NUMBER SYSTEMS

We are familiar with the following sets of numbers

i) N = Set of natural numbers
$= \{1, 2, 3, \dots\}$ (Counting set)

ii) W = Set of whole numbers
$= \{0, 1, 2, \dots\}$

iii) Z = Set of all integers
$= \{0, \pm 1, \pm 2, \dots\}$

iv) Q = Set of rational numbers
$= \{p/q \,/\, p, q \in Z \wedge q \neq 0\}$
($\wedge$ stands for "and")

v) Q' = Set of irrational numbers
$= \{\pm\sqrt{a} \,/\, a$ is not a perfect square $\}$

vi) $\mathbb{R}$ = Set of all real numbers
$= Q \cup Q'$

Decimal Representation of Rational and Irrational Numbers

1. Terminating Decimals

A decimal which has only a finite number of decimal places, is called a Terminating Decimal. e.g. 202.04, 0.00415, 1000, 1236 etc.
Since every terminating decimal can be converted into a common fraction, therefore it is a rational number.

2. Recurring or Periodic Decimals:

A decimal in which one or more digits repeat indefinitely, is called a recurring or periodic decimal. e.g. $1.333\dots = 1.\overline{3}$, $0.818181\dots = 0.\overline{81}$.
Since every recurring decimal can be converted into a common fraction, therefore it is a rational number.

3. Non-Terminating, Non-Recurring Decimals:

A decimal which neither terminates nor it is recurring is called non-terminating, non-recurring decimal.
A decimal of such type is an irrational number.

Example

i) $0.25 \left(= \frac{25}{100}\right)$ is a rational number. (Terminating decimal)

ii) $0.333\dots (= 0.\overline{3}) (= \frac{1}{3})$ is a rational number (Recurring decimal)

iii) $2.333\dots (= 2.\overline{3})$ is a rational number (Recurring decimal)

iv) $0.412857412857\dots (= 0.\overline{412857})$ is a rational no. (Recurring decimal)

v) $0.01001000100001\dots$ is an irrational number (non-terminating, non-recurring)

vi) $2.1412112211122211112222\dots$ is also an irrational number. (non-terminating, non-recurring)

vii) 3.141592654... is an irrational number. (non-terminating, non-recurring) called π which denotes the constant ratio of the circumference of any circle to the length of its diameter. i.e;

$$\pi = \frac{\text{Circumference of any circle}}{\text{length of its diameter}}$$

An approximate value of π is $\frac{22}{7}$, a better approximation is $\frac{355}{113}$ and still a better approximation is 3.14159. All of these are rational numbers. These approximations make the calculation of problems involving π simpler and easier.

EXAMPLE

Prove $\sqrt{2}$ is an irrational number.

Sol: Suppose $\sqrt{2}$ is rational, then $\sqrt{2} = \frac{p}{q}$ where $p, q \in Z$, $q \neq 0$ and suppose that p and q have no common divisor other than 1.

$\because \sqrt{2} = \frac{p}{q}$, $q \neq 0$

Squaring both sides, we get

$2 = \frac{p^2}{q^2}$

$\Rightarrow 2q^2 = p^2$ or $p^2 = 2q^2$

$\Rightarrow p^2$ is divisible by 2

$\Rightarrow p$ is divisible by 2

$\Rightarrow p^2$ is divisible by 4

$\Rightarrow 2q^2$ is divisible by 4

$\Rightarrow q^2$ is divisible by 2

$\Rightarrow q$ is divisible by 2

$\Rightarrow$ both p & q are divisible by 2

$\Rightarrow$ both p & q have a common divisor 2 and $2 \neq 1$

$\Rightarrow$ a contradiction

Thus our supposition is wrong and hence $\sqrt{2}$ is an irrational number.

EXAMPLE

Prove $\sqrt{3}$ is an irrational number.

Solution:

Suppose $\sqrt{3}$ is rational, then $\sqrt{3} = \frac{p}{q}$ where $p, q \in Z$, $q \neq 0$ and suppose that p & q have no common divisor other than 1.

$\because \sqrt{3} = \frac{p}{q}$

Squaring both sides, we get

$3 = \frac{p^2}{q^2}$

$\Rightarrow 3q^2 = p^2$ or $p^2 = 3q^2$

$\Rightarrow p^2$ is divisible by 3

$\Rightarrow p$ is divisible by 3

$\Rightarrow p^2$ is divisible by 9

$\Rightarrow 3q^2$ is divisible by 9

$\Rightarrow q^2$ is divisible by 3

$\Rightarrow q$ is divisible by 3

$\Rightarrow$ both p & q are divisible by 3.

$\Rightarrow$ both p & q have a common divisor 3 and $3 \neq 1$

$\Rightarrow$ a contradiction

Thus our supposition is wrong

and hence $\sqrt{3}$ is an irrational number.

Properties of Real Numbers:

Binary Operation

A binary operation in a set S is a rule denoted by * that assigns to any pair of elements of S a unique element of S. i.e; for $a, b \in S$, * is a Binary operation if $*(a,b) = a*b \in S$

Following are the properties or laws of real numbers.

1. Addition Laws

i) Closure Law of Addition

$\forall\ a, b \in \mathbb{R},\ a+b \in \mathbb{R}$

ii) Associative Law

$\forall\ a, b, c \in \mathbb{R}$

$a+(b+c) = (a+b)+c$

iii) Additive Identity

$\forall\ a \in \mathbb{R}, \exists\ 0 \in \mathbb{R}$ such that

$a+0 = 0+a = a$

0 is called additive identity.

iv) Additive Inverse

$\forall\ a \in \mathbb{R}\ \exists\ -a \in \mathbb{R}$ such that

$a+(-a) = 0 = -a+a$

a and $-a$ are called additive inverse of each other.

v) Commutative Law

$\forall\ a, b \in \mathbb{R}$

$a+b = b+a$

2. Multiplication Laws

vi) Closure Law

$\forall\ a, b \in \mathbb{R},\ a \cdot b \in \mathbb{R}$

vii) Associative Law

$\forall\ a, b, c \in \mathbb{R}$

$a(bc) = (ab)c$

viii) Multiplicative Identity

$\forall\ a \in \mathbb{R}, \exists\ 1 \in \mathbb{R}$ such that

$a \cdot 1 = 1 \cdot a = a$. 1 is called multiplicative identity

ix) Multiplicative Inverse

$\forall\ a \in \mathbb{R}, a \neq 0\ \exists\ a^{-1} \in \mathbb{R}$ such that $a \cdot a^{-1} = 1 = a^{-1} \cdot a$ where $a^{-1} = \frac{1}{a}$

a and $\frac{1}{a}$ are called multiplicative inverse of each other.

x) Commutative Law

$\forall\ a, b \in \mathbb{R},\ ab = ba$

3. Multiplication-Addition Law

xi) $\forall\ a, b, c \in \mathbb{R}$

$a \cdot (b+c) = ab+ac$ (left distributive law)

and

$(a+b) \cdot c = ac+bc$ (Right dist. law)

4. Properties of Equality

i) Reflexive Property

$\forall\ a \in \mathbb{R},\ a = a$

ii) Symmetric Property

$\forall a,b \in \mathbb{R}$ $a=b \Rightarrow b=a$

iii) Transitive Property

$\forall a,b,c \in \mathbb{R}$

$a=b \wedge b=c \Rightarrow a=c$

iv) Additive Property

$\forall a,b,c \in \mathbb{R}$

$a=b \Rightarrow a+c=b+c$

v) Multiplicative Property

$\forall a,b,c \in \mathbb{R}$

$a=b \Rightarrow ac=bc \wedge ca=cb$

vi) Cancellation Law w.r.t. '+'

$\forall a,b,c \in \mathbb{R}$ $a+c=b+c \Rightarrow a=b$

vii) Cancellation Law w.r.t. '·'

$\forall a,b,c \in \mathbb{R}, ac=bc \Rightarrow a=b, c \neq 0$

5. Properties of Inequalities (order properties)

i) Trichotomy Property

$\forall a,b \in \mathbb{R}$

either $a=b$ or $a<b$ or $a>b$

ii) Transitive Property

$\forall a,b,c \in \mathbb{R}$

a) $a>b \wedge b>c \Rightarrow a>c$

b) $a<b \wedge b<c \Rightarrow a<c$

iii) Additive Property

$\forall a,b,c \in \mathbb{R}$

a) $a>b \Rightarrow a+c>b+c$ and
$a<b \Rightarrow a+c<b+c$

b) $a>b \wedge c>d \Rightarrow a+c>b+d$
and $a<b \wedge c<d \Rightarrow a+c<b+d$

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iv) Multiplicative Property

a) $\forall a,b,c \in \mathbb{R}$ and $c>0$

$a>b \Rightarrow ac>bc$ and

$a<b \Rightarrow ac<bc$

b) $\forall a,b,c \in \mathbb{R}$ and $c<0$

$a>b \Rightarrow ac<bc$ and

$a<b \Rightarrow ac>bc$

c) $\forall a,b,c,d \in \mathbb{R}$ (all +ve)

$a>b \wedge c>d \Rightarrow ac>bd$ and

$a<b \wedge c<d \Rightarrow ac<bd$

Example 4

Prove that for any real numbers a, b

i) $a \cdot 0 = 0$

ii) $ab=0 \Rightarrow a=0 \vee b=0$

Proof

(i) $a \cdot 0 = a[1+(-1)]$ (additive inverse)

$= a(1-1)$ (Def. of subtraction)

$= a\cdot 1 - a\cdot 1$ (Distributive Law)

$= a - a$ (1 is multiplicative identity)

$= a+(-a)$ (Def. of Sub.)

$a\cdot 0 = 0$ (Additive inverse)

ii) Given that

$ab=0$ ——— ①

Suppose $a \neq 0$ then $\frac{1}{a}$ exists

From ①

$\frac{1}{a}(ab) = \frac{1}{a}\cdot 0$ (multiplicative Property)

$\frac{1}{a}(ab) = 0$ $\because a\cdot 0 = 0$

$(\frac{1}{a}\cdot a)b = 0$ (Assoc. Law of '·')

$1\cdot b = 0$ (Multiplicative inverse)

$b = 0$ (1 is multiplicative identity)

Thus if $ab=0$, $a \neq 0$ then $b=0$

Similarly it can be proved that if $ab=0$ and $b\neq 0$ then $a=0$.

Hence $ab=0 \Rightarrow a=0$ or $b=0$

EXAMPLE 5.

For real numbers a, b show the following by stating the properties used.

i) $(-a)b = a(-b) = -ab$

ii) $(-a)(-b) = ab$

Proof

i) $(-a)b+ab = (-a+a)b$ (dist. law)

$= 0\cdot b$ (additive Inverse)

$= 0$

$\Rightarrow (-a)b$ and ab are additive inverse of each other.

$\therefore (-a)b = -(ab) = -ab$

Now

$a(-b)+ab = a(-b+b)$ (Distributive law)

$= a\cdot 0$ (additive inverse)

$= 0$

$\Rightarrow a(-b)$ and ab are additive inverse of each other.

$\therefore a(-b) = -(ab) = -ab$

ii) $(-a)(-b) + [-(ab)]$

$= (-a)(-b) + (-ab)$

$= (-a)(-b) + (-a)(b)$ (By (i))

$= (-a)\cdot(-b+b)$ (Distributive law)

$= (-a)\cdot 0 = 0$

$\Rightarrow (-a)(-b)$ and $-(ab)$ are additive inverse of each other.

$\therefore (-a)(-b) = ab$

EXAMPLE 6.

Sol: (i) Suppose $\frac{a}{b} = \frac{c}{d}$

$\frac{a}{b}\times 1 = 1\times\frac{c}{d}$ (Multiplicative identity)

$\frac{a}{b}\times\left(d\times\frac{1}{d}\right) = \left(b\times\frac{1}{b}\right)\times\frac{c}{d}$ (mult. inverse)

$\frac{a}{b}\times\frac{d}{d} = \frac{b}{b}\times\frac{c}{d}$ $\left(\because \frac{a}{b} = a\times\frac{1}{b}\right)$

$\frac{ad}{bd} = \frac{bc}{bd}$

$ad\times\frac{1}{bd} = bc\times\frac{1}{bd}$ $\quad\because \frac{a}{b} = a\times\frac{1}{b}$

$ad = bc$ (Cancellation law)

Conversely, suppose that

$ad = bc$

$(ad)\times\left(\frac{1}{bd}\right) = (bc)\times\left(\frac{1}{bd}\right)$ [mult. property]

$(ad)\times\left(\frac{1}{b}\times\frac{1}{d}\right) = (bc)\times\left(\frac{1}{b}\cdot\frac{1}{d}\right)$

$(ad)\times\left(\frac{1}{d}\times\frac{1}{b}\right) = (cb)\times\left(\frac{1}{b}\cdot\frac{1}{d}\right)$ [Closure Law]

$a\cdot\left(d\cdot\frac{1}{d}\right)\cdot\frac{1}{b} = c\cdot\left(b\cdot\frac{1}{b}\right)\cdot\frac{1}{d}$ (Associative Law)

$a\cdot 1\cdot\frac{1}{b} = c\cdot 1\cdot\frac{1}{d}$ [mult. inverse]

$a\cdot\frac{1}{b} = c\cdot\frac{1}{d}$ [mult. identity]

$\frac{a}{b} = \frac{c}{d}$ (proved)

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To prove

ii) $\frac{1}{a}\cdot\frac{1}{b}=\frac{1}{ab}$

Consider

$(ab)\left(\frac{1}{a}\cdot\frac{1}{b}\right)=(ba)\left(\frac{1}{a}\cdot\frac{1}{b}\right)$ [Closure law]

$= b\cdot\left(a\cdot\frac{1}{a}\right)\cdot\frac{1}{b}$ (Assoc. law)

$= b\cdot 1\cdot\frac{1}{b} = b\cdot\frac{1}{b} = 1$

$\Rightarrow ab$ and $\frac{1}{a}\cdot\frac{1}{b}$ are multiplicative inverse of each other.

$\therefore \frac{1}{a}\cdot\frac{1}{b}=\frac{1}{ab}$

iii) To prove $\frac{a}{b}\cdot\frac{c}{d}=\frac{ac}{bd}$

L.H.S. $=\frac{a}{b}\cdot\frac{c}{d}$

$=\left(a\cdot\frac{1}{b}\right)\cdot\left(c\cdot\frac{1}{d}\right)$

$=a\cdot\left(\frac{1}{b}\cdot c\right)\cdot\frac{1}{d}$ (Assoc. law)

$=a\cdot\left(c\cdot\frac{1}{b}\right)\cdot\frac{1}{d}$ (Commutative law)

$=(ac)\left(\frac{1}{b}\cdot\frac{1}{d}\right)$ (Assoc. law)

$=ac\cdot\frac{1}{bd}$

$=\frac{ac}{bd}$ = R.H.S.

iv) To prove $\frac{a}{b}=\frac{ka}{kb}$ $(k\neq 0)$

L.H.S. $=\frac{a}{b}$

$=\frac{a}{b}\cdot 1$ (multiplicative identity)

$=\frac{a}{b}\cdot\left(k\cdot\frac{1}{k}\right)$ (multiplicative inverse)

$=\frac{a}{b}\cdot\frac{k}{k}$

$=\frac{ak}{bk}$ = R.H.S.

v) To prove

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{ad}{bc}$$

$\boxed{6}$ L.H.S. $=\frac{\frac{a}{b}}{\frac{c}{d}}$

$=\frac{\frac{a}{b}\times 1}{1\times\frac{c}{d}}$ [Multiplicative Identity]

$=\frac{\frac{a}{b}\times\left(d\times\frac{1}{d}\right)}{\left(b\times\frac{1}{b}\right)\times\frac{c}{d}}$ [Multiplicative Inverse]

$=\frac{\frac{a}{b}\times\frac{d}{d}}{\frac{b}{b}\times\frac{c}{d}}=\frac{\frac{ad}{bd}}{\frac{bc}{bd}}$

$=\frac{ad\times\frac{1}{bd}}{bc\times\frac{1}{bd}}$

$=\frac{ad}{bc}$ [Cancellation Law]

= R.H.S.

* EXERCISE 1.1 *

① Which of the following sets have closure property w.r.t. '+' and '×'

i) $\{0\}$

Addition Table

+	0
0	0

$\because 0+0=0\in\{0\}\Rightarrow\{0\}$ has closure property w.r.t. '+'

Multiplication Table

×	0
0	0

$\because 0\times 0=0\in\{0\}\Rightarrow\{0\}$ has closure property w.r.t. '×'

ii) $\{1\}$

Addition Table

+	1
1	2

$\because 1+1=2\notin\{1\}\Rightarrow\{1\}$ does not have closure property w.r.t. '+'

Multiplication Table

×	1
1	1

$\because 1\times 1 = 1 \in \{1\} \Rightarrow \{1\}$ has closure property w.r.t. '×'

iii) $\{0, -1\}$

Addition Table

+	0	−1
0	0	−1
−1	−1	−2

$\because 0+0 = 0 \in \{0,-1\}$

$0+(-1) = -1 \in \{0,-1\}$

$(-1)+0 = -1 \in \{0,-1\}$

$(-1)+(-1) = -2 \notin \{0,-1\}$

$\Rightarrow \{0,-1\}$ does not have closure property w.r.t. '+'

Multiplication Table

×	0	−1
0	0	0
−1	0	1

$\because 0\times 0 = 0 \in \{0,-1\}$

$0\times(-1) = 0 \in \{0,-1\}$

$(-1)\times 0 = 0 \in \{0,-1\}$

$(-1)\times(-1) = 1 \notin \{0,-1\}$

$\Rightarrow \{0,-1\}$ does not have closure property w.r.t. '×'

iv) $\{1, -1\}$

Addition Table

+	1	−1
1	2	0
−1	0	−2

$\because 1+1 = 2 \notin \{1,-1\}$

$1+(-1) = 0 \notin \{1,-1\}$

$(-1)+1 = 0 \notin \{1,-1\}$

$(-1)+(-1) = -2 \notin \{1,-1\}$

$\Rightarrow \{1,-1\}$ does not have closure property w.r.t. '+'

Multiplication Table

×	1	−1
1	1	−1
−1	−1	1

$\because 1\times 1 = 1 \in \{1,-1\}$

$1\times(-1) = -1 \in \{1,-1\}$

$(-1)\times 1 = -1 \in \{1,-1\}$

$(-1)\times(-1) = 1 \in \{1,-1\}$

$\Rightarrow \{1,-1\}$ has closure property w.r.t. '×'

② Name the properties used in the following questions.

i) $4+9 = 9+4$ (Commutative property w.r.t. '+')

ii) $(a+1)+\frac{3}{4} = a+\left(1+\frac{3}{4}\right)$ (Assoc. property w.r.t. '+')

iii) $(\sqrt{3}+\sqrt{5})+\sqrt{7} = \sqrt{3}+(\sqrt{5}+\sqrt{7})$ (")

iv) $100+0 = 100$ (Additive Identity)

v) $1000\times 1 = 1000$ (Multiplicative Identity)

vi) $4.1+(-4.1) = 0$ (Additive inverse)

vii) $a-a = 0$ (Additive inverse)

viii) $\sqrt{2}\times\sqrt{5} = \sqrt{5}\times\sqrt{2}$ (Commutative property w.r.t '×')

ix) $a(b-c) = ab-ac$ (Left distributive property)

x) $(x-y)z = xz-yz$ (Right distributive property)

xi) $4\times(5\times 8) = (4\times 5)\times 8$ (Associative property w.r.t. '×')

xii) $a(b+c-d) = ab+ac-ad$ (Left distributive property)

③ Name the properties used in the following inequalities.

i) $-3 < -2 \Rightarrow 0 < 1$ (Additive property)

ii) $-5 < -4 \Rightarrow 20 > 16$ (multiplicative property)

iii) $1 > -1 \Rightarrow -3 > -5$ (Additive property)

iv) $a < 0 \Rightarrow -a > 0$ (multiplicative property)

v) $a > b \Rightarrow \frac{1}{a} < \frac{1}{b}$ (")

vi) $a > b \Rightarrow -a < -b$ (")

④ Prove the following rules of addition.

i) $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

$L.H.S. = \frac{a}{c} + \frac{b}{c}$

$= a \times \frac{1}{c} + b \times \frac{1}{c}$

$= (a+b) \times \frac{1}{c}$ (Right dist. property)

$= \frac{a+b}{c} = R.H.S.$

ii) $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$

$L.H.S. = \frac{a}{b} + \frac{c}{d}$

$= \frac{a}{b} \times 1 + 1 \times \frac{c}{d}$

$= \frac{a}{b} \times (d \times \frac{1}{d}) + (b \times \frac{1}{b}) \times \frac{c}{d}$

$= \frac{a}{b} \times \frac{d}{d} + \frac{b}{b} \times \frac{c}{d}$

$= \frac{ad}{bd} + \frac{bc}{bd}$

$= ad \times \frac{1}{bd} + bc \times \frac{1}{bd}$

$= (ad+bc) \times \frac{1}{bd}$

$= \frac{ad+bc}{bd} = R.H.S.$

⑤ Prove that

$-\frac{7}{12} - \frac{5}{18} = \frac{-21-10}{36}$

$L.H.S. = -\frac{7}{12} - \frac{5}{18}$

$= -\frac{7}{12} \times 1 - \frac{5}{18} \times 1$

$= -\frac{7}{12} \times (3 \times \frac{1}{3}) - \frac{5}{18} \times (2 \times \frac{1}{2})$

$= -\frac{7}{12} \times \frac{3}{3} - \frac{5}{18} \times \frac{2}{2}$

$= -\frac{21}{36} - \frac{10}{36}$

$= -21 \times \frac{1}{36} - 10 \times \frac{1}{36}$

$= (-21-10) \times \frac{1}{36}$

$= \frac{-21-10}{36} = R.H.S.$

⑥ Simplify by justifying each step.

i) $\frac{4+16x}{4} = \frac{1}{4} \times (4+16x)$ $\because \frac{a}{b} = \frac{1}{b} \times a$

$= \frac{1}{4} \times (4 \times 1 + 4 \times 4x)$ (multiplicative Identity)

$= \frac{1}{4} \times 4 \times (1+4x)$ (Distributive Property)

$= 1 \times (1+4x)$ (multiplicative inverse)

$= 1+4x$ (Multiplicative Identity)

ii) $\frac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}} = \frac{\frac{1}{4} \times 1 + \frac{1}{5} \times 1}{\frac{1}{4} \times 1 - \frac{1}{5} \times 1}$ (mult. identity)

$= \frac{\frac{1}{4} \times (5 \times \frac{1}{5}) + \frac{1}{5} \times (4 \times \frac{1}{4})}{\frac{1}{4} \times (5 \times \frac{1}{5}) - \frac{1}{5} \times (4 \times \frac{1}{4})}$ (mult. inverse)

$= \frac{\frac{1}{4} \times \frac{5}{5} + \frac{1}{5} \times \frac{4}{4}}{\frac{1}{4} \times \frac{5}{5} - \frac{1}{5} \times \frac{4}{4}}$ $\because \frac{a}{b} = a \times \frac{1}{b}$

$= \frac{\frac{5}{20} + \frac{4}{20}}{\frac{5}{20} - \frac{4}{20}}$ $\because \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

$$= \frac{5\times\frac{1}{20}+4\times\frac{1}{20}}{5\times\frac{1}{20}-4\times\frac{1}{20}} \quad \because \frac{a}{b}=a\cdot\frac{1}{b}$$

$$= \frac{(5+4)\times\frac{1}{20}}{(5-4)\times\frac{1}{20}} \text{ (Dist. property)}$$

$$= \frac{5+4}{5-4} \text{ (Cancellation law)}$$

$$= \frac{9}{1} = 9$$

iii) $$\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}} = \frac{\frac{a}{b}\times 1+1\times\frac{c}{d}}{\frac{a}{b}\times 1-1\times\frac{c}{d}}$$
(multiplicative Identity)

$$= \frac{\frac{a}{b}\times(d\times\frac{1}{d})+(b\times\frac{1}{b})\times\frac{c}{d}}{\frac{a}{b}\times(d\times\frac{1}{d})-(b\times\frac{1}{b})\times\frac{c}{d}}$$
(multiplicative inverse)

$$= \frac{\frac{a}{b}\times\frac{d}{d}+\frac{b}{b}\times\frac{c}{d}}{\frac{a}{b}\times\frac{d}{d}-\frac{b}{b}\times\frac{c}{d}} \quad \because \frac{a}{b}=a\times\frac{1}{b}$$

$$= \frac{\frac{ad}{bd}+\frac{bc}{bd}}{\frac{ad}{bd}-\frac{bc}{bd}} \quad \because \frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$$

$$= \frac{ad\times\frac{1}{bd}+bc\times\frac{1}{bd}}{ad\times\frac{1}{bd}-bc\times\frac{1}{bd}} \quad \because \frac{a}{b}=a\times\frac{1}{b}$$

$$= \frac{(ad+bc)\times\frac{1}{bd}}{(ad-bc)\times\frac{1}{bd}} \text{ (Dist. Law)}$$

$$= \frac{ad+bc}{ad-bc} \text{ (Cancellation Law)}$$

iv) $$\frac{\frac{1}{a}+\frac{1}{b}}{1-\frac{1}{a}\cdot\frac{1}{b}}$$

$$= \frac{\frac{1}{a}\times 1+1\times\frac{1}{b}}{1-\frac{1}{a}\cdot\frac{1}{b}} \text{ (Mult. Ident.)}$$

$$= \frac{\frac{1}{a}\times(b\times\frac{1}{b})+(a\times\frac{1}{a})\times\frac{1}{b}}{1-\frac{1}{a}\cdot\frac{1}{b}}$$
(Multiplicative inverse)

$$= \frac{\frac{1}{a}\times\frac{b}{b}+\frac{a}{a}\times\frac{1}{b}}{1-\frac{1}{a}\cdot\frac{1}{b}} \quad \because \frac{a}{b}=a\times\frac{1}{b}$$

$$= \frac{\frac{b}{ab}+\frac{a}{ab}}{1-\frac{1}{ab}} \quad \because \frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$$

$$= \frac{b\times\frac{1}{ab}+a\times\frac{1}{ab}}{1-\frac{1}{ab}} \quad \because \frac{a}{b}=a\times\frac{1}{b}$$

$$= \frac{b\times\frac{1}{ab}+a\times\frac{1}{ab}}{ab\times\frac{1}{ab}-1\cdot\frac{1}{ab}}$$
(Multiplicative inverse and multi. Identity)

$$= \frac{(b+a)\times\frac{1}{ab}}{(ab-1)\times\frac{1}{ab}} \text{ (Dist. property)}$$

$$= \frac{b+a}{ab-1} \text{ (Cancellation Law)}$$

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These notes are compiled by Fahad Hameed

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