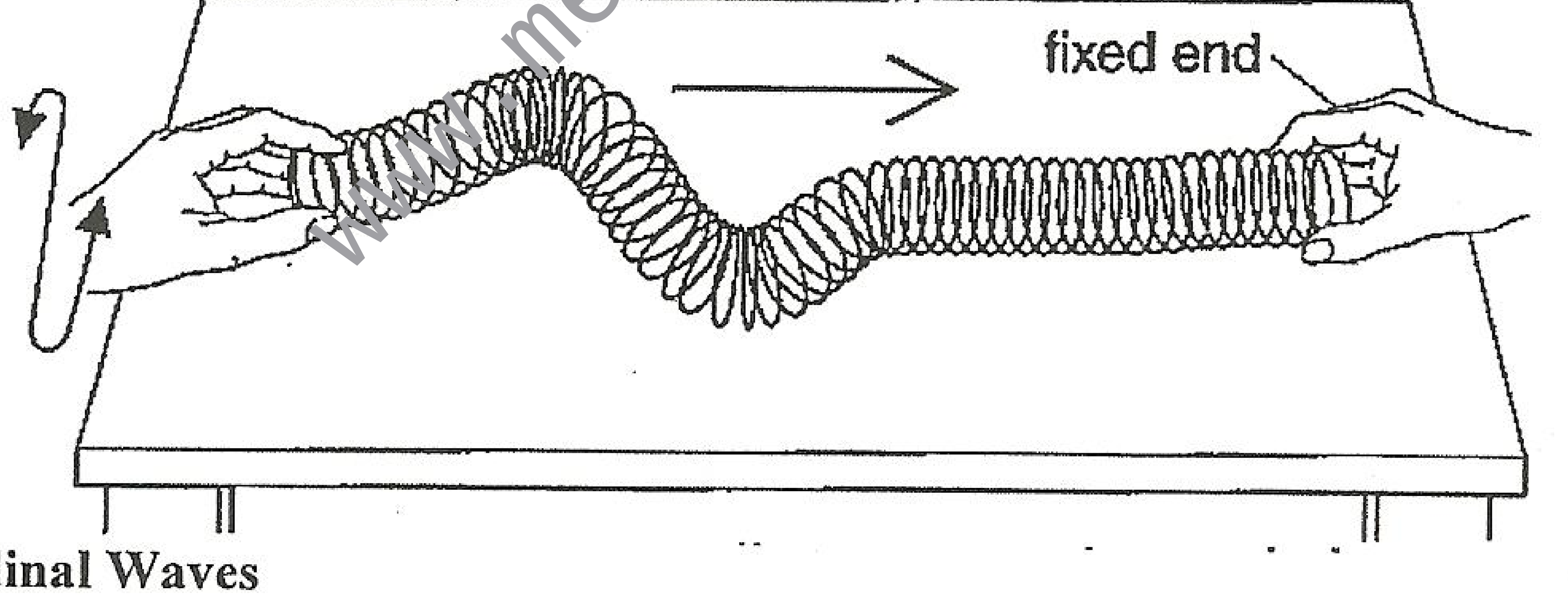
Waves

Wave Motion
Waves Motion is a disturbance created by vibrations in which energy is transferred from one point to the other without the medium being transferred.

Mechanical Waves
Mechanical waves are those which require a material medium for its propagation e.g. Sound waves, water waves, waves in strings. These waves are produced by a disturbance in a material medium and are transmitted by the particles of the medium oscillating to and fro.

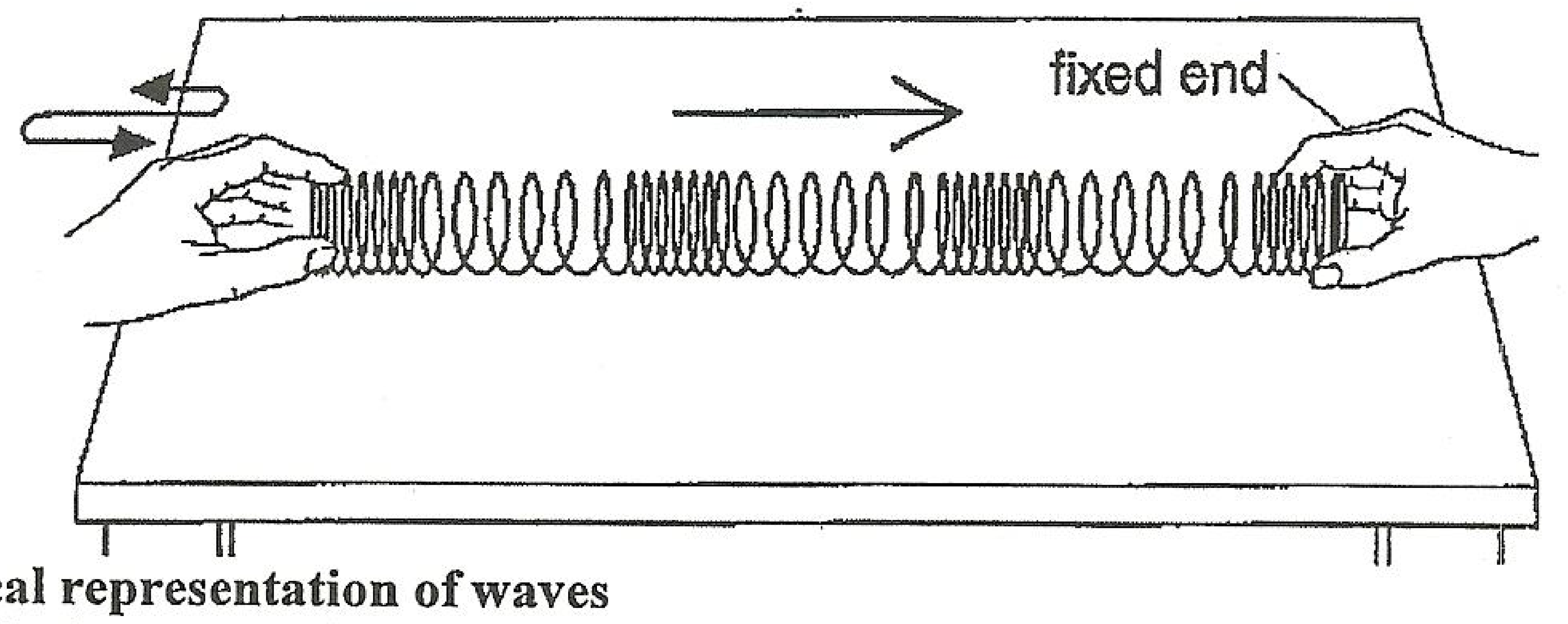
Electromagnetic Waves
Electromagnetic waves do not require a material medium for propagation; they consist of a disturbance in the form of varying electrical and magnetic fields. They travel more easily and have the same speed in a vacuum (max speed, the speed of light $3\times10^{8}\,m/s$), except in other mediums where the speed changes. E.g. light.

Transverse Waves
In transverse waves the vibrations are perpendicular to the direction of propagation of the wave, e.g., all electromagnetic waves, waves in a string when one end is moved up and down.
Experimental e.g. a rope fixed at one end B whilst the other end is given a vertical oscillation. Energy is transferred from A to B. there is no transfer of matter.



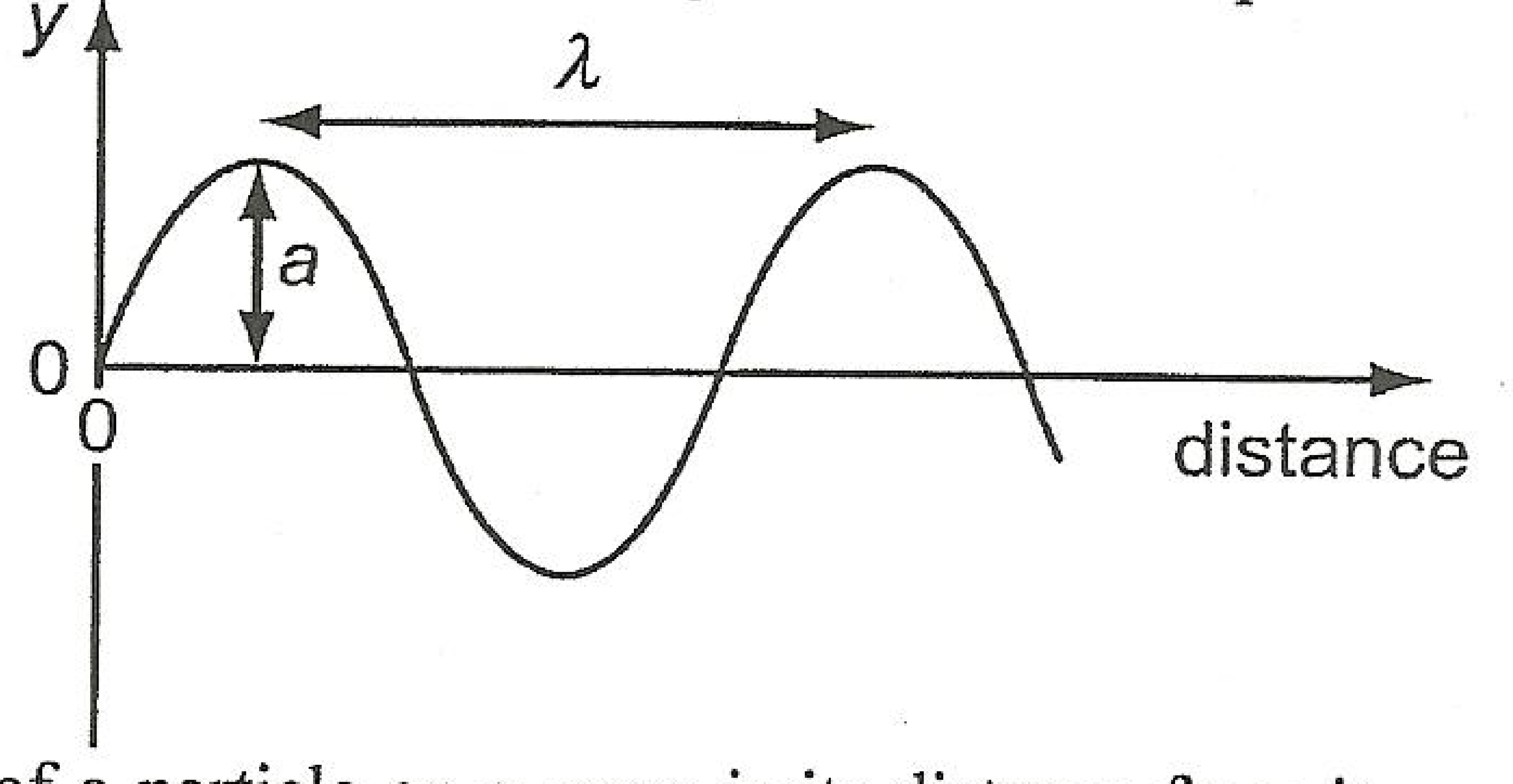
Longitudinal Waves
In longitudinal waves the vibrations are parallel to the direction of propagation of waves e.g. Sound waves, heat waves, waves set up in a string when one end is moved to and fro along the length of the string. In longitudinal waves we have compressions and rarefactions.
Compressions are regions where particles/molecules move towards one another creating a region where the pressure is above atmospheric while rarefactions are regions where they move away from one another creating a region where the pressure is less than atmospheric.
Experimental e.g. a loose slinky coil is fixed at the ends of A and B. The end A is given a longitudinal vibration.

Waves



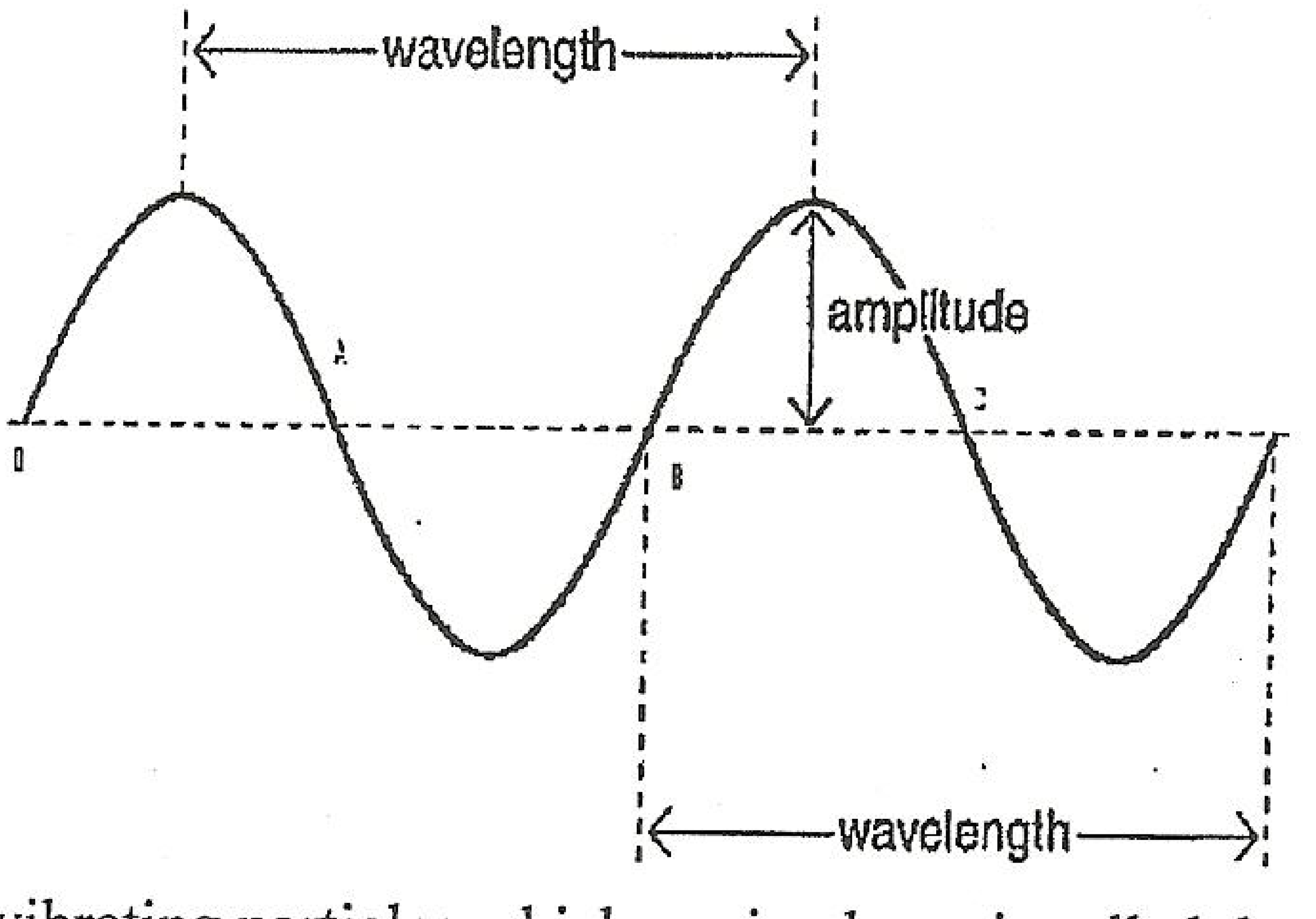
Graphical representation of waves

Whether a wave is transverse or longitudinal we can represent it graphically.



The displacement of a particle on a wave is its distance from its rest position, and the amplitude of a wave is its maximum displacement from its rest position.
A transverse wave can be plotted, by representing displacement on the y-axis and distance on the x-axis.

Wavelength (λ) is the distance between two successive points which are in the same phase.
For a longitudinal wave displacement of a particle is along the direction of the energy travelled. However if a graph of displacement on the y-axis verses distance on the x-axis is plotted the graph is exactly the same shape, hence one graph represents both types of waves.



This graph shows how different particles along the path of the wave are displaced at a given instance.
The maximum displacement a particle can achieve is called the amplitude of vibration.
Two vibrating particles with identical motion are said to be in set or in phase
E.g. O & B / A & C

The distance between two successive vibrating particles which are in phase, is called the wave length λ.

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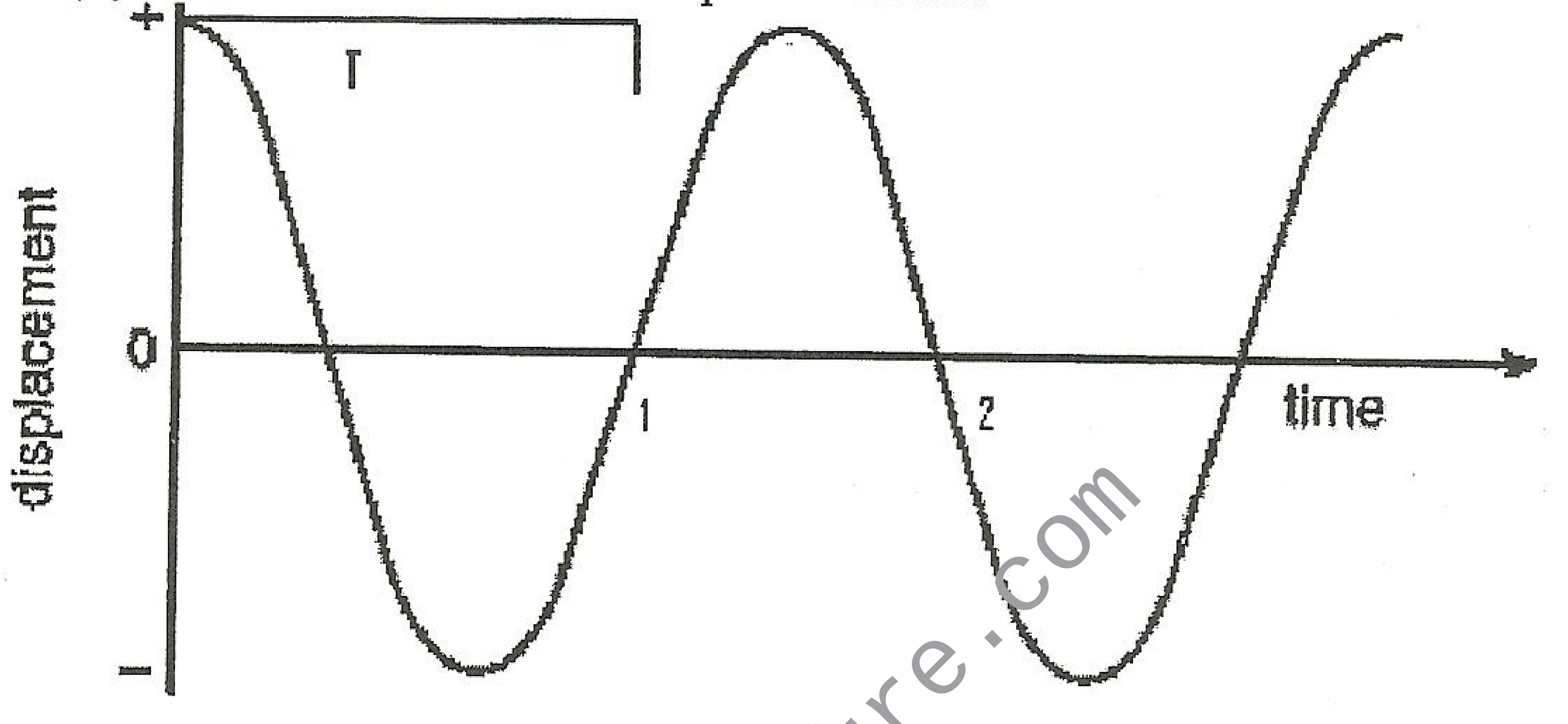
Particles which are separated by a distance **n** λ where **n = 1, 2, 3, 4...** are in step with the phase angle of vibration.

Wavelength (λ) is the distance between two successive points which are in the same phase.

Period (T) is the time taken for one complete vibration.



Frequency is the number of vibrations per second $f = \frac{1}{T}$.

We know that $\textbf{speed} = \frac{\textbf{distance}}{\textbf{time}}$

Therefore $v = \frac{\lambda}{T}$

$v = f\lambda$

So, the speed, frequency and wavelength of a wave are linked by this equation
$speed = frequency \times wavelength \therefore v = f\lambda$

$\frac{\omega T}{4} = \frac{\pi}{2}$

$\omega = \frac{2\pi}{T}$

or

$T = \frac{2\pi}{\omega}$

Displacement-time graph may be represented by:

$y = Sin\omega t$

$Sin\frac{\omega t}{4} = 1$

now

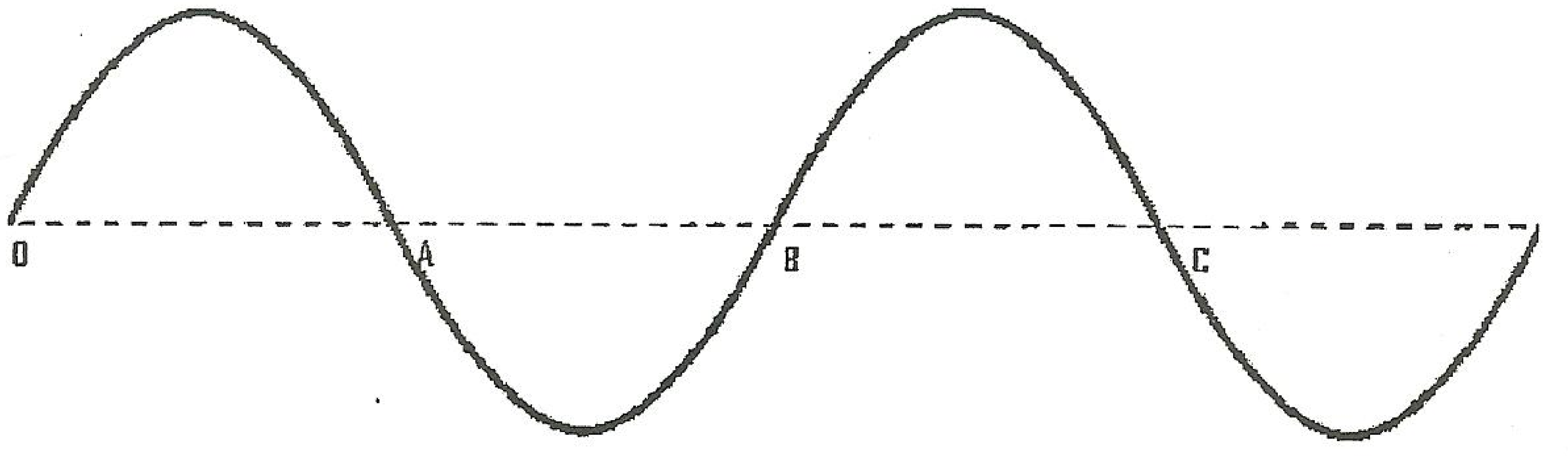
when $\frac{\omega t}{4} = 90^o = \frac{\pi}{2}$

$T = \frac{2\pi}{20}$ $\therefore \omega = \frac{2\pi}{T}$

In one vibration, energy travels through a distance of one wavelength in time T.

In time **T** distance traveled is λ.∴ In one second distance traveled = $\frac{\lambda}{T}$, which is equal to speed $v = f\lambda$

Waves



'A' vibrates 'T' seconds after 'O' because distance between them is λ and time is T.
When a wave passes, the particles along the path of the wave starts to vibrate with time, there is lagging behind in the vibrations. For e.g. the particle B will start to vibrate T seconds later than O and they are separated by a distance λ. O & B are said to have a phase difference.
This difference is given in terms of an angle. When two points are separated by λ, the phase difference is said to be 2π

Phase difference
$Sin\theta = Sin(\theta + 2\pi)$ Expressed in terms of $\angle$

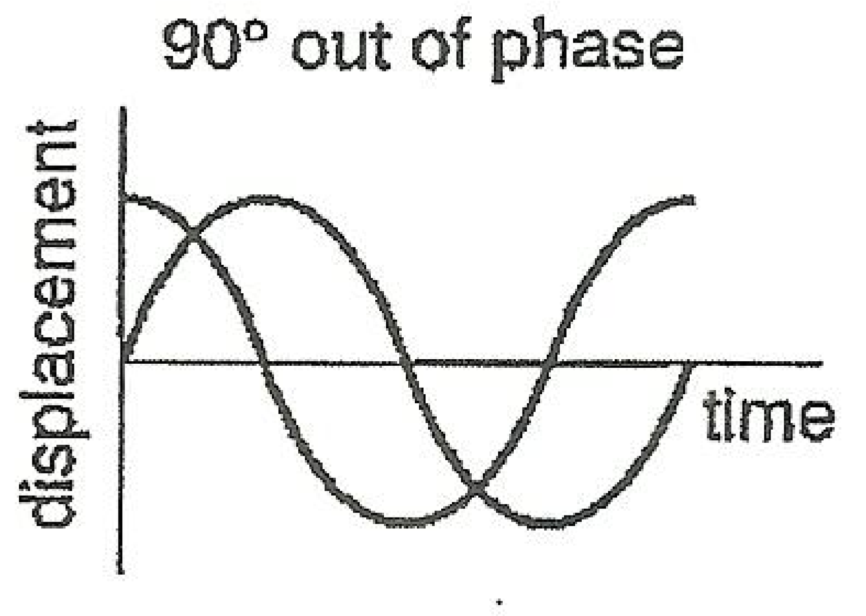
Phase difference between O & A is 2π
When distance $\lambda \longrightarrow 2\pi$ is phase difference
When distance $\frac{\lambda}{2} \longrightarrow \pi$ is phase difference
When distance $\frac{\lambda}{4} \longrightarrow \frac{\pi}{2}$ is phase difference

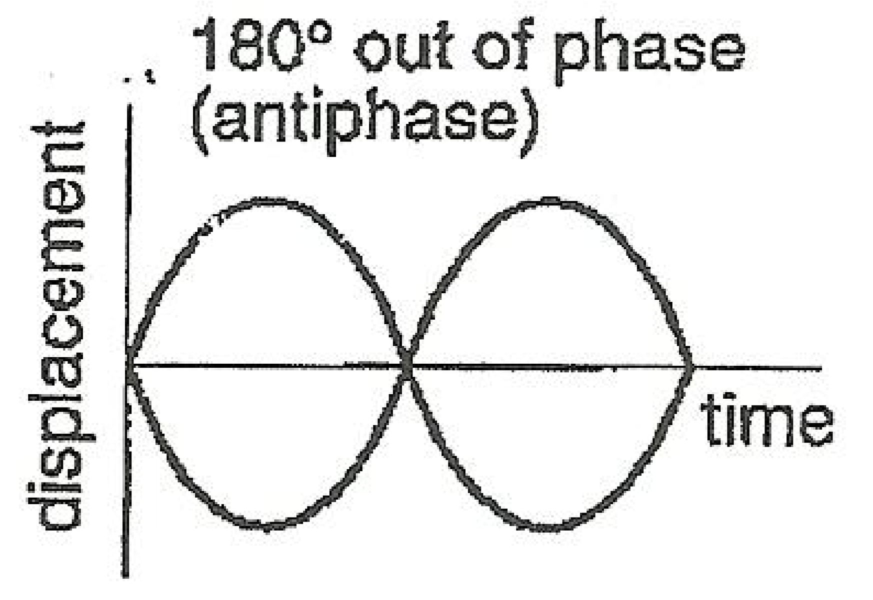
According to Time

λ	$\longrightarrow 2\pi$	T
$\frac{\lambda}{2}$	$\longrightarrow \pi$	$\frac{T}{2}$
$\frac{\lambda}{4}$	$\longrightarrow \frac{\pi}{2}$	$\frac{T}{4}$

Distance b/w two points	Phase difference	Time
0	0	0
$\frac{\lambda}{4}$	$\frac{\pi}{2}$	$\frac{T}{4}$
$\frac{\lambda}{2}$	π	$\frac{T}{2}$
$\frac{3}{4}\lambda$	$\frac{3}{2}\pi$	$\frac{3}{4}T$
λ	2π	T

Waves





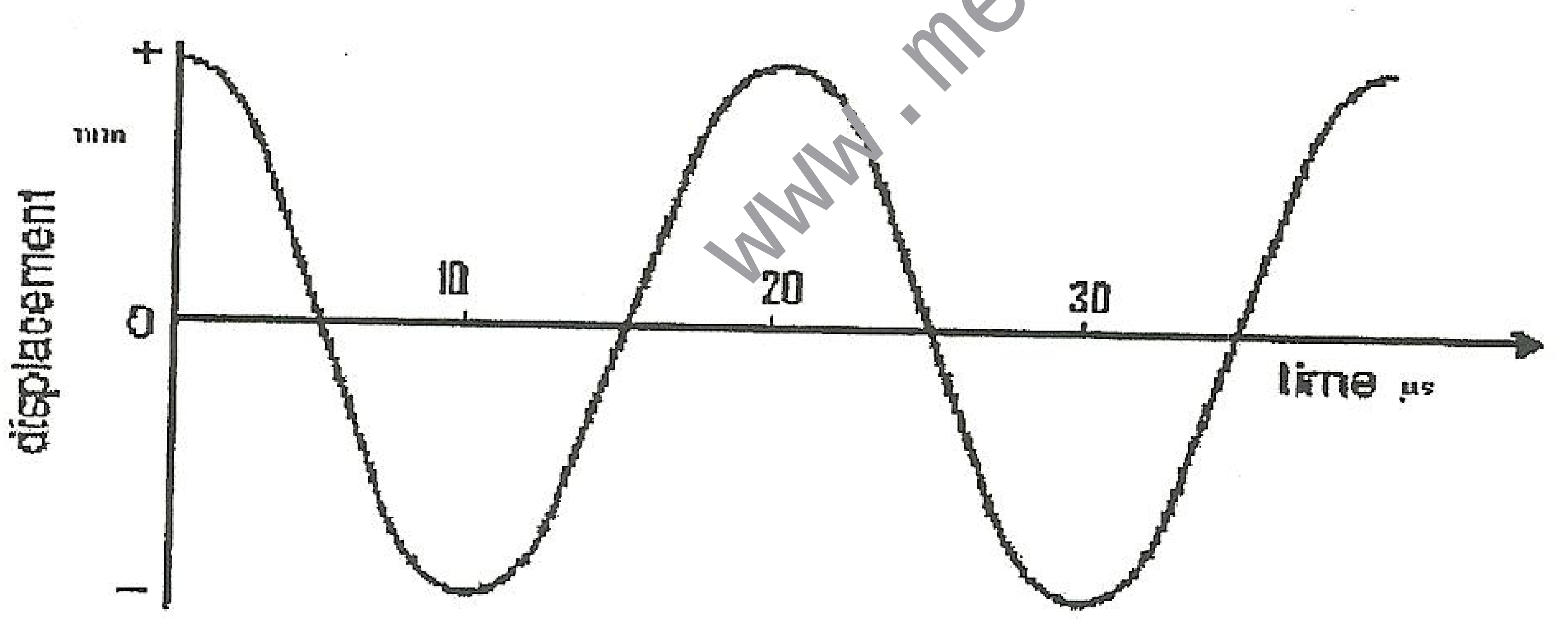
?. A sound wave of frequency 400Hz is traveling at a speed of 320m/s. What is the phase difference between two points on the wave 0.2m apart in the direction of travel?

$v = f\lambda \quad \lambda = \frac{320}{400} = 0.8m$

$0.2m = \frac{\lambda}{4} = \frac{0.8}{4}$

phase difference $= \frac{\pi}{2} = 90^o$

The wave shown in the diagram is traveling at the speed of $5m/s$. Find its amplitude and its wavelength.
Amplitude = 2 mm



$20\mu s = \frac{20}{1000000} = \frac{1}{50000} = T$

$f = \frac{1}{T} = 50000Hz$

now

$v = 5kms^{-1} = 5000ms^{-1}$

now

$v = f\lambda$

so

$5000 = 50000 \times \lambda$

$\lambda = 0.1m$

Intensity

The energy incident normally per unit time per unit area, on area A (m^2), E joules of energy falls normally in time T. Intensity is inversely proportional to the square of the distance.

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$I = \frac{E}{AT} \qquad = Jm^{-2}s^{-1}$

$\text{power} = W = \frac{E}{T} = Js^{-1}$

$\therefore I = Wm^{-2}$

Variation of intensity with distance

S: source emitting waves in all directions — E: energy emitted per second

S.A. : surface area of sphere = $4\pi r^2$ — energy falling per unit area per second = $\frac{E}{4\pi r^2}$

$I = \frac{E}{4\pi r^2} \qquad I \propto \frac{1}{r^2}$

$\therefore$ As you go away from the source, the intensity of the source decreases.

Variation of intensity with amplitude of vibration

Intensity $\propto$ (amplitude)^2

?. A small source of sound radiates energy equally in all directions, at a particular frequency. The intensity of sound 1m from the source is $1.0 \times 10^{-5}\ Wm^{-2}$ corresponding to amplitude of oscillation of one molecule of 70 μm. Assuming that the sound is propagated without the loss of energy, what will be the:

1. Intensity of sound?
2. Amplitude of oscillation of the air molecules at a distance 5m from the source?

1. $I = k_1 a^2 \qquad I = \frac{k_2}{r^2}$

$I_1 r_1^2 = I_2 r_2^2 \qquad 1\times10^{-5} \times 1^2 = I_2 \times 5^2$

$\therefore I_2 = 4\times10^{-7}\ Wm^{-2}$

2. $\frac{I}{a^2} = k \qquad \frac{I_1}{a_1^2} = \frac{I_2}{a_2^2}$

$\frac{1\times10^{-5}}{70^2} = \frac{4\times10^{-7}}{a^2} \qquad a^2 = 196$

$\therefore a = 14\mu m$

?. A point source of sound emits energy equally in all directions at a constant rate. And a person 8m from the source listens. After a while, the intensity of the source is decreased by 50%.

If the person wishes to convert the sound back to its original intensity, how far should he be from the source?

Let E be the energy emitted per second.

$I = \frac{E}{4\pi r^2} = \frac{E/2}{4\pi r^2} = \frac{E}{8\pi r^2}$

$\frac{E}{64} = \frac{E}{2r^2} \qquad r^2 = 32$

$\therefore r = 5.66m$

Properties of waves

Though there are many different types of waves, there are a number of basic properties which they all have in common. All waves can be reflected, refracted, diffracted and produce interference patterns.

Wave profile

Vertical section representing the displaced particles from their equilibrium position along the path of the wave is called a wave profile.

Wave fronts

A wave front is a line or a surface on which all the vibrating particles are in phase. Wave fronts can be of three kinds:

1. circular (2D) e.g. water waves in a ripple tank
2. spherical (3D) e.g. light waves
3. plane

When a circular wave or spherical wave moves a large distance from the source, they become plane wave fronts. A wave front is always parallel and at right angles to the direction of the plane.

Progressive waves

Wave profile moves along with energy moving outward from the source.

1. Vibrating particles of a progressive wave have the same amplitude.
2. If the two points are taken within 1λ, no two particles will be in step i.e. All vibrating particles are out of phase.
3. If two points are separated by a distance λ or $n\lambda$ all particles will be in phase. When two identical progressive waves traveling in opposite direction, overlap in a region, a stationary wave pattern is obtained.
4. There are two types of progressive waves transverse and longitudinal.

Stationary waves

Within a loop, vibrating particles:

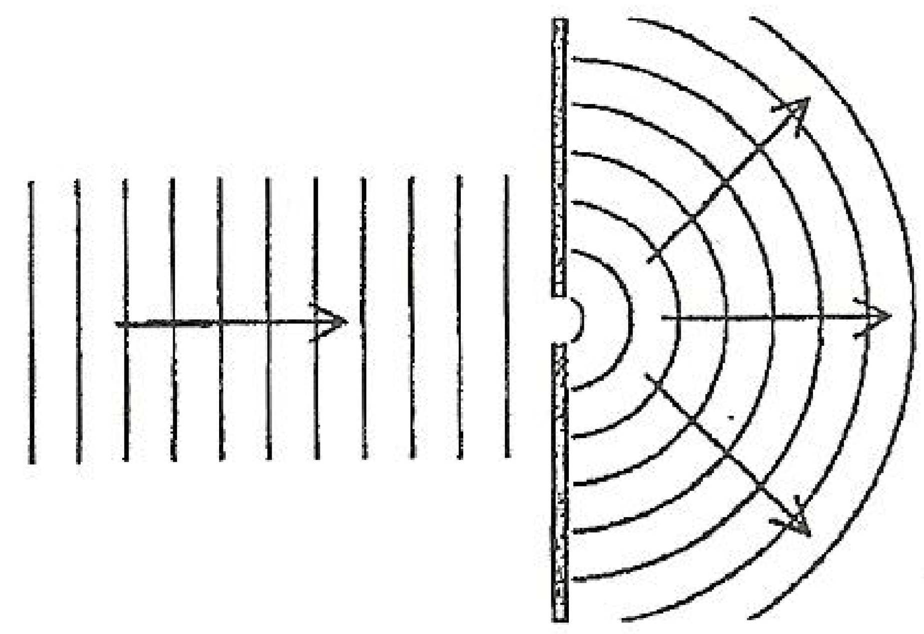
- are in step
- have different amplitude

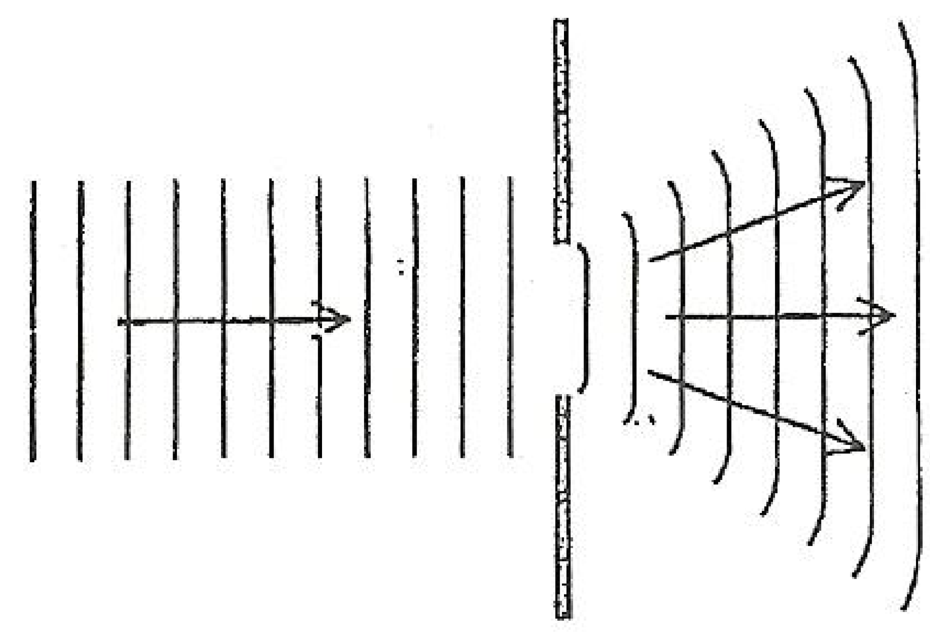
Points where particles are permanently at rest or at minimum displacement are called nodes like wise points where particles are at maximum displacement (amplitude) are called antinodes. Vibrating particles in the neighboring loop is anti-phase.

In a stationary wave, energy is trapped within a loop; there is no net transfer of energy because wave profile does not move.

Diffraction

When waves pass through a small opening or pass around an obstacle, they spread out and reach areas where they not normally expected. This process is called diffraction. For diffraction to take place, the wave length of the wave must be comparable with the size of the opening (narrower the opening greater the diffraction).





Superposition:

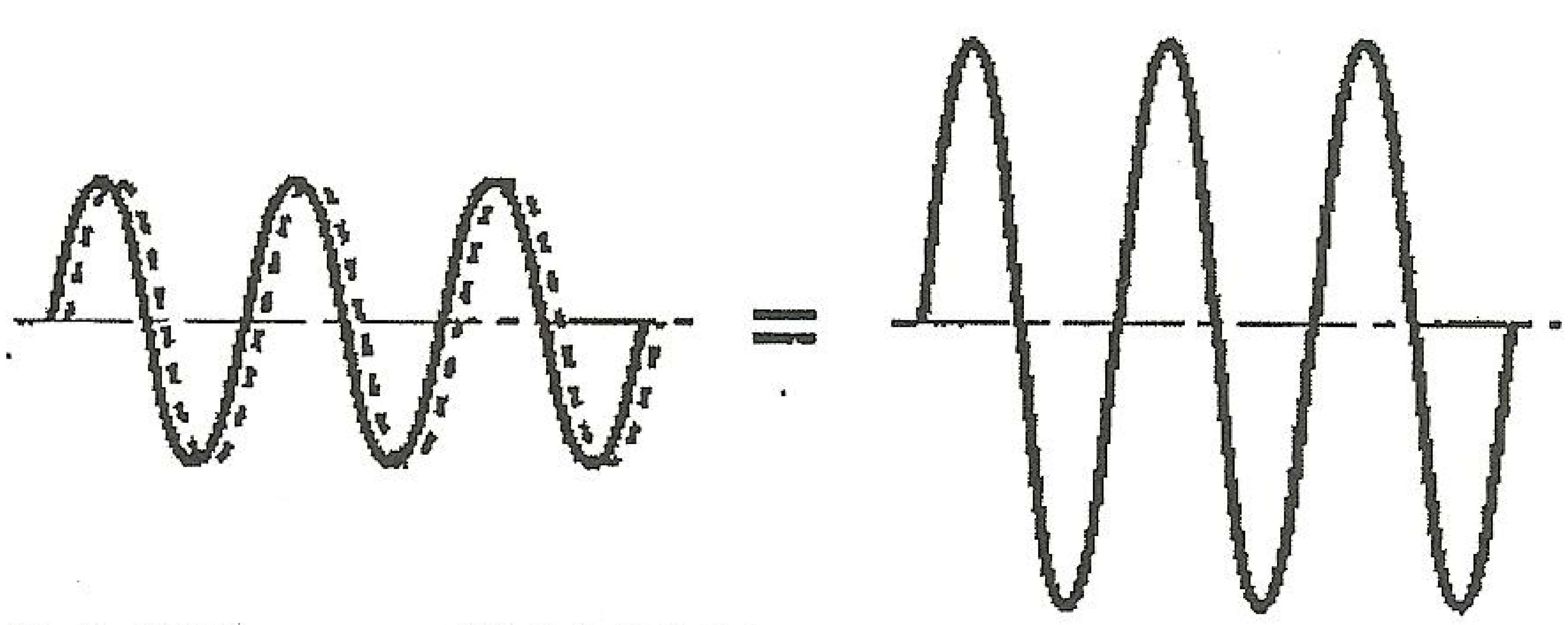
When waves overlap in a region, places where you get maximum effect are called constructive superposition whereas those with minimum effect are called destructive superposition.

When waves from two different sources superimpose in a region, if the two sources do not have a constant phase relationship with respect to time the positions of constructive and destructive interference change. So a clear pattern can not be observed, but if the two sources are coherent (identical), the two sources have a constant phase relationship with time, the places of constructive and destructive interference do not change so a clear pattern of maximum and minimum effect can be observed. Such a pattern is called an interference pattern.

Path difference

The extra distance one wave motion travels further than another, the two distances are measured from points where the two waves are in phase and end at the point where the two waves meet.

Constructive interference

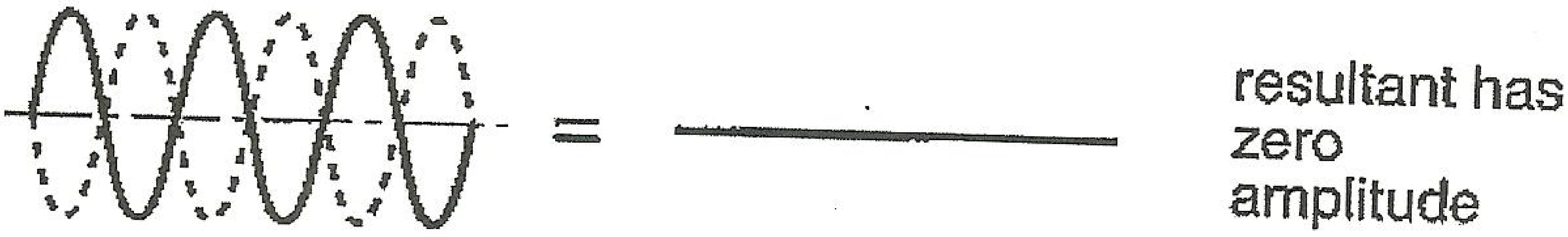


Path Difference $= 0\lambda, 1\lambda, 2\lambda, 3\lambda...$

$PD = n\lambda$ where $n = 0,1,2,3...$

The constructive interference will occur along the antinodal line where the path difference is the multiple of λ $(n\lambda)$.

Destructive interference



Path Difference $= \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots$

$PD = (2n+1)\frac{\lambda}{2}$ *where* $n = 0,1,2,3\ldots$

Destructive interference occurs along the nodal line where the path difference is equal to the multiple of λ $(2n+1)\frac{\lambda}{2}$. PD is an odd multiple of half a wavelength.

Principle of superposition:

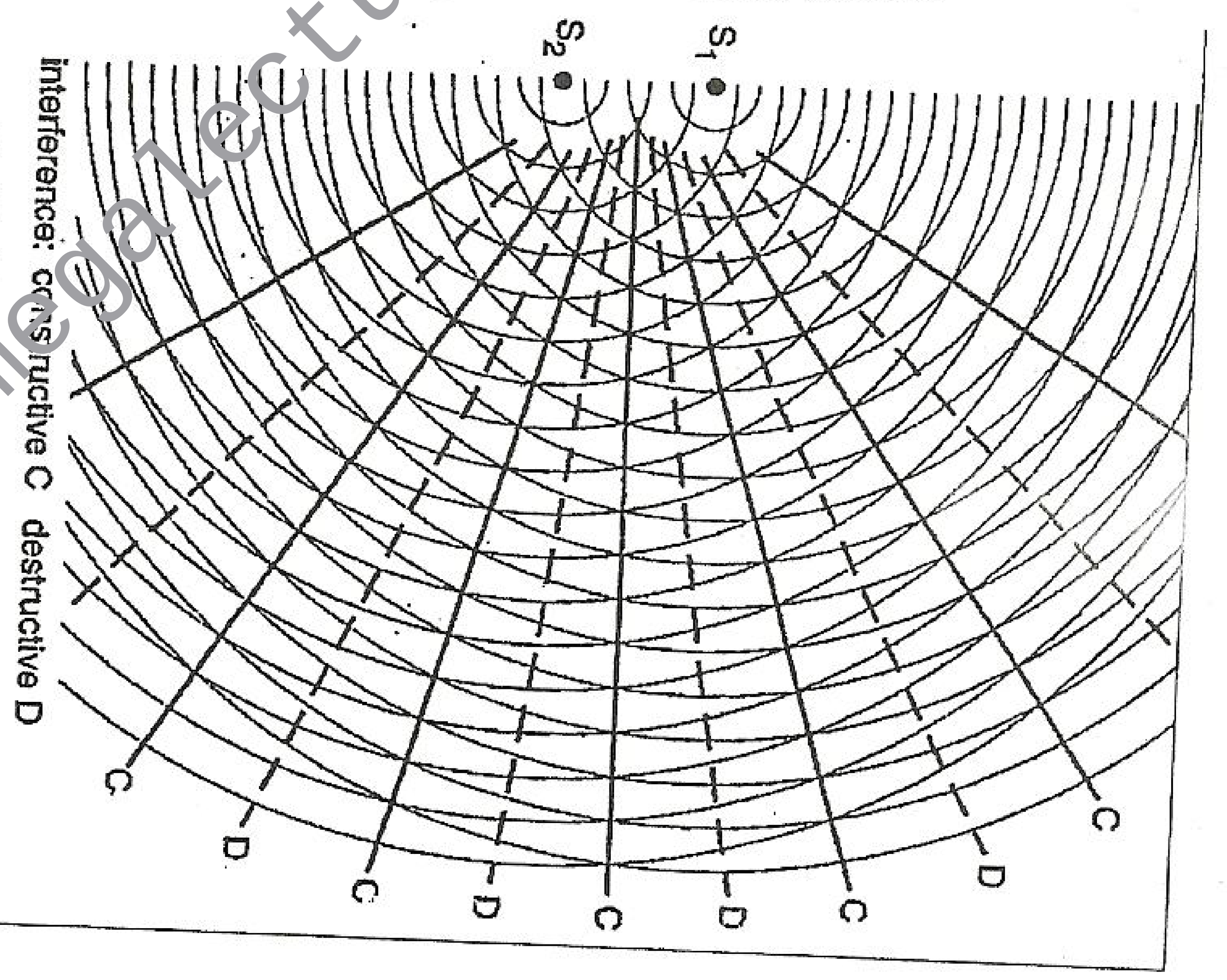
Resultant displacement of a particle is the algebraic sum of the individual displacements of the particles due to the 2 different waves.

The resultant displacement of the vibrating particles from their equilibrium position is equal to the algebraic sum of the displacement caused by the individual waves.

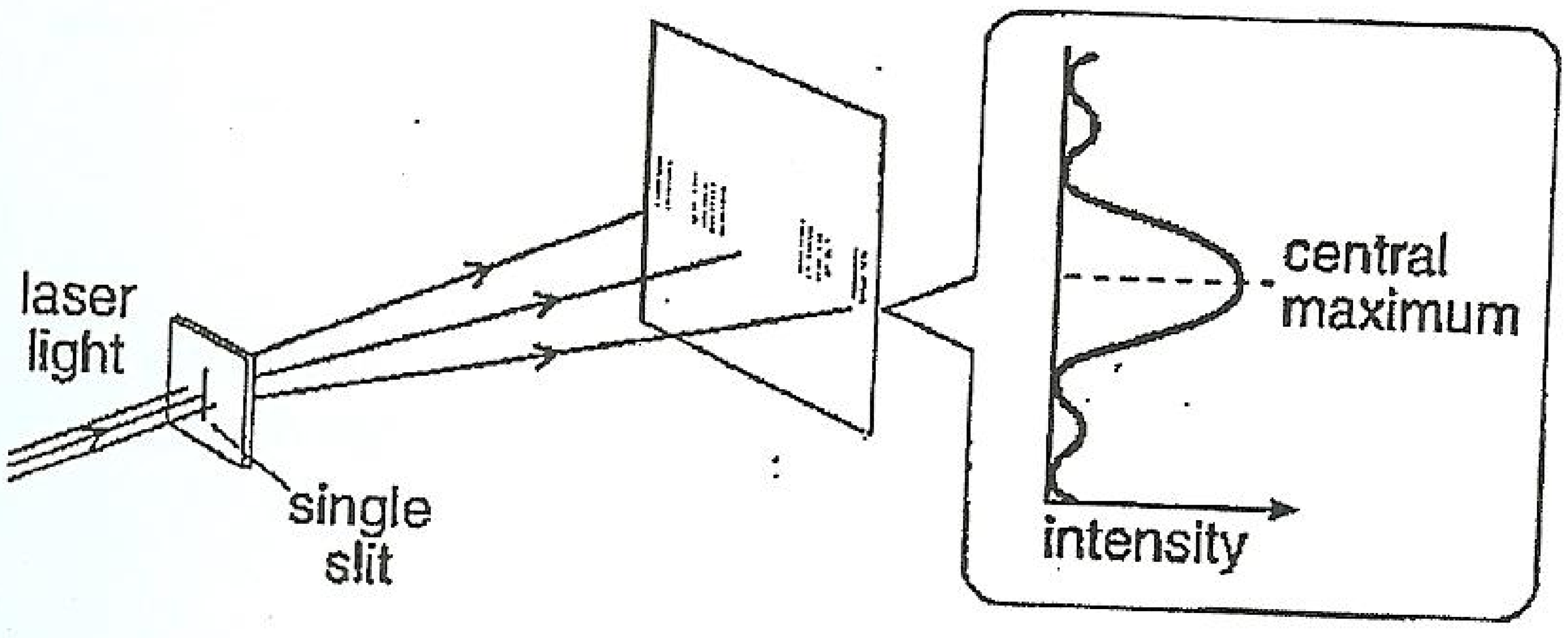
What is an interference pattern?

In a region where wave from coherent sources cross superposition occurs giving reinforcements of the wave at the same points and cancellation at the others.

The resulting effect is called an interference pattern.

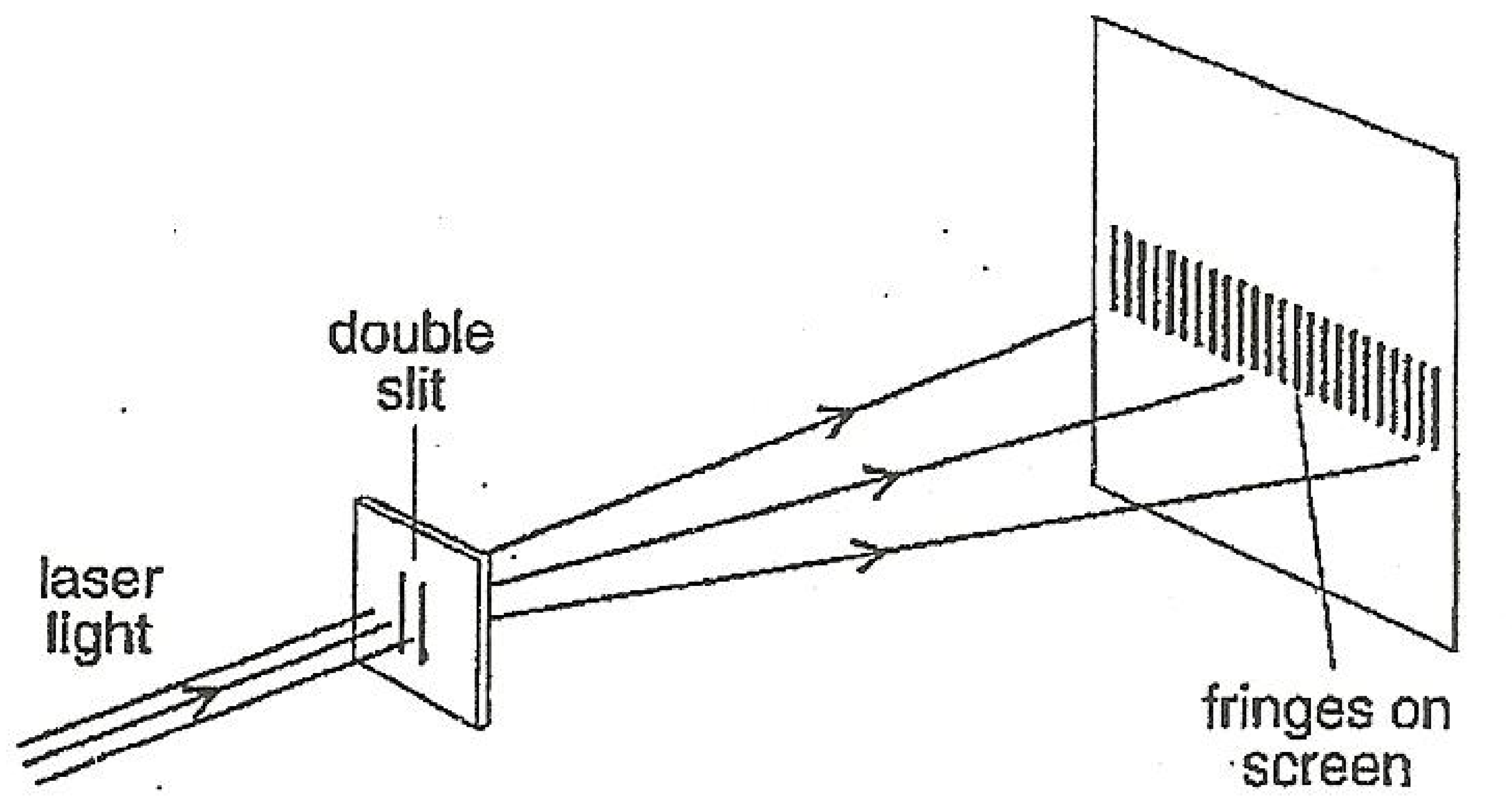


Young's double slit experiment



When a point of source of light is viewed through a narrow slit alternate bright and dark fringes parallel to the slit are observed. They consist of a centre bright fringe followed by other subsidiary bright fringes on either side. These bright fringes are separated by dark fringes. The centre fringe is the brightest and is twice the width of the other bright fringes, narrower the slit greater the diffraction. The fringes are more widely spaced out (as long as the wavelength is constant) if two colored filters are used in turn, over the light source. The fringes obtained by red light compared with blue light has a longer wavelength, greater is the diffraction (as long as the slit width is constant).

Young's double slit experiment



Using two slits instead of a single slit gives a fringe pattern with alternate bright and dark fringes as observed by Thomas Young in his double slit experiment. The maximum intensity of the double slit double slit remains the same, the fringe width is narrower hence it is sharper and the double slit pattern fades away at every single slit.

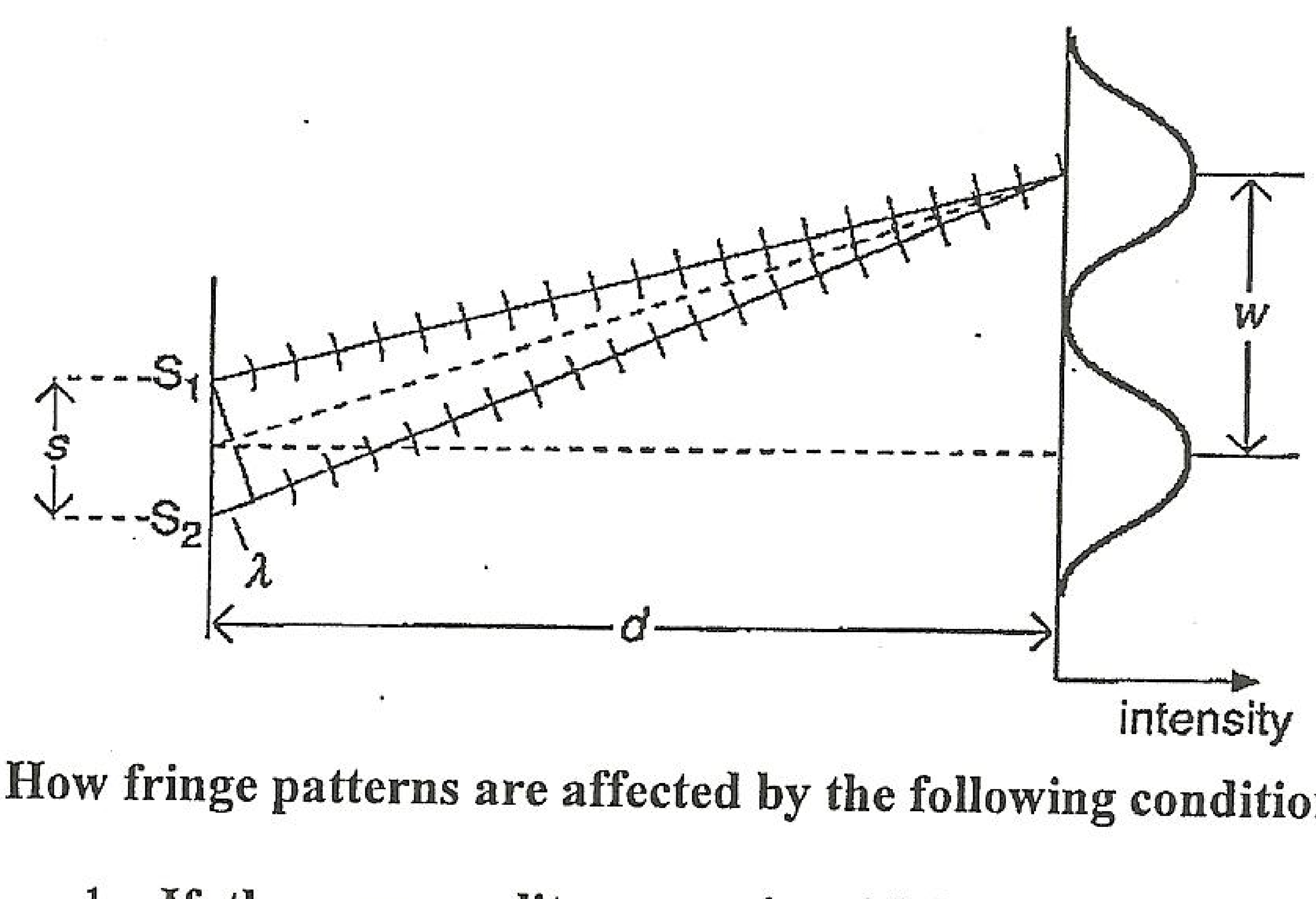
Equations

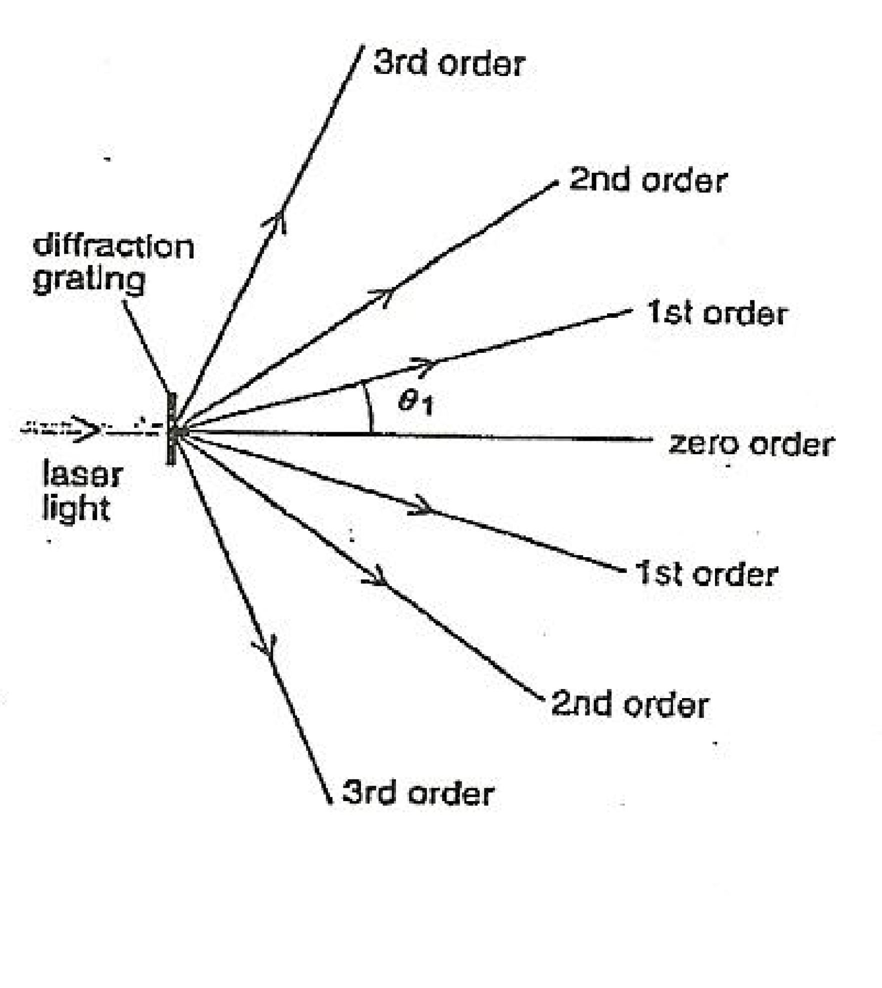
$d\sin\theta = n\lambda$ $\quad \sin\theta = \frac{n\lambda}{d} \le 1$ $\quad \tan\theta = \frac{xn}{D}$ $\quad \therefore \frac{xn}{D} = \frac{n\lambda}{d}$

$y = \frac{\lambda D}{d}$

D = distance between slits and screen $\quad$ y = fringe seperation $\quad$ θ = small angles

xn = distance of n^{th} bright fringe from centre C $\quad$ λ = wavelength





How fringe patterns are affected by the following conditions:

1. **If the source slit screen is withdrawn (moved away from the double slit screen):** no change in the fringe pattern but the brightness of the fringe decreases.

2. **If the distance between the double slits (d) is increased:** ~~no~~ fringe separation and the brightness of the fringe decreases.
3. **If the wavelength (λ) of the light is changed from red light to blue light (decrease in λ):** the fringe separation (y) decreases and an increase in the number of fringes.
4. **If distance between the double slits and the screen (D) is increased:** the fringe separation increases and the fringe will be less bright.
5. **If the widths of the slits (s_1 and s_2) are decreased:** greater will be the diffraction through the two slits, more points of interference and increase in number of fringes.
6. **If one slit is closed:** the double slit interference patterns changes to a single slit diffraction pattern
7. **If the frequency (f) of the incident light is increased:** the wavelength (λ) decreases, fringe separation (y) decreases and there is an increase in number of fringes.
8. **If the monochromatic source is changed to white light:** the centre fringe on the screen appears white and all other fringes above and below the centre fringe will be spectra.
9. **If the space between the double slits and the screen is filled with a medium of a high refractive index:** the number of fringes increases but brightness remains the same.
10. **If the screen is replaced by a smoke screen:** there is no change in fringe pattern but the fringes will appear to be less bright due to the scattering of light by the soot particles.
11. **If the screen is inclined the upper end tilted backwards:** the fringe separation (y) will increase and as you go up the screen the brightness decrease.
12. **If one slit was covered by a sheet of glass:** the fringe pattern on the screen is shifted towards the covered slit.
13. **If the two sources (S_1 and S_2) are exactly opposite in phase (having phase difference of π):** the fringes would be dark.

?. In a young double slit experiment, mercury green light of wave length 0.54μm was used with a pair of parallel slits of separation 0.60mm. The fringes were observed at a distance if 40cm from the slits. Calculate fringe separation.

We have $\lambda = 0.54\times10^{-6}$, $d = 0.60\times10^{-3}$ **and** $D = 0.40$

Using $y = \dfrac{\lambda D}{d} = \dfrac{0.54\times10^{-6}\times0.40}{0.60\times10^{-3}} = 0.36\times10^{-3} \quad = 0.36mm$

Note : If there are 600 lines in a mm then

$600 lines \quad 1\times10^{-3}$

:

$1 \quad d \quad \therefore d = \dfrac{1\times10^{-3}}{600} = 1.67\times10^{-6} = 1.67\mu m$

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?. Light from a white source passes through a filter that transmits only the band of wavelengths from $400\eta m - 600\eta m$ when this filtered light is incident normally on a diffraction grating the $400\eta m$ light in one order of the spectrum is diffracted at the same angle 30^o as $600\eta m$ light in adjacent order (n+1). Find the spacing between the lines of the grating (d).

$$d\sin\theta = n\lambda = d\sin 30 = n\times 600\times 10^{-9}$$

$$d = \frac{n\times 600\times 10^{-9}}{\sin 30} \_\_(1) \qquad d\sin 30 = (n+1)\times 400\times 10^{-9} \_\_(2)$$

(1) **into** (2)

$$\frac{n\times 600\times 10^{-9}}{\sin 30}\times \sin 30 = n\times 400\times 10^{-9} + 400\times 10^{-9}$$

$$n(600\times 10^{-9} - 400\times 10^{-9}) = 400\times 10^{-9} \qquad \therefore n = \frac{400\times 10^{-9}}{2\times 10^{-7}} = 2$$

$$\therefore d = \frac{2\times 600\times 10^{-9}}{\sin 30} = 2.4\times 10^{-6} \qquad = 2.4\mu m$$

$$\textbf{LHS} = ms^{-1} \qquad \textbf{RHS} = \sqrt{\frac{kgms^{-2}}{kgm^{-1}}} = \sqrt{\frac{m^2}{s^2}} = ms^{-1}$$

LHS = RHS ∴ equation is homogenous

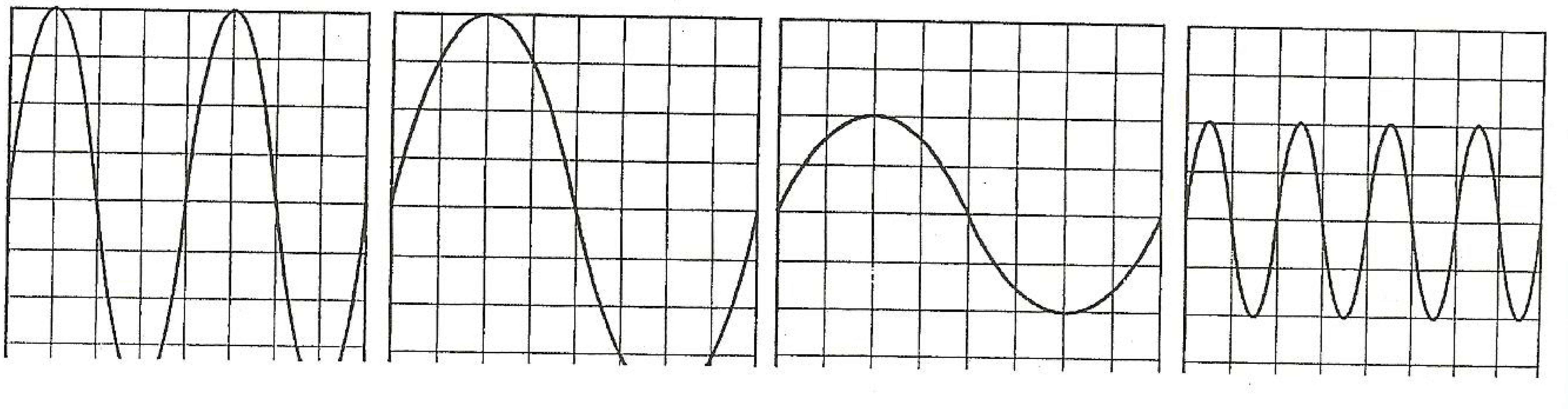
The frequency for which the wave vibrates in one direction only is called the fundamental frequency for vibration and that vibration is called fundamental mode of vibration.
For the fundamental mode of vibration:

$$\lambda = 2L \qquad v = f\lambda \qquad v = f_o 2L \qquad f_o = \frac{v}{2L}$$

The frequency of the periodic forces is the vibrating frequency. Any system has its own natural frequency which depends on the mass and geometrical shape of the system. To force a system to vibrate with its natural frequency by applying a driving force, we say resonance is achieved. The particles vibrate with its maximum amplitude.

When a vibrating system oscillates with its natural frequency, its amplitude is maximum the condition is called resonance.
When you keep on increasing the frequency of the signal generator, it will eventually vibrate with two loops. This is called first overtone.

CRO:
Anode is at a higher positive potential with respect to the cathode so $PD = V$. Electrons



will pass through the anode, forming a beam of electrons. If no potential difference is applied to the X and Y plates, the beam will fall on the screen, forming a bright spot. If a potential difference is applied across X plates, the spot will move horizontally where as if applied across Y plates, the spot will move vertically. A potential difference is applied across the X plates; the electron beam would oscillate between the two plates.

Transducer is a device that converts electrical signals into other forms.

Measuring sound waves
To find the wavelength of the sound wave; move the microphone towards the loudspeaker, counting the number of nodes that you pass, and reach another node say n. note the distance moved by the microphone and divide it by the number of nodes.

$$\frac{R_n - R_o}{n} = \frac{\lambda}{2}$$

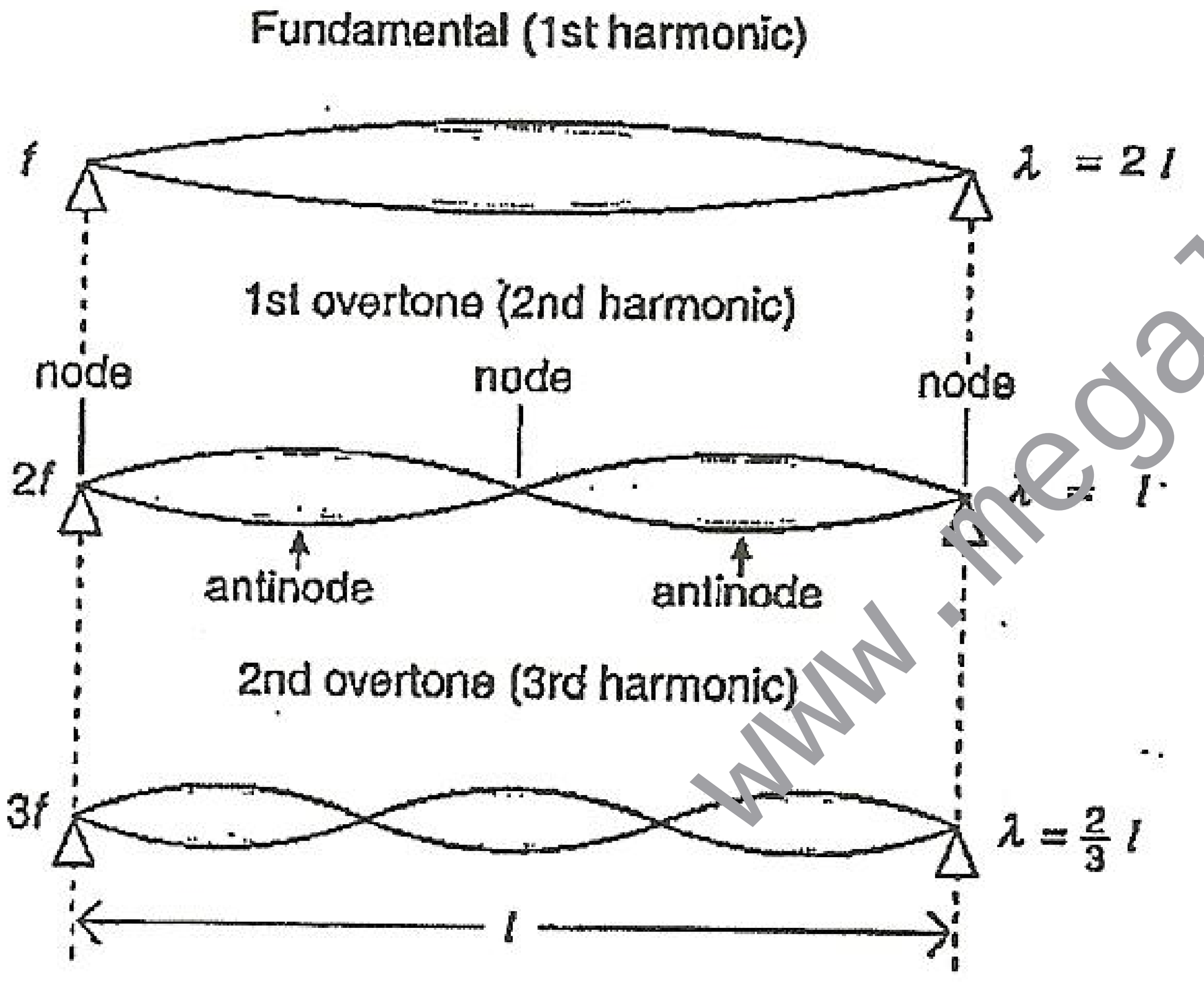
Frequency can be calculated from the sound generator itself if it's a reliable one.

$x = \sec/cm$ $\quad$ $y\,cm$ **represents** 1 **oscillations**

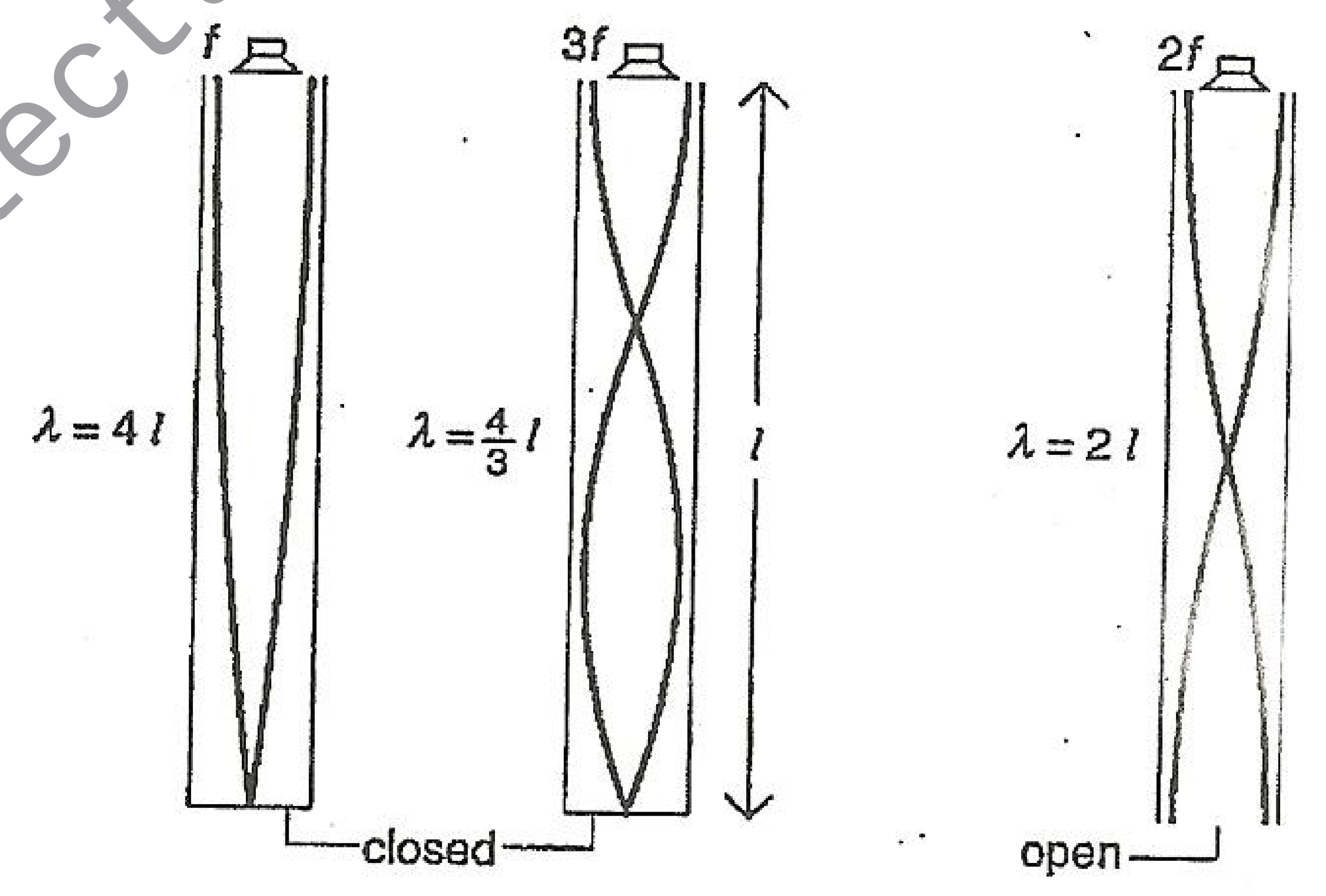
period $= xy$ $\quad$ **frequency** $= \frac{1}{xy}$

Stationary waves

Transverse waves – vibration of strings



Longitudinal waves – vibration in air columns

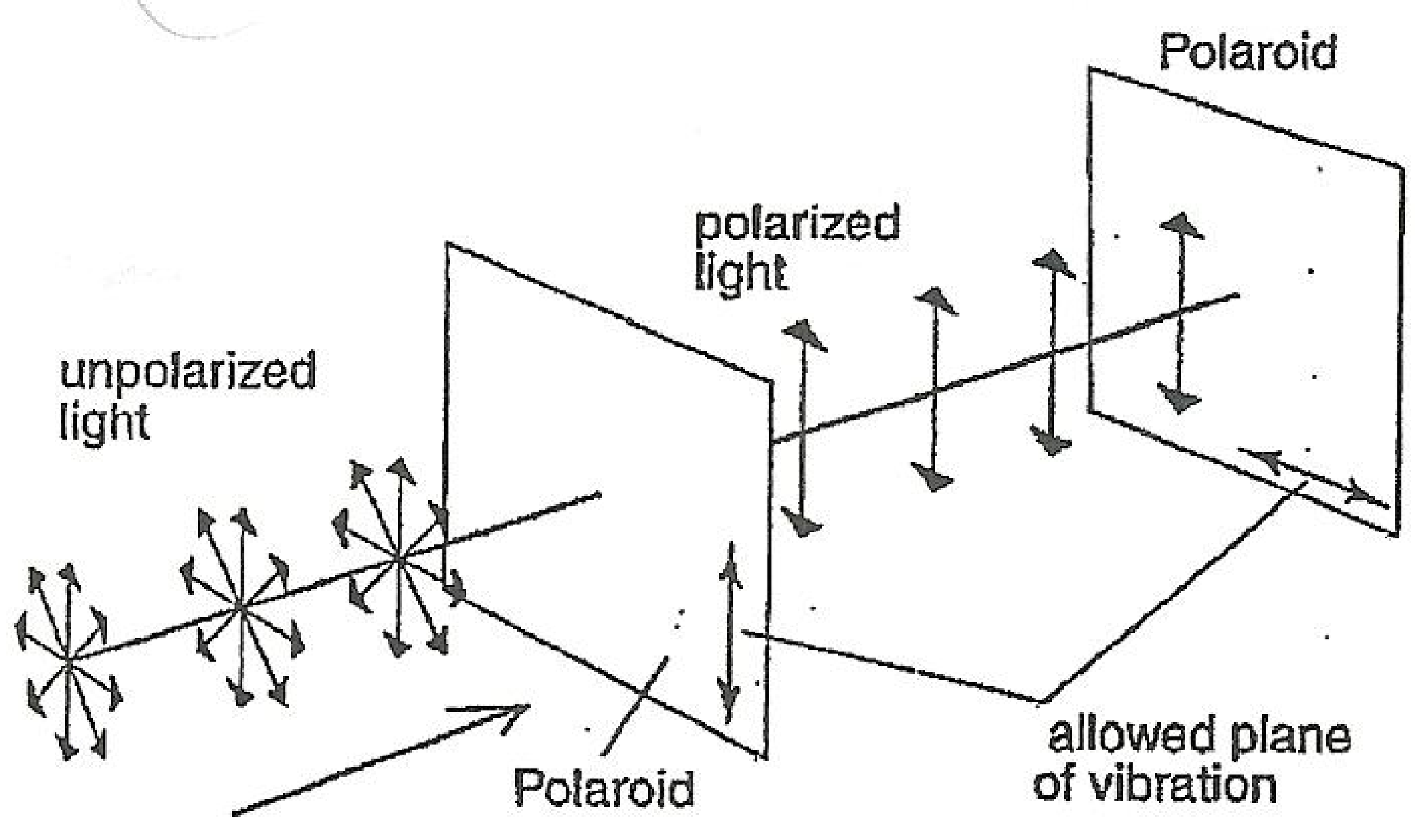


Note: $\lambda = \frac{2l}{1}, \frac{2l}{2}, \frac{2l}{3}, \frac{2l}{4}, \frac{2l}{5} \ldots$

Note: $\frac{4l}{1}, \frac{4l}{3}, \frac{4l}{5}, \frac{4l}{7} \ldots$

Polarization
Light from a point source vibrates in many (directions) planes. This is called unpolarized light, whereas light that vibrates in a single plane and contains the direction of travel of the wave is known as Plane polarized light.

(14)



Unpolarized light may be polarized by passing it through a thin plane. Hence for a plane to be polarized is for the vibration to be contained in one plane only.

- Polarization is a phenomenon associated only with transverse waves.
- Longitudinal waves cannot be plane polarized.

Uses of polaroids:

- In sunglasses - to cut out glare
- Stress analysis – to detect flaws in metal casting.

Reflected light is partially plane-polarized (Polarized Horizontally). If the reflected and refracted light is at 90^o then the light is completely horizontally polarized. This is known as Brewster's law