$$\alpha+\beta+\gamma=180$$

$$\Rightarrow \gamma=180-\alpha-\beta =180^\circ-38^\circ20'-30^\circ26' \qquad \Rightarrow \boxed{\gamma=111^\circ14'}$$

Question # 3

Solve the following triangle ABC in which:

$$a=\sqrt{3}-1=0.732 \quad , \quad b=\sqrt{3}+1=2.732 \quad , \quad \gamma=60^\circ$$

Solution

$$a=\sqrt{3}-1=0.732 \quad , \quad b=\sqrt{3}+1=2.732 \quad , \quad \gamma=60^\circ$$

By law of cosine

$$c^2=a^2+b^2-ab\cos\gamma$$

$$=(0.732)^2+(2.732)^2-2(0.732)(2.732)\cos 60^\circ$$

$$=0.5358+7.4638-1.9998=5.9998\approx 6$$

$$\Rightarrow \boxed{c=\sqrt{6}=2.449}$$

Again by law of cosines

$$\cos\alpha=\frac{b^2+c^2-a^2}{2bc}=\frac{(2.732)^2+(2.449)^2-(0.732)}{2(2.732)(2.449)}$$

$$=\frac{7.4638+5.9976-0.5358}{13.3813}=\frac{12.9256}{13.3813}=0.9659$$

$$\Rightarrow \alpha=\cos^{-1}(0.9659) \qquad \Rightarrow \boxed{\alpha=15^\circ}$$

Since in any triangle

$$\alpha+\beta+\gamma=180$$

$$\Rightarrow \beta=180-\alpha-\gamma =180-15-60 \qquad \Rightarrow \boxed{\beta=105^\circ}$$

Question # 4

Solve the following triangle ABC in which:

$$a=3 \quad , \quad b=6 \quad , \quad \gamma=36^\circ20'$$

Solution

Do yourself as above

Question # 5

Solve the following triangle ABC in which:

$$a=7 \quad , \quad b=3 \quad , \quad \gamma=38^\circ13'$$

Solution

Do yourself as above

Question # 6

Solve the following triangle, using first law of tangent and then law of sines:

$$a=36.21 \quad , \quad b=42.09 \quad , \quad \gamma=44^\circ29'$$

Solution

Since $\qquad \alpha+\beta+\gamma=180$

$$\Rightarrow \alpha+\beta=180-\gamma$$

$$= 180 - 44^\circ 29'$$
$$\Rightarrow \alpha + \beta = 135^\circ 31' \ldots\ldots\ldots\ldots\text{(i)}$$

By law of tangent

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)} \qquad \Rightarrow \frac{36.21-42.09}{36.21+42.09} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{135^\circ 31'}{2}\right)}$$

$$\Rightarrow \frac{-5.88}{78.3} = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(67^\circ 45'\right)} \qquad \Rightarrow -0.0751 = \frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{2.4443}$$

$$\Rightarrow \tan\left(\frac{\alpha-\beta}{2}\right) = -0.0751(2.4443)$$
$$= -0.1836$$
$$\Rightarrow \frac{\alpha-\beta}{2} = \tan^{-1}(-0.1836)$$
$$\Rightarrow \frac{\alpha-\beta}{2} = -10^\circ 24' \qquad \Rightarrow \alpha-\beta = -20^\circ 48' \ldots\ldots\ldots\ldots\text{(ii)}$$

Adding (i) & (ii)

$$\alpha + \beta = 135^\circ 31'$$
$$\alpha - \beta = -20^\circ 48'$$
$$2\alpha = 114^\circ 43' \qquad \Rightarrow \boxed{\alpha = 57^\circ 22'}$$

Putting value of α in eq. (i)

$$57^\circ 22' + \beta = 135^\circ 22'$$
$$\Rightarrow \beta = 135^\circ 22' - 57^\circ 22' \qquad \Rightarrow \boxed{\beta = 78^\circ 9'}$$

Now by law of sine

$$\frac{c}{\sin\gamma} = \frac{a}{\sin\alpha} \qquad \Rightarrow \frac{c}{\sin 44^\circ 29'} = \frac{36.21}{\sin 57^\circ 22'}$$
$$\Rightarrow c = \frac{36.21}{\sin 57^\circ 22'}\cdot \sin 44^\circ 29'$$
$$= \frac{36.21}{0.8421}\cdot 0.7007 \qquad \Rightarrow \boxed{c = 30.13}$$

Question # 7, 8 & 9

Solve the following triangle, using first law of tangent and then law of sines:

(7) $a = 93,\ \ c = 101,\ \ \alpha = 80^\circ$ (8) $b = 14.8,\ \ c = 16.1,\ \ \alpha = 42^\circ 45'$

(9) $a = 319,\ \ b = 168,\ \ \alpha = 110^\circ 22'$

Solution

Do yourself as above

Question # 10

Solve the following triangle, using first law of tangent and then law of sines:

$b=61 \quad , \quad c=32 \quad , \quad \alpha=59^\circ 30'$

Solution

$b=61 \quad , \quad c=32 \quad , \quad \alpha=59^\circ 30'$

Since $\alpha+\beta+\gamma=180$

$\Rightarrow \beta+\gamma=180-\alpha$

$=180-59^\circ 30'$

$\Rightarrow \beta+\gamma=120^\circ 30'$ ………….. (i)

By law of tangent

$$\frac{b-c}{b+c}=\frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{\beta+\gamma}{2}\right)} \quad \Rightarrow \frac{61-32}{61+32}=\frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(\frac{120^\circ 30'}{2}\right)}$$

$$\Rightarrow \frac{29}{93}=\frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{\tan\left(60^\circ 15'\right)} \quad \Rightarrow 0.3118=\frac{\tan\left(\frac{\beta-\gamma}{2}\right)}{1.7496}$$

$$\Rightarrow \tan\left(\frac{\beta-\gamma}{2}\right)=0.3118(1.7496)$$

$$=0.5455$$

$$\Rightarrow \frac{\beta-\gamma}{2}=\tan^{-1}(0.5455)$$

$$\Rightarrow \frac{\beta-\gamma}{2}=28^\circ 37' \quad \Rightarrow \beta-\gamma=57^\circ 14' \text{ ………….. (ii)}$$

Adding (i) & (ii)

$$\beta+\gamma = 120^\circ 30'$$

$$\beta-\gamma = 57^\circ 14'$$

$$2\beta = 177^\circ 44' \quad \Rightarrow \boxed{\beta=88^\circ 52'}$$

Putting value of α in eq. (*i*)

$$88^\circ 52'+\gamma=120^\circ 30'$$

$$\Rightarrow \gamma=120^\circ 30'-88^\circ 52' \quad \Rightarrow \boxed{\gamma=31^\circ 38'}$$

Now by law of sine

$$\frac{c}{\sin\gamma}=\frac{a}{\sin\alpha} \quad \Rightarrow \frac{32}{\sin 31^\circ 38'}=\frac{a}{\sin 59^\circ 30'}$$

$$\Rightarrow c=\frac{32}{\sin 31^\circ 38'}\cdot\sin 59^\circ 30'$$

$$=\frac{32}{0.5244}\cdot 0.8616 \quad \Rightarrow \boxed{c=52.57}$$

Question # 11
Measure of two sides of the triangle are in the ratio $3:2$ and they include an angle of measure 57°. Find the remaining two angles.
Solution

Let $a:b=3:2$

i.e. $\dfrac{a}{b}=\dfrac{3}{2} \quad \Rightarrow \; a=\dfrac{3}{2}b$

and $\gamma=57^\circ$

Since $\alpha+\beta+\gamma=180$

$\Rightarrow \; \alpha+\beta=180-\gamma$

$=180-57 \quad \Rightarrow \; \alpha+\beta=123^\circ$ ……….. (i)

By law of tangent

$$\frac{a-b}{a+b}=\frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{\alpha+\beta}{2}\right)} \qquad \Rightarrow \; \frac{\frac{3}{2}b-b}{\frac{3}{2}b+b}=\frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(\frac{123^\circ}{2}\right)}$$

$$\Rightarrow \; \frac{\frac{1}{2}b}{\frac{5}{2}b}=\frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{\tan\left(61^\circ 30'\right)} \qquad \Rightarrow \; \frac{1}{5}=\frac{\tan\left(\frac{\alpha-\beta}{2}\right)}{1.8418}$$

$$\Rightarrow \; \tan\left(\frac{\alpha-\beta}{2}\right)=\frac{1}{5}(1.8418)$$

$$=0.3684$$

$$\Rightarrow \; \frac{\alpha-\beta}{2}=\tan^{-1}(0.3684)=20^\circ 13'$$

$\Rightarrow \; \alpha-\beta=40^\circ 26'$ ……….. (ii)

Adding (i) & (ii)

$\alpha+\beta = 123^\circ$

$\alpha-\beta = 40^\circ 27'$

$2\alpha = 163^\circ 27' \quad \Rightarrow \; \boxed{\alpha=81^\circ 44'}$

Putting value of α in eq. (i)

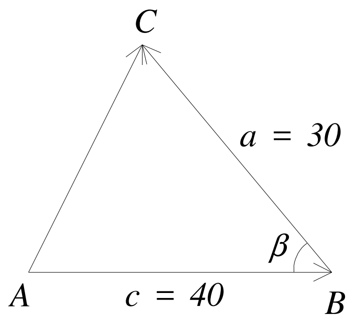
$81^\circ 44'+\beta=123^\circ$

$\Rightarrow \; \beta=123^\circ-81^\circ 44' \quad \Rightarrow \; \boxed{\beta=41^\circ 16'}$

Question # 12
Two forces of 40N and 30N are represented by $\overrightarrow{AB}$ and $\overrightarrow{BC}$ which are inclined at an angle of $147^\circ 25'$. Find $\overrightarrow{AC}$, the resultant of $\overrightarrow{AB}$ and $\overrightarrow{BC}$.
Solution

Since $\overrightarrow{AB}=c=40N$

$$\overrightarrow{BC} = a = 30N$$
$$m\angle B = \beta = 147^\circ 25'$$
$$\overrightarrow{AC} = b = ?$$



By law of cosine

$$b^2 = c^2 + a^2 - 2ca\cos\beta$$
$$= (40)^2 + (30)^2 - 2(40)(30)\cos 147^\circ 25'$$
$$= 1600 + 900 - 2400(-0.8426)$$
$$= 4522.26$$
$$\Rightarrow \; b = \sqrt{4522.26} = 67.248$$

i.e. $\overrightarrow{AC} = 67.248N$
