

Question # 1

Without using the tables, find the value of :

- (i) $\sin(-780^\circ)$ (ii) $\cot(-855^\circ)$ (iii) $\csc(2040^\circ)$
(iv) $\csc(2040^\circ)$ (v) $\tan(1110^\circ)$ (iv) $\sin(-300^\circ)$

Solution

- (i) $\sin(-780^\circ) = -\sin 780^\circ = -\sin(8(90) + 60)$
 $= -\sin(60) = -\frac{\sqrt{3}}{2}$ $\therefore 780$ is in the Ist quad.
- (ii) $\cot(-855^\circ) = -\cot 855^\circ = -\cot(9(90) + 45)$
 $= -(-\tan 45^\circ) = \tan 45^\circ = 1$ $\therefore 855$ is in the IIInd quad.
- (iii) $\csc(2040^\circ) = \csc(22(90) + 60) = -\csc(60)$ $\therefore 2040^\circ$ is in the Ist quad.
 $= -\frac{1}{\sin(60)} = -\frac{1}{\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$
- (iv) $\sec(-960) = \sec(960) = \sec(10(90) + 60) = -\sec 60^\circ$ $\therefore 960^\circ$ is in the IIIrd quad.
 $= -\frac{1}{\cos 60^\circ} = -\frac{1}{\frac{1}{2}} = -2$
- (v) $\tan(1110) = \tan(12(90) + 30) = \tan(30) = \frac{1}{\sqrt{3}}$ $\therefore 1110^\circ$ is in the Ist quad
- (vi) $\sin(-300) = -\sin(300) = -\sin(3(90) + 30)$
 $= -(-\cos 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$ $\therefore 300^\circ$ is in the IIIrd quad.

Question # 2

Express each of the following as a trigonometric function of an angle of positive degree measure of less than 45° .

- (i) $\sin 196^\circ$ (ii) $\cos 147^\circ$ (iii) $\sin 319^\circ$ (iv) $\cos 254^\circ$
(v) $\tan 294^\circ$ (vi) $\cos 728^\circ$ (vii) $\sin(-625^\circ)$ (viii) $\cos(-435^\circ)$

Solution

- (i) $\sin 196^\circ = \sin(180 + 16) = \sin 180^\circ \cos 16^\circ + \cos 180^\circ \sin 16^\circ$
 $= (0)\cos 16^\circ + (-1)\sin 16^\circ = -\sin 16^\circ$
- (ii) $\cos 147^\circ = \cos(180 - 33) = \cos 180^\circ \cos 33^\circ + \sin 180^\circ \sin 33^\circ$
 $= (-1)\cos 33^\circ + (0)\sin 33^\circ = -\cos 33^\circ$

(iii) $\sin 319^\circ = \sin(360 - 41) = \sin 360^\circ \cos 41^\circ - \cos 360^\circ \sin 41^\circ$ *Now Do yourself*

(iv) $\cos 254^\circ = \cos(270 - 16)$ *Do yourself*

(v)
$$\begin{aligned} \tan 294^\circ &= \frac{\sin 294^\circ}{\cos 294^\circ} = \frac{\sin(270 + 24)}{\cos(270 + 24)} \\ &= \frac{\sin 270^\circ \cos 24^\circ + \cos 270^\circ \sin 24^\circ}{\cos 270^\circ \cos 24^\circ - \sin 270^\circ \sin 24^\circ} = \frac{(-1)\cos 24^\circ + (0)\sin 24^\circ}{(0)\cos 24^\circ - (-1)\sin 24^\circ} \\ &= \frac{-\cos 24^\circ + 0}{0 + \sin 24^\circ} = \frac{-\cos 24^\circ}{\sin 24^\circ} = -\cot 24^\circ \end{aligned}$$

Alternative Method:

$$\begin{aligned} \tan 294^\circ &= \tan(270 + 24) = \frac{\tan 270^\circ + \tan 24^\circ}{1 - \tan 270^\circ \tan 24^\circ} \\ &= \frac{\tan 270^\circ \left(1 + \frac{\tan 24^\circ}{\tan 270^\circ}\right)}{\tan 270^\circ \left(\frac{1}{\tan 270^\circ} - \tan 24^\circ\right)} = \frac{\left(1 + \frac{\tan 24^\circ}{\infty}\right)}{\left(\frac{1}{\infty} - \tan 24^\circ\right)} \\ &= \frac{(1+0)}{(0 - \tan 24^\circ)} = -\frac{1}{\tan 24^\circ} = -\cot 24^\circ \quad \square \end{aligned}$$

(vi) $\cos 728^\circ = \cos(720 + 8)$ *Now Do yourself*

(vii)
$$\begin{aligned} \sin(-625^\circ) &= -\sin 625^\circ = -\sin(630 - 5) \\ &= -(\sin 630^\circ \cos 5^\circ - \cos 630^\circ \sin 5^\circ) = -((-1)\cos 5^\circ - (0)\sin 5^\circ) \\ &= -(-\cos 5^\circ - 0) = \cos 5^\circ \end{aligned}$$

(viii) $\cos(-435^\circ) = \cos 435^\circ$
 $= \cos(450 - 15)$ *Now Do yourself*

Question # 3

Prove the following:

- (i) $\sin(180 + \alpha)\sin(90 - \alpha) = -\sin \alpha \cos \alpha$
- (ii) $\sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 30^\circ = \frac{1}{2}$
- (iii) $\sin 306^\circ + \cos 234^\circ + \cos 162^\circ + \cos 18^\circ = 0$
- (iv) $\cos 330^\circ \sin 600^\circ + \cos 120^\circ \sin 150^\circ = -1$

Solution

(i) L.H.S = $\sin(180 + \alpha)\sin(90 - \alpha)$
 $= (\sin 180^\circ \cos \alpha + \cos 180^\circ \sin \alpha)(\sin 90^\circ \cos \alpha - \cos 90^\circ \sin \alpha)$
 $= ((0)\cos \alpha + (-1)\sin \alpha)((1)\cos \alpha - (0)\sin \alpha)$

$$= (0 - \sin \alpha)(\cos \alpha - 0) = -\sin \alpha \cos \alpha = \text{R.H.S} \quad \square$$

(ii) First we calculate

$$\sin 780^\circ = \sin(720 + 60) = \sin(2 \times 360 + 60) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \sin 480^\circ &= \sin(450 + 30) = \sin 450^\circ \cos 30^\circ + \cos 450^\circ \sin 30^\circ \\ &= (1) \cos 30 + (0) \sin 30 = \cos 30 + 0 = \frac{\sqrt{3}}{2} \end{aligned}$$

$$\cos 120^\circ = -\frac{1}{2} \quad \text{and} \quad \sin 30^\circ = \frac{1}{2}.$$

So L.H.S = $\sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 30^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} = \text{R.H.S} \quad \square$$

(iii) First we calculate

$$\begin{aligned} \cos 306^\circ &= \cos(270 + 36) = \cos 270^\circ \cos 36^\circ - \sin 270^\circ \sin 36^\circ \\ &= (0) \cos 36^\circ - (-1) \sin 36^\circ = 0 + \sin 36^\circ = \sin 36^\circ \end{aligned}$$

$$\begin{aligned} \cos 234^\circ &= \cos(270 - 36) = \cos 270^\circ \cos 36^\circ + \sin 270^\circ \sin 36^\circ \\ &= (0) \cos 36^\circ + (-1) \sin 36^\circ = 0 - \sin 36^\circ = -\sin 36^\circ \end{aligned}$$

$$\begin{aligned} \cos 162^\circ &= \cos(180 - 18) = \cos 180^\circ \cos 18^\circ + \sin 180^\circ \sin 18^\circ \\ &= (-1) \cos 18 + (0) \sin 18 = -\cos 18 + 0 = -\cos 18 \end{aligned}$$

So L.H.S = $\sin 306^\circ + \cos 234^\circ + \cos 162^\circ + \cos 18^\circ$

$$= \sin 36^\circ - \sin 36^\circ - \cos 18^\circ + \cos 18^\circ = 0 = \text{R.H.S} \quad \square$$

(iv) First we calculate (*Alternative Method*)

$$\cos 330^\circ = \cos(360 - 30) = \cos(-30^\circ) = \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$$\sin 600^\circ = \sin(6 \times 90 + 60) = -\sin 60 = -\frac{\sqrt{3}}{2} \quad \because 600^\circ \text{ is in the IIIrd quad}$$

$$\cos 120^\circ = \cos(90 + 30) = -\sin 30 = -\frac{1}{2} \quad \because 120^\circ \text{ is in the IIInd quad}$$

$$\sin 150^\circ = \sin(90 + 60) = \cos 60^\circ = \frac{1}{2} \quad \because 150^\circ \text{ is in the IIInd quad}$$

So L.H.S = $\cos 330^\circ \sin 600^\circ + \cos 120^\circ \sin 150^\circ$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = -\frac{3}{4} - \frac{1}{4} = -\frac{4}{4} = -1 = \text{R.H.S}$$

Question # 4

Prove that;

$$(i) \frac{\sin^2(\pi + \theta) \tan\left(\frac{3\pi}{2} + \theta\right)}{\cot^2\left(\frac{3\pi}{2} - \theta\right) \cos^2(\pi - \theta) \operatorname{cosec}(2\pi - \theta)} = \cos \theta$$

$$(ii) \frac{\cos(90^\circ + \theta) \sec(-\theta) \tan(180^\circ - \theta)}{\sec(360^\circ - \theta) \sin(180^\circ + \theta) \cot(90^\circ - \theta)} = -1$$

Solution

(i) First we calculate

$$\begin{aligned} \sin(\pi + \theta) &= \sin \pi \cos \theta + \cos \pi \sin \theta = (0) \cos \theta + (-1) \sin \theta \\ &= 0 - \sin \theta = -\sin \theta \end{aligned}$$

$$\tan\left(\frac{3\pi}{2} + \theta\right) = \tan\left(3 \cdot \frac{\pi}{2} + \theta\right) = -\cot \theta \quad \because \frac{3\pi}{2} + \theta \text{ is in the IVth quad}$$

$$\cot\left(\frac{3\pi}{2} - \theta\right) = \cot\left(3 \cdot \frac{\pi}{2} - \theta\right) = \tan \theta \quad \because \frac{3\pi}{2} - \theta \text{ is in the IIIrd quad}$$

$$\begin{aligned} \cos(\pi - \theta) &= \cos \pi \cos \theta + \sin \pi \sin \theta = (-1) \cos \theta + (0) \sin \theta \\ &= -\cos \theta + 0 = -\cos \theta \end{aligned}$$

$$\operatorname{csc}(2\pi - \theta) = \operatorname{csc}(-\theta) = -\operatorname{csc} \theta$$

Now

$$\begin{aligned} \text{L.H.S} &= \frac{\sin^2(\pi + \theta) \tan\left(\frac{3\pi}{2} + \theta\right)}{\cot\left(\frac{3\pi}{2} - \theta\right) \cos^2(\pi - \theta) \operatorname{csc}(2\pi - \theta)} \\ &= \frac{(-\sin \theta)^2 (-\cot \theta)}{(\tan \theta)^2 (-\cos \theta)^2 (-\operatorname{csc} \theta)} = \frac{\sin^2 \theta (-\cot \theta)}{\tan^2 \theta \cos^2 \theta (-\operatorname{csc} \theta)} \\ &= \frac{\sin^2 \theta \frac{\cos \theta}{\sin \theta}}{\frac{\sin^2 \theta}{\cos^2 \theta} \cos^2 \theta \frac{1}{\sin \theta}} = \frac{\sin \theta \cos \theta}{\sin \theta} = \cos \theta = \text{R.H.S} \end{aligned}$$

(ii) First we calculate

$$\cos(90 + \theta) = -\sin \theta \quad \because 90 + \theta \text{ is in the IInd quad.}$$

$$\sec(-\theta) = \sec \theta$$

$$\tan(180 - \theta) = \tan(2(90) - \theta) = -\tan \theta \quad \because 180 - \theta \text{ is in the IInd quad.}$$

$$\sec(360 - \theta) = \sec(-\theta) = \sec \theta$$

$$\sin(180 + \theta) = \sin(2(90) + \theta) = -\sin \theta \quad \because 180 + \theta \text{ is in the IIIrd quad.}$$

$$\cot(90 - \theta) = \tan \theta \quad \because 90 - \theta \text{ is in the Ist quad.}$$

Now

$$\begin{aligned} \text{L.H.S} &= \frac{\cos(90 + \theta) \sec(-\theta) \tan(180 - \theta)}{\sec(360 - \theta) \sin(180 + \theta) \cot(90 - \theta)} \\ &= \frac{(-\sin \theta) \sec \theta (-\tan \theta)}{\sec \theta (-\sin \theta) (-\tan \theta)} = 1 = \text{R.H.S} \end{aligned}$$

Question # 5

If α, β, γ are the angles of a triangle ABC , then prove that;

- (i) $\sin(\alpha + \beta) = \sin \gamma$ (ii) $\cos\left(\frac{\alpha + \beta}{2}\right) = \sin \frac{\gamma}{2}$
 (iii) $\cos(\alpha + \beta) = \cos \gamma$ (iv) $\tan(\alpha + \beta) + \tan \gamma = 0$

Solution

(i) Since α, β and γ are angles of triangle therefore

$$\alpha + \beta + \gamma = 180 \Rightarrow \alpha + \beta = 180 - \gamma$$

$$\begin{aligned} \text{Now L.H.S} &= \sin(\alpha + \beta) = \sin(180 - \gamma) \\ &= \sin 180 \cos \gamma - \cos 180 \sin \gamma \\ &= (0) \cos \gamma - (-1) \sin \gamma = 0 + \sin \gamma = \sin \gamma = \text{R.H.S} \end{aligned}$$

(ii) Since α, β and γ are angles of triangle therefore

$$\alpha + \beta + \gamma = 180$$

$$\Rightarrow \alpha + \beta = 180 - \gamma \Rightarrow \frac{\alpha + \beta}{2} = \frac{180 - \gamma}{2}$$

$$\begin{aligned} \text{Now L.H.S} &= \cos\left(\frac{\alpha + \beta}{2}\right) = \cos\left(\frac{180 - \gamma}{2}\right) = \cos\left(90 - \frac{\gamma}{2}\right) \\ &= \cos 90 \cos \frac{\gamma}{2} + \sin 90 \sin \frac{\gamma}{2} \\ &= (0) \cos \frac{\gamma}{2} + (1) \sin \frac{\gamma}{2} = 0 + \sin \frac{\gamma}{2} = \sin \frac{\gamma}{2} = \text{R.H.S} \quad \square \end{aligned}$$

(iii) Since α, β and γ are angles of triangle therefore

$$\alpha + \beta + \gamma = 180 \Rightarrow \alpha + \beta = 180 - \gamma$$

$$\begin{aligned} \text{Now L.H.S} &= \cos(\alpha + \beta) = \cos(180 - \gamma) \\ &= \cos 180 \cos \gamma + \sin 180 \sin \gamma \\ &= (-1) \cos \gamma + (0) \sin \gamma = -\cos \gamma + 0 = -\cos \gamma = \text{R.H.S} \end{aligned}$$

(iv) Since α, β and γ are angles of triangle therefore

$$\alpha + \beta + \gamma = 180 \Rightarrow \alpha + \beta = 180 - \gamma$$

$$\begin{aligned} \text{Now L.H.S} &= \tan(\alpha + \beta) + \tan \gamma = \tan(180 - \gamma) + \tan \gamma \\ &= \frac{\tan 180 - \tan \gamma}{1 + \tan 180 \tan \gamma} + \tan \gamma \end{aligned}$$

$$\begin{aligned} &= \frac{(0) - \tan \gamma}{1 + (0) \tan \gamma} + \tan \gamma = \frac{-\tan \gamma}{1 + 0} + \tan \gamma \\ &= -\tan \gamma + \tan \gamma = 0 = \text{R.H.S} \quad \square \end{aligned}$$

Remember:

- $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$
- $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$
- $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$
- $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$
- $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$
- $\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$

Three Steps to solve $\sin\left(n \cdot \frac{\pi}{2} \pm \theta\right)$

Step I: First check that n is even or odd

Step II: If n is even then the answer will be in *sin* and if the n is odd then *sin* will be converted to *cos* and vice versa (i.e. *cos* will be converted to *sin*).

Step III: Now check in which quadrant $n \cdot \frac{\pi}{2} \pm \theta$ is lying if it is in *Ist* or *IInd* quadrant the answer will be positive as *sin* is positive in these quadrant and if it is in the *IIIrd* or *IVth* quadrant the answer will be negative.

e.g. $\sin 667^\circ = \sin(7(90) + 37)$

Since $n = 7$ is odd so answer will be in *cos* and 667 is in *IVth* quadrant and *sin* is -ive in *IVth* quadrant therefore answer will be in negative. i.e $\sin 667^\circ = -\cos 37$
Similar technique is used for other trigonometric ratios. i.e $\tan \rightleftharpoons \cot$ and $\sec \rightleftharpoons \csc$.

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