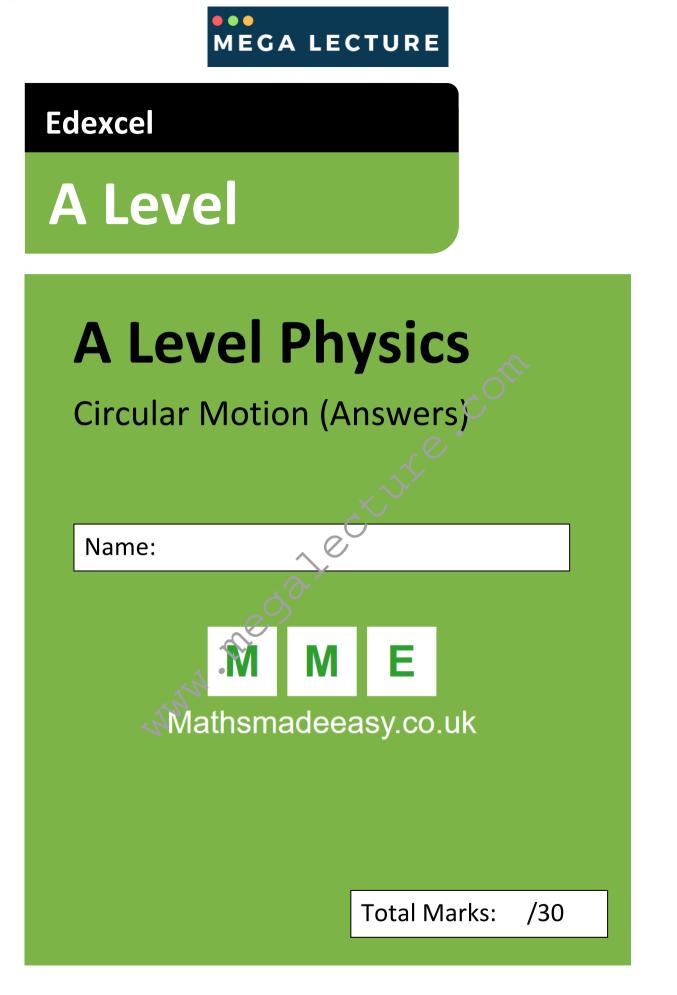
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Total for Question 1: 13

[1]

(a) Define angular velocity.

1.

**Solution:** Rate of change of angular displacement:  $\omega = \theta/t$ 

(b) Calculate the angular velocity of a car travelling at  $30 \text{ kmhr}^{-1}$  around a roundabout whose radius [3]is 50 m.

Solution:  $\frac{1}{6}$  rad s<sup>-1</sup>

(c) Give three examples of situations in which centripetal forces arise, detailing precisely which forces [3]contribute to the centripetal force.

Solution: Anything valid e.g. vehicles on banked turns (reaction/friction force), satellites in orbit (gravity), yoyos being whirled (tension).





(d) The diagram below shows a bob tracing out a circle in a vertical plane. Points A and C are separated from B - the point at which the string is horizontal - by the angle  $\theta$ . Show that the horizontal acceleration is given by  $a_x = \frac{2v \sin \theta}{t}$ , where v is the speed of the bob and t is the time taken to get from A to C.

θ

θ

С

В

Solution:

A and C are equal distances above B and so the bob has the same vertical velocity at each:  $v_y = v \cos \theta$ 

Since acceleration is the rate of change of velocity,  $a_y = 0$ .

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The horizontal velocities must also have the same magnitude, but will be of opposite polarity. Since  $v_x = v \sin \theta$ , the change in velocity will be  $2v \sin \theta$ . Hence, the acceleration will be  $2v \sin \theta/t$ .





(e) Using this result, demonstrate that the acceleration of a mass moving around a circle with a radius [3] of r at a speed of v is  $a = \frac{v^2}{r}$ .

**Solution:** The velocity is simply the arc length divided by the time taken:  $v = r\theta/t \rightarrow t = r\theta/v$ . Here this is  $2r\theta/v$  (since we have an angle of  $2\theta$ ). Inserting this into the above:  $a_x = \frac{v^2 \sin \theta}{r\theta}$ . Small angle approximation:  $\sin \theta \approx \theta$ . Thus,  $a = \frac{v^2}{r}$ .



MEGA LECTURE

2. A cyclist is travelling around a bend with a radius of 15 m on a horizontal road. The frictional force is related to the reaction force from the ground and the coefficient of friction by the equation  $F = \mu R$ , where  $\mu$  is the coefficient of friction and R is the reaction force.

Total for Question 2: 10

(a) In dry conditions  $\mu = 0.5$ . Calculate the maximum speed at which the cyclist can travel if he is not to fall off. [3]

Solution:  $8.6 \text{ ms}^{-1}$ 

(b) The reaction of the surface and the frictional force both act on the cyclist, but at a distance from [3] the centre of mass. They therefore provide a torque. Qualitatively, explain why a cyclist leaning inwards when cycling around bends helps to prevent these torques destabilising the bike.

Solution: 3 forces acting on the bike: weight, friction and reaction of the surface. The first acts through the COM and therefore doesn't provide a torque. The other two act from the same point. Given that their magnitudes can't be changed, the only way to balance them is to change the angle between the direction of the force and the line intersecting the COM and the point through which they act.



Page 5 of 8

MEGA LECTURE

(c) Rosie is feeling particularly brave and decides to conduct an experiment to calculate the coefficient of friction when the road surface is wet. She uses five different bends, each with a different radius. For each, she records her speed at the point her wheels begin to slide. Using the data in the table below, plot a graph and calculate the coefficient of friction.

Solution: Should plot  $v^2$  against radius. Best fit line should go through origin. Gradient is  $\mu g$ .  $\mu \approx 0.2$ 

bend radius / m	speed / $\rm ms^{-1}$
9	45
4.5	15
11	60
6.5	20
3	5



[4]



3. A conical pendulum is simply a mass suspended from a point that traces out a horizontal circle, rather than one that swings back and forth.

(a) Draw a free-body diagram for the mass.

**Solution:** Two forces: weight mg acting downwards and tension T acting along the suspending string towards the attachment point, at an angle  $\theta$  from the vertical.

(b) What provides the centripetal force in this situation?

Solution: Tension in the attaching string.

(c) Express the tension in the string in terms of the mass, the mass's velocity and the radius of the [2] circle in which it moves.

<b>Solution:</b> $\frac{mv^2}{r} = T\sin\theta$	
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Total for Question 3: 7

[1]

[1]



(d) By balancing the weight with the tension in the string, show that the speed of the bob is given by  $v = \sqrt{rg \tan \theta}$  [3]

## Solution:

Balance:  $mg = T \cos \theta$ Divide this equation by answer to previous question:  $\frac{\sin \theta}{\cos \theta} = \frac{v^2}{rg}$ Rearrange to get required format.

