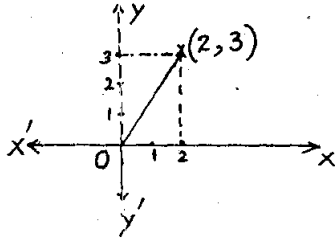
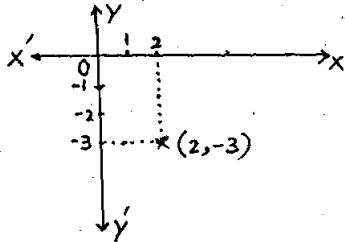


**EXERCISE 1.3**

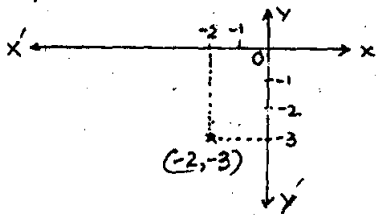
i)  $2+3i = (2, 3)$



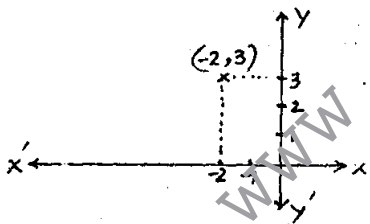
ii)  $2-3i = (2, -3)$



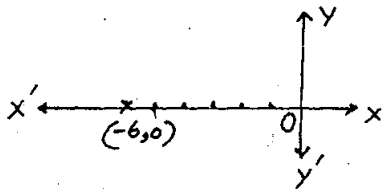
iii)  $-2-3i = (-2, -3)$



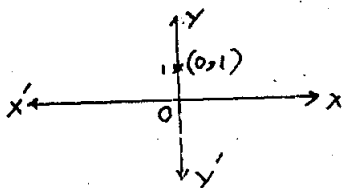
iv)  $-2+3i = (-2, 3)$



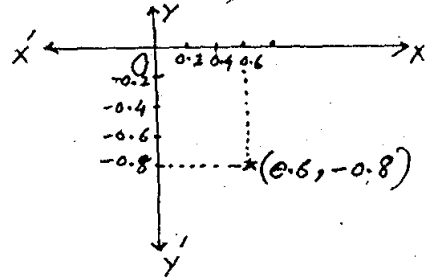
v)  $-6 = -6 + i \cdot 0 = (-6, 0)$



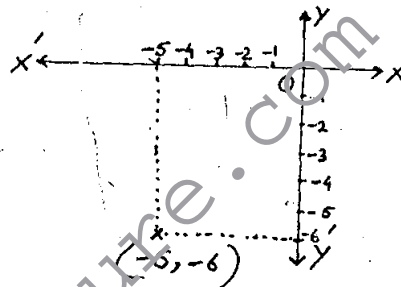
vi)  $i = 0 + i = (0, 1)$



vii)  $\frac{3}{5} - \frac{4}{5}i = 0.6 - 0.8i$   
 $= (0.6, -0.8)$



viii)  $-5-6i = (-5, -6)$



② i) let  $z = -3i$

Multiplicative inverse of  $z$   
 $= \frac{1}{z} = \frac{1}{-3i} = \frac{1}{-3i} \times \frac{i}{i} = \frac{i}{-3i^2}$   
 $= \frac{i}{-3(-1)} = \frac{i}{3}$  Ans.

ii) let  $z = 1-2i$

Multiplicative inverse of  $z = \frac{1}{z}$   
 $= \frac{1}{1-2i} = \frac{1}{1-2i} \times \frac{1+2i}{1+2i}$   
 $= \frac{1+2i}{(1)^2 - (2i)^2} = \frac{1+2i}{1-4i^2} = \frac{1+2i}{1-4(-1)}$   
 $= \frac{1+2i}{1+4} = \frac{1+2i}{5} = \frac{1}{5} + \frac{2}{5}i$  Ans.

iii) let  $z = -3-5i$

Multiplicative inverse of  $z = \frac{1}{z}$   
 $= \frac{1}{-3-5i} = \frac{1}{-3-5i} \times \frac{-3+5i}{-3+5i}$   
 $= \frac{-3+5i}{(-3)^2 - (5i)^2}$

$$\frac{-3+5i}{9-25i^2} = \frac{-3+5i}{9+25}$$

$$= \frac{-3+5i}{34} = \frac{-3}{34} + \frac{5i}{34} \text{ Ans.}$$

iv) let  $z = (1, 2) = 1+2i$   
 Multiplicative inverse of  $z = \frac{1}{z}$   
 $= \frac{1}{1+2i} = \frac{1}{1+2i} \times \frac{1-2i}{1-2i} = \frac{1-2i}{(1)^2 - (2i)^2}$   
 $= \frac{1-2i}{1-4i^2} = \frac{1-2i}{1-4(-1)} = \frac{1-2i}{1+4} = \frac{1-2i}{5}$   
 $= \frac{1}{5} - \frac{2}{5}i = \left(\frac{1}{5}, -\frac{2}{5}\right) \text{ Ans.}$

③ i)  $i^{101} = (i^2)^{50} \cdot i = (-1)^{50} \cdot i = 1 \cdot i = i$   
 $= i \text{ Ans.}$

ii)  $(-ai)^4 = (-a)^4 i^4 = a^4 (i^2)^2$   
 $= a^4 (-1)^2 = a^4 \cdot 1 = a^4 \text{ Ans.}$

iii)  $i^{-3} = \frac{1}{i^3} = \frac{1}{i^2 \cdot i} = \frac{1}{(-1)(i)} = \frac{1}{-i}$   
 $= \frac{1}{-i} \times \frac{i}{i} = \frac{i}{-i^2} = \frac{i}{-(-1)} = \frac{i}{1} = i$   
 $= i \text{ Ans.}$

iv)  $i^{-10} = \frac{1}{i^{10}} = \frac{1}{(i^2)^5} = \frac{1}{(-1)^5} = \frac{1}{-1} = -1$   
 $= -1 \text{ Ans.}$

④ Let  $z = a+bi, a, b \in \mathbb{R}$   
 Suppose that  $\bar{z} = z$  (1)

We shall prove that  $z$  is real

$$\therefore \bar{z} = z$$

$$\Rightarrow a+ib = a+ib$$

$$\Rightarrow a-ib = a+ib$$

$$\Rightarrow a-ib-a-ib=0$$

$$\Rightarrow -2ib=0$$

$$\Rightarrow b=0 \quad \because -2i \neq 0$$

$\therefore$  (1) becomes

$$z = a+i(0) = a+0 = a$$

$\Rightarrow z$  is real.

Conversely suppose that  $z$  is real.

Then we shall prove that

$$\bar{z} = z$$

$\therefore z$  is real

$\Rightarrow$  Imaginary part of  $z = 0$

$$\Rightarrow b=0$$

$\therefore$  (1) becomes

$$z = a+i(0) = a+0 = a$$

$$\Rightarrow z = a \quad \text{--- (2)}$$

Taking conjugate on both sides

$$\bar{z} = \bar{a} \Rightarrow \bar{z} = a \quad \text{--- (3)} \quad \because a \in \mathbb{R}$$

$\therefore$  From (2) and (3), we get

$$\bar{z} = z \quad (\text{proved})$$

⑤ i)  $5+2\sqrt{-4} = 5+2\sqrt{(-1)(4)}$   
 $= 5+2\sqrt{-1}\sqrt{4} = 5+2(i)(2) = 5+4i$   
 $\text{Ans.}$

ii)  $(2+\sqrt{-3})(3+\sqrt{-3})$   
 $= (2+\sqrt{(-1)(3)})(3+\sqrt{(-1)(3)})$   
 $= (2+i\sqrt{3})(3+i\sqrt{3})$   
 $= 6 + i2\sqrt{3} + i3\sqrt{3} + i^2(\sqrt{3})^2$   
 $= 6 + 5\sqrt{3}i + (-1)(3)$   
 $= 6 + 5\sqrt{3}i - 3 = 3 + 5\sqrt{3}i \text{ Ans.}$

iii)  $\frac{2}{15+\sqrt{-8}} = \frac{2}{15+\sqrt{(-1)(8)}} = \frac{2}{15+i\sqrt{8}}$   
 $= \frac{2}{15+i(2\sqrt{2})} \left\{ \begin{array}{l} \sqrt{8} = \sqrt{2 \times 2 \times 2} \\ = \sqrt{2^2 \times 2} \\ = 2\sqrt{2} \end{array} \right.$

$$\begin{aligned}
 &= \frac{2}{\sqrt{5} + 2\sqrt{2}i} \quad \text{[24]} \\
 &= \frac{2}{\sqrt{5} + 2\sqrt{2}i} \times \frac{\sqrt{5} - 2\sqrt{2}i}{\sqrt{5} - 2\sqrt{2}i} \\
 &= \frac{2(\sqrt{5} - 2\sqrt{2}i)}{(\sqrt{5})^2 - (2\sqrt{2}i)^2} = \frac{2(\sqrt{5} - 2\sqrt{2}i)}{5 - 4(2)i^2} \\
 &= \frac{2(\sqrt{5} - 2\sqrt{2}i)}{5 + 8} = \frac{2\sqrt{5} - 4\sqrt{2}i}{13} \\
 &= \frac{2\sqrt{5}}{13} - \frac{4\sqrt{2}}{13}i \quad \text{Ans.}
 \end{aligned}$$

$$\begin{aligned}
 \text{iv)} \quad \frac{3}{\sqrt{6} - \sqrt{-12}} &= \frac{3}{\sqrt{6} - \sqrt{(-1)(12)}} = \frac{3}{\sqrt{6} - \sqrt{-1}\sqrt{12}} \\
 &= \frac{3}{\sqrt{6} - i\sqrt{12}} = \frac{3}{\sqrt{6} - i(2\sqrt{3})} \quad \left\{ \begin{array}{l} \sqrt{12} = \sqrt{2 \times 2 \times 3} \\ = \sqrt{2^2 \times 3} \\ = 2\sqrt{3} \end{array} \right. \\
 &= \frac{3}{\sqrt{6} - 2\sqrt{3}i} \times \frac{\sqrt{6} + 2\sqrt{3}i}{\sqrt{6} + 2\sqrt{3}i} \\
 &= \frac{3(\sqrt{6} + 2\sqrt{3}i)}{(\sqrt{6})^2 - (2\sqrt{3}i)^2} = \frac{3(\sqrt{6} + 2\sqrt{3}i)}{6 - 4(3)i^2} \\
 &= \frac{3(\sqrt{6} + 2\sqrt{3}i)}{6 + 12} = \frac{3(\sqrt{6} + 2\sqrt{3}i)}{18} \\
 &= \frac{\sqrt{6} + 2\sqrt{3}i}{6} = \frac{\sqrt{6}}{6} + \frac{2\sqrt{3}}{6}i \\
 &= \frac{\sqrt{6}}{6} + \frac{\sqrt{3}}{3}i = \frac{\sqrt{6}}{\sqrt{6} \times \sqrt{6}} + \frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}}i \\
 &= \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{3}}i \quad \text{Ans.}
 \end{aligned}$$

vi)  $z^2 + \bar{z}^2$  is a real number.

Let  $z = a + ib$ ,  $a, b \in \mathbb{R}$   
 then  $\bar{z} = a - ib$   
 $z^2 + \bar{z}^2 = (a + ib)^2 + (a - ib)^2$   
 $= a^2 + i^2b^2 + 2abi + a^2 + i^2b^2 - 2abi$   
 $= 2a^2 + 2i^2b^2$   
 $= 2a^2 + 2(-1)b^2 = 2a^2 - 2b^2$   
 which is real.

ii)  $z^2 - \bar{z}^2$  is Imaginary.

Let  $z = a + ib$ ,  $a, b \in \mathbb{R}$   
 then  $\bar{z} = a - ib$

Now  $z^2 - \bar{z}^2 = (a + ib)^2 - (a - ib)^2$   
 $= (a^2 + i^2b^2 + 2abi) - (a^2 + i^2b^2 - 2abi)$   
 $= a^2 + i^2b^2 + 2abi - a^2 - i^2b^2 + 2abi$   
 $= 4abi$  which is imaginary number.

7) Simplify

$$\begin{aligned}
 \text{i)} \quad &\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^3 \\
 &= \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2 \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\
 &= \left[\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}i\right)^2 + 2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right)\right] \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\
 &= \left(\frac{1}{4} + \frac{3}{4}i^2 - \frac{\sqrt{3}}{2}i\right) \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\
 &= \left(\frac{1}{4} - \frac{3}{4} - \frac{\sqrt{3}}{2}i\right) \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \quad \left\{ \begin{array}{l} \frac{1}{4} - \frac{3}{4} \\ = \frac{1-3}{4} \\ = -\frac{2}{4} = -\frac{1}{2} \end{array} \right. \\
 &= \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\
 &= \left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}i\right)^2 = \frac{1}{4} - \frac{3}{4}i^2 \\
 &= \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1 \quad \text{Ans.}
 \end{aligned}$$

ii)  $\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^3$

$$\begin{aligned}
 &= \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^2 \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \\
 &= \left[\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}i\right)^2 - 2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right)\right] \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \\
 &= \left(\frac{1}{4} - \frac{3}{4} + \frac{\sqrt{3}}{2}i\right) \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \\
 &= \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \\
 &= \left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}i\right)^2 \\
 &= \frac{1}{4} - \frac{3}{4}i^2 = \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1
 \end{aligned}$$

Ans

$$\begin{aligned} \text{iii)} & \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{-2} \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \\ &= \frac{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}{\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^2} \\ &= \frac{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}i\right)^2 + 2\left(-\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}i\right)} \\ &= \frac{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}{\frac{1}{4} - \frac{3}{4} - \frac{\sqrt{3}}{2}i} = \frac{-\frac{1}{2} - \frac{\sqrt{3}}{2}i}{-\frac{1}{2} - \frac{\sqrt{3}}{2}i} = 1 \end{aligned}$$

$$\begin{aligned} \text{iii)} & \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{-2} \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \\ & \text{(according to book)} \\ &= \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)^{-1} = \frac{1}{-\frac{1}{2} - \frac{\sqrt{3}}{2}i} \\ &= \frac{1}{-\frac{1}{2} - \frac{\sqrt{3}}{2}i} \times \frac{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}{-\frac{1}{2} + \frac{\sqrt{3}}{2}i} \\ &= \frac{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}{\left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}i\right)^2} = \frac{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}{\frac{1}{4} + \frac{3}{4}} \\ &= \frac{-\frac{1}{2} + \frac{\sqrt{3}}{2}i}{\frac{4}{4}} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \\ &= \frac{-1 + \sqrt{3}i}{2} \text{ Ans} \end{aligned}$$

$$\begin{aligned} \text{iv)} (a+bi)^2 &= a^2 + b^2i^2 + 2abi \\ &= a^2 + b^2(-1) + 2abi \\ &= a^2 - b^2 + 2abi \text{ Ans} \end{aligned}$$

$$\begin{aligned} \text{v)} (a+bi)^{-2} &= \frac{1}{(a+bi)^2} = \frac{1}{a^2 + b^2i^2 + 2abi} \\ &= \frac{1}{a^2 - b^2 + 2abi} \\ &= \frac{1}{(a^2 - b^2) + 2abi} \times \frac{(a^2 - b^2) - 2abi}{(a^2 - b^2) - 2abi} \end{aligned}$$

$$\begin{aligned} \text{v)} & \frac{(a^2 - b^2) - 2abi}{(a^2 - b^2)^2 - (2abi)^2} \\ &= \frac{(a^2 - b^2) - 2abi}{a^4 + b^4 - 2a^2b^2 - 4a^2b^2i^2} \\ &= \frac{(a^2 - b^2) - 2abi}{a^4 + b^4 - 2a^2b^2 + 4a^2b^2} \\ &= \frac{(a^2 - b^2) - 2abi}{a^4 + b^4 + 2a^2b^2} = \frac{(a^2 - b^2) - 2abi}{(a^2 + b^2)^2} \\ &= \frac{a^2 - b^2}{(a^2 + b^2)^2} - \frac{2abi}{(a^2 + b^2)^2} \text{ Ans} \end{aligned}$$

$$\begin{aligned} \text{vi)} (a+bi)^3 &= a^3 + (bi)^3 + 3(a)(bi)(a+bi) \\ &= a^3 + b^3i^3 + 3abi(a+bi) \\ &= a^3 + b^3(-i) + 3a^2bi + 3ab^2i^2 \\ &= a^3 - b^3i + 3a^2bi - 3ab^2 \\ &= a^3 - 3ab^2 + 3a^2bi - b^3i \\ &= a^3 - 3ab^2 + i(3a^2b - b^3) \text{ Ans} \end{aligned}$$

$$\begin{aligned} \text{vii)} (a-bi)^3 &= (a)^3 - (bi)^3 - 3(a)(bi)(a-bi) \\ &= a^3 - b^3i^3 - 3abi(a-bi) \\ &= a^3 - b^3(-i) - 3a^2bi + 3ab^2i^2 \\ &= a^3 + b^3i - 3a^2bi - 3ab^2 \\ &= a^3 - 3ab^2 - i(3a^2b - b^3) \text{ Ans} \end{aligned}$$

$$\begin{aligned} \text{viii)} (3-\sqrt{-4})^{-3} &= \frac{1}{[3-\sqrt{-4}]^3} = \frac{1}{[3-\sqrt{(-1)(4)}]^3} \\ &= \frac{1}{(3-\sqrt{-1} \cdot 2)^3} = \frac{1}{[3-i(2)]^3} = \frac{1}{(3-2i)^3} \\ &= \frac{1}{(3)^3 - (2i)^3 - 3(3)(2i)(3-2i)} \\ &= \frac{1}{27 - 8i^3 - 18i(3-2i)} \end{aligned}$$

$$= \frac{1}{27 - 8(-i) - 54i + 36i^2}$$

$$= \frac{1}{27 + 8i - 54i - 36}$$

$$= \frac{1}{-9 - 46i}$$

$$= \frac{1}{-9 - 46i} \times \frac{-9 + 46i}{-9 + 46i}$$

$$= \frac{-9 + 46i}{(-9)^2 - (46i)^2} = \frac{-9 + 46i}{81 - 2116i^2}$$

$$= \frac{-9 + 46i}{81 + 2116} = \frac{-9 + 46i}{2197}$$

$$= \frac{-9}{2197} + \frac{46i}{2197} \quad \underline{\underline{\text{Ans}}}$$

End of Chapter No. 1.

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