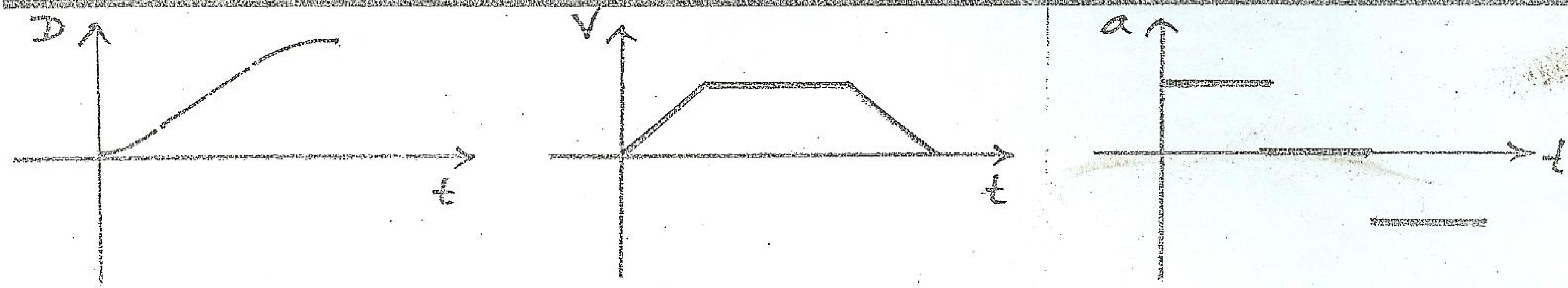
احمد مقصود حمیدی

— MECHANICS — (QUICK NOTES)

MISC NOTES: GO OVER EACH BOX VERY CAREFULLY. NOTHING IS TRIVIAL. THEY ARE ALL IMPORTANT. YOU SHOULD KNOW EACH!

1) THE 3 GRAPHS BELOW SHOW THE SAME MOTION



2) EQUATIONS OF UNIFORMLY ACCELERATED MOTION

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$2as = v^2 - u^2$

$s = \frac{1}{2}(u+v)t$

VERTICAL MOTION UNDER GRAVITY

RULES:

If thrown up, $a = -10\,m/s^2$

If Dropped or thrown Down, $a = +10\,m/s^2$

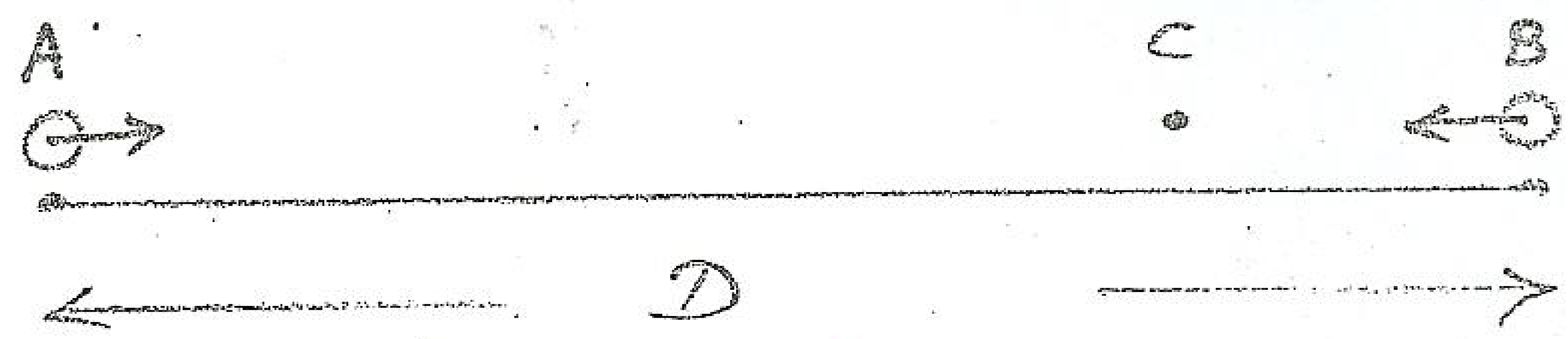
) MISC. RESULTS FOR VERTICAL MOTION UNDER GRAVITY

To find time taken to reach max. height use
$v = u + at$
and put $v = 0$

The time taken to come back to the launch point is DOUBLE of this.

To find Max height above launch point use $2as = v^2 - u^2$ and put $v = 0$.

4) WHEN 2 PARTICLES START FROM OPPOSITE ENDS AND MOVE TOWARDS EACH OTHER



Write Down $S_A = ut + \frac{1}{2}at^2$, $S_B = ut + \frac{1}{2}at^2$

If they collide at C $\therefore S_A + S_B = D$.

This gives you the value of time of Collision. The same analysis holds if the motion is Vertical

WHEN TWO PARTICLES ARE LAUNCHED UPWARDS FROM THE GROUND AND FROM A TOWER (5

Remember S is measured from the launch point

$\therefore$ if you want to find the time when they will be at the same height you cannot put $S_A = S_B$!

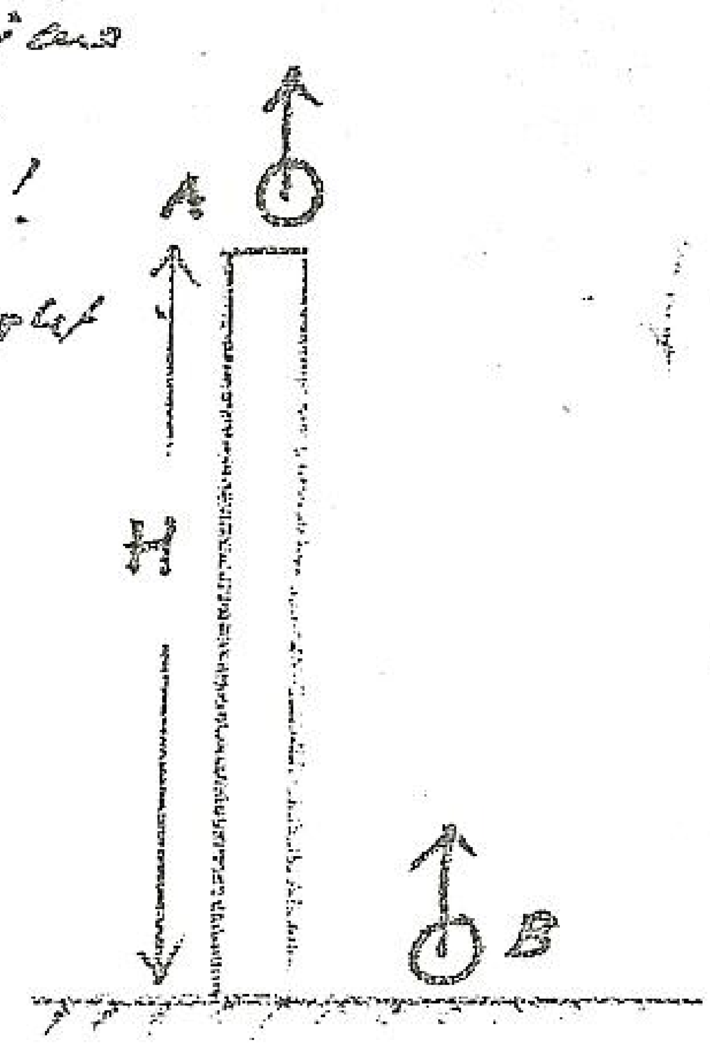
When they are at the same height

$S_B - S_A = H$

OR you could measure S_A from the ground as:

$S_A = H + ut + \frac{1}{2}at^2$

if you do it this way then at the same height $S_A = S_B$



To find the time interval for which a particle will be above a given height H, use $s = ut + \frac{1}{2}at^2$. Just put $s = H$. You get a quadratic in t. The DIFFERENCE in the two values is the time for which the particle was above that height.

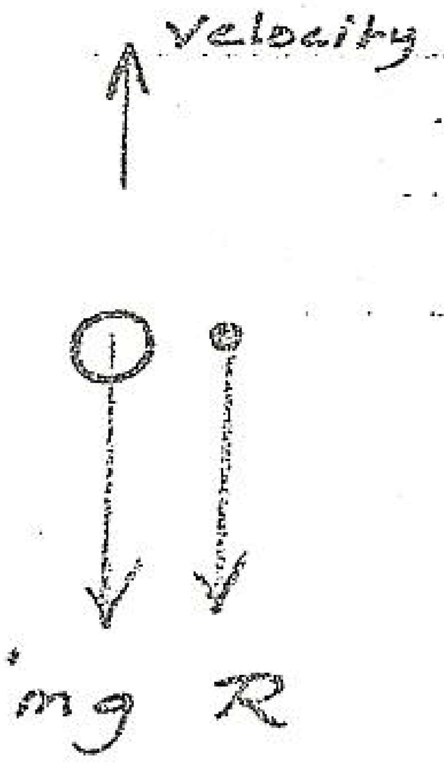
احمد مقصود حمیدی

7) VERTICAL MOTION WITH AIR RESISTANCE

Remember in this case the accelerations are DIFFERENT going up and coming Down. You will have to split the problem into two parts: The upward phase and the Downward phase.

GOING UP:

Let R = resistance as the particle is going up, R points down

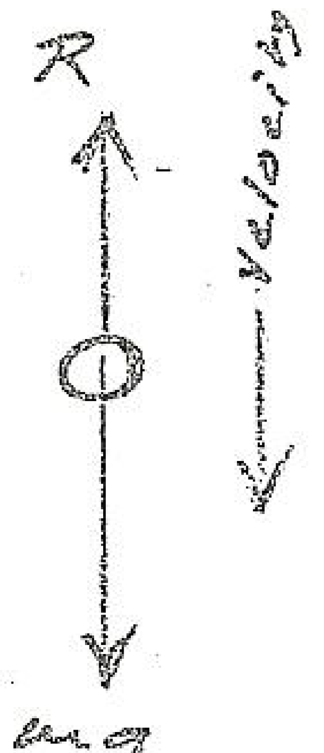


$\therefore mg + R = ma$

The 'a' value you get must be used with a − sign as it is a deceleration.

COMING DOWN:

as velocity is Down, R points up



$\therefore mg - R = ma$

and the 'a' value that you get must be used with a + sign as the particle is accelerating.

8) ALL ABOUT FRICTION

$F = \mu R$ is true only when the body is in limiting equilibrium.

Normal Reaction is always perpendicular to the surface on which the particle is placed which could be a table or rod.

The phrase "contact force" refers to the frictional force and Normal reaction.

$\therefore$ horizontal component of contact force = friction

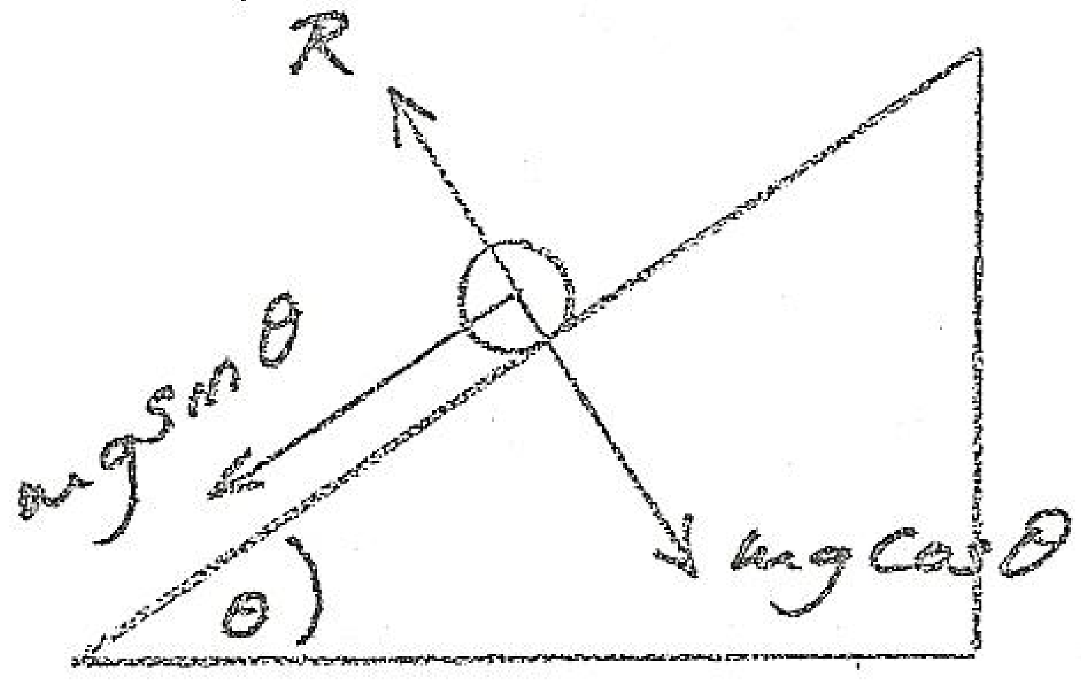
Vertical component of contact force = Normal Reaction

Magnitude of contact force

$$= \sqrt{(\text{friction})^2 + (\text{Nor. Reaction})^2}$$

BREAKING UP WEIGHT ON AN INCLINED PLANE. (9)

The weight has to be resolved into a component Down the plane ($mg\sin\theta$) and into a component going into the plane ($mg\cos\theta$)



ACCELERATION OF A FREE BODY ON INCLINED PLANE (10)

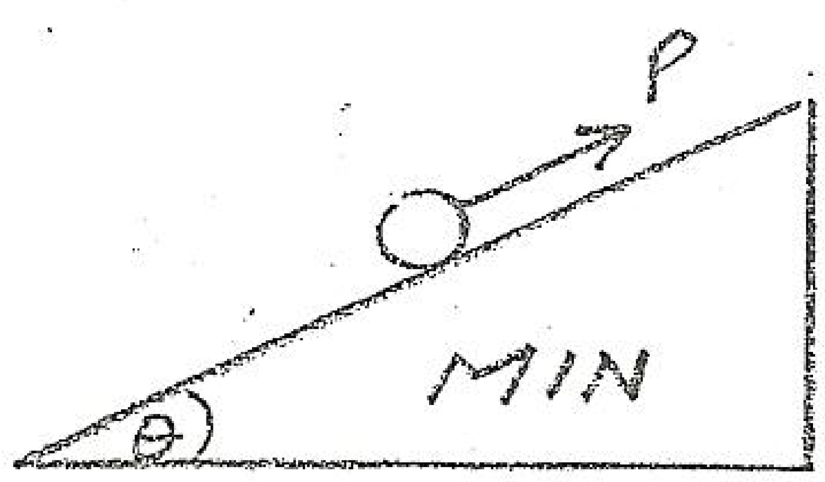
The acceleration of a free particle on a plane (with angle θ) is $\pm g\sin\theta$

+ : if coming Down

− : if going up

PLEASE NOTE: The plane is SMOOTH!

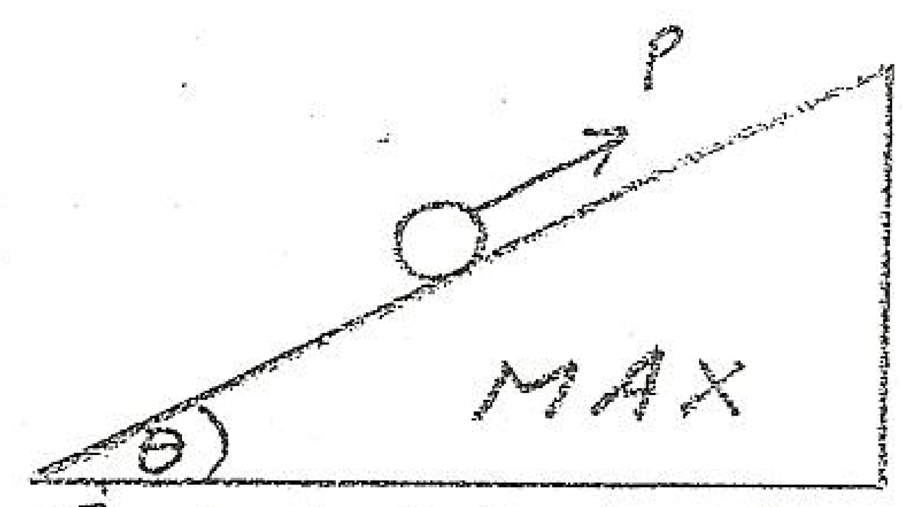
TO FIND THE MAX AND MIN VALUE OF A FORCE REQUIRED TO KEEP A PARTICLE IN EQUILIBRIUM ON A ROUGH PLANE. (11)



The particle is about to slip DOWN

$\therefore \mu R$ acts up

$\therefore \mu R + P = mg\sin\theta$



The particle is about to slip UP

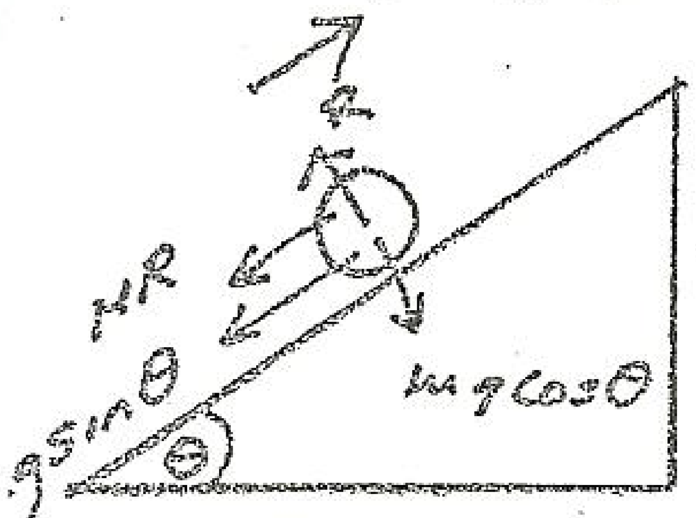
$\therefore \mu R$ Acts DOWN

$\Rightarrow P = \mu R + mg\sin\theta$

احمد مقصود حمیدی

FREE PARTICLE GOING UP OR DOWN A ROUGH INCLINED PLANE (12)

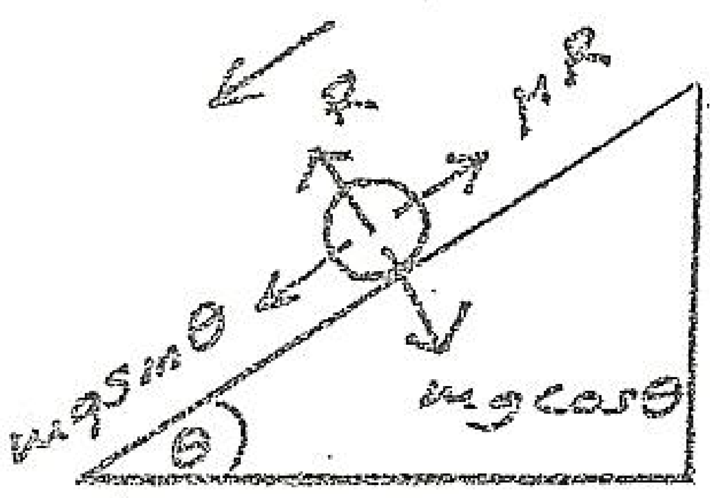
going up



$\mu R + mg\sin\theta = ma$

This 'a' is a Deacceleration

coming down

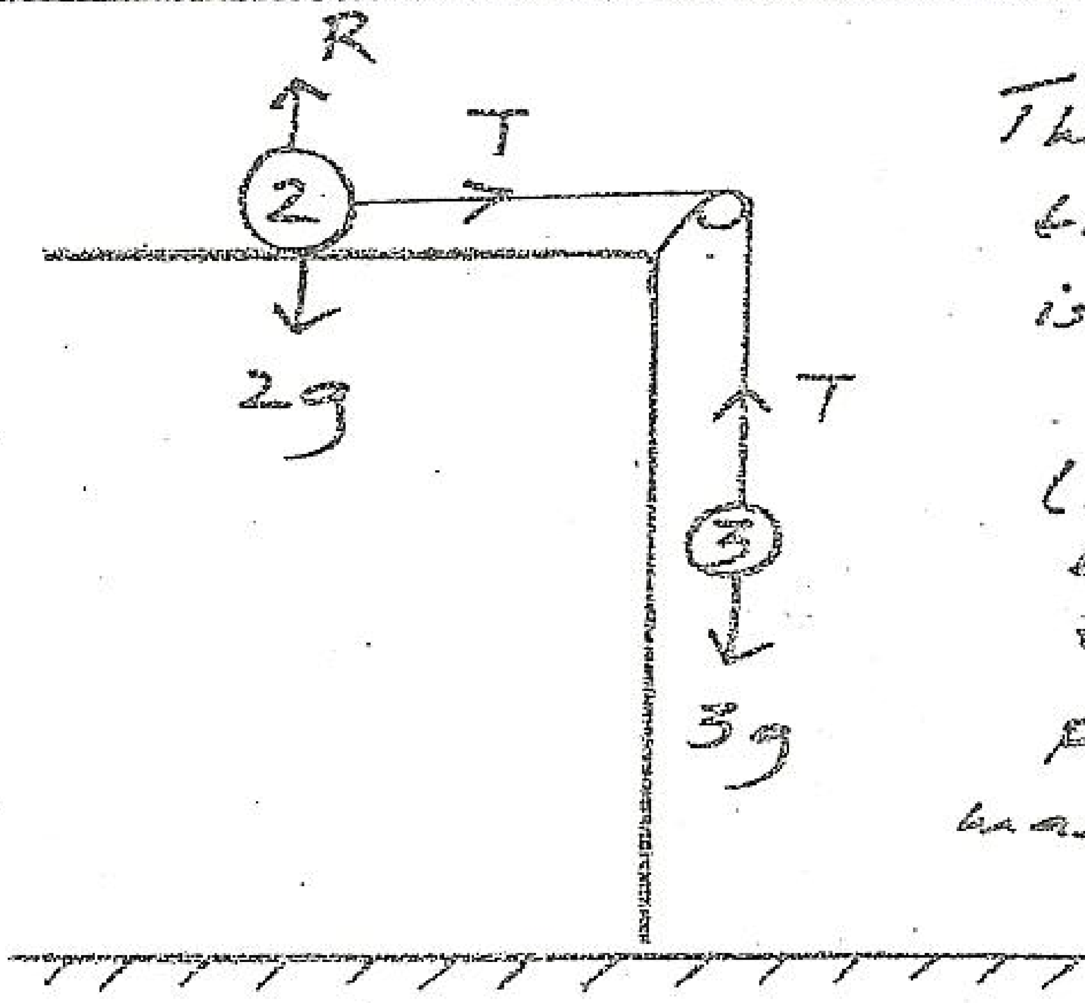


$mg\sin\theta - \mu R = ma$

This 'a' is a acceleration

Note: $R = mg\cos\theta$ in both cases.

A COMMON CONFUSION INVOLVING A TABLE PULLEY SYSTEM (SMOOTH!) (13)



The equation for the 2kg mass is Just

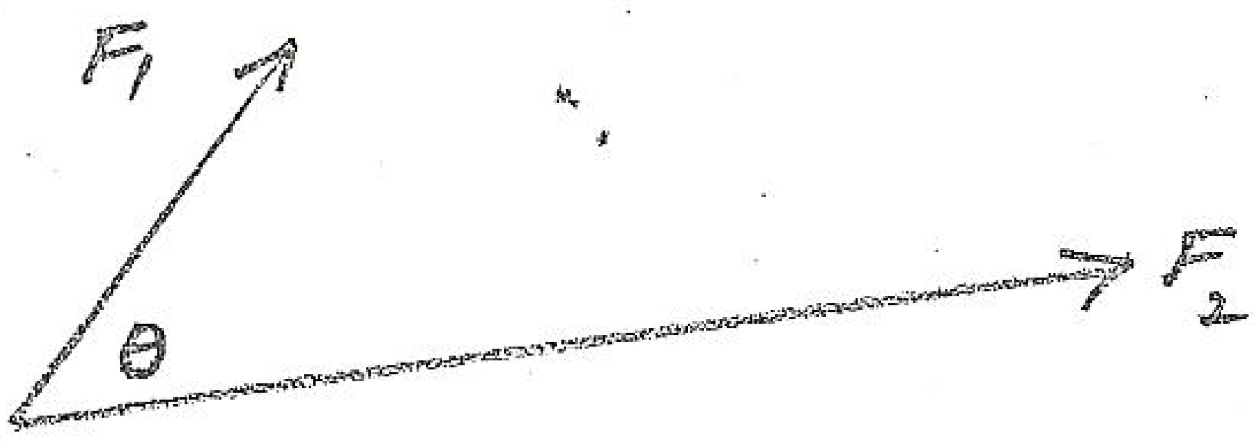
$T = 2a$

(There is no backward force to subtract!)

For the 3kg mass obviously:

$3g - T = 3a$

A VERY IMPORTANT RESULT CONCERNING THE RESULTANT OF 2 FORCES. (14)



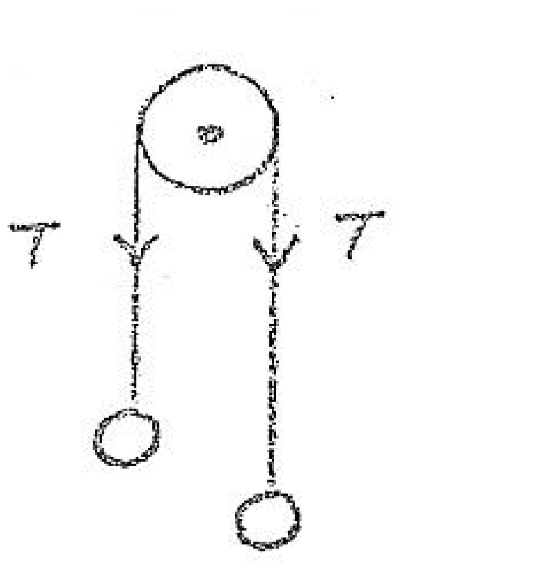
if R is the magnitude of the resultant

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$$

yes! the sign is + !

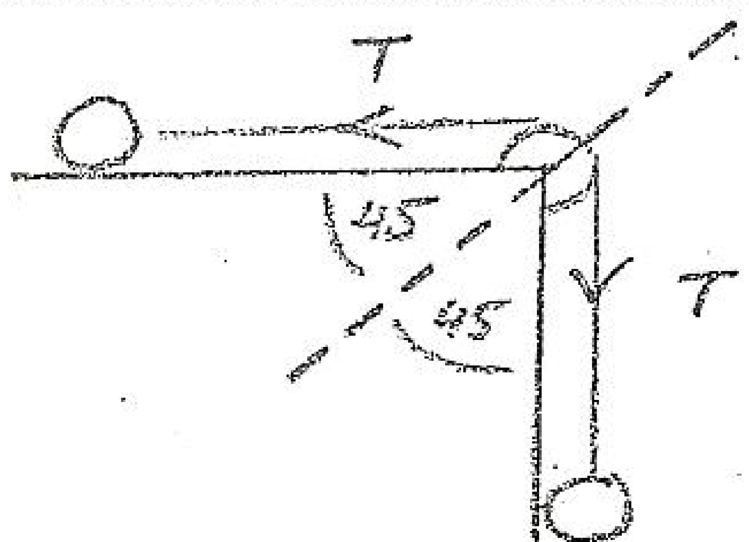
NOTE: θ can be obtuse as well!

FORCE EXERTED BY A STRING ON THE PULLEY (15)



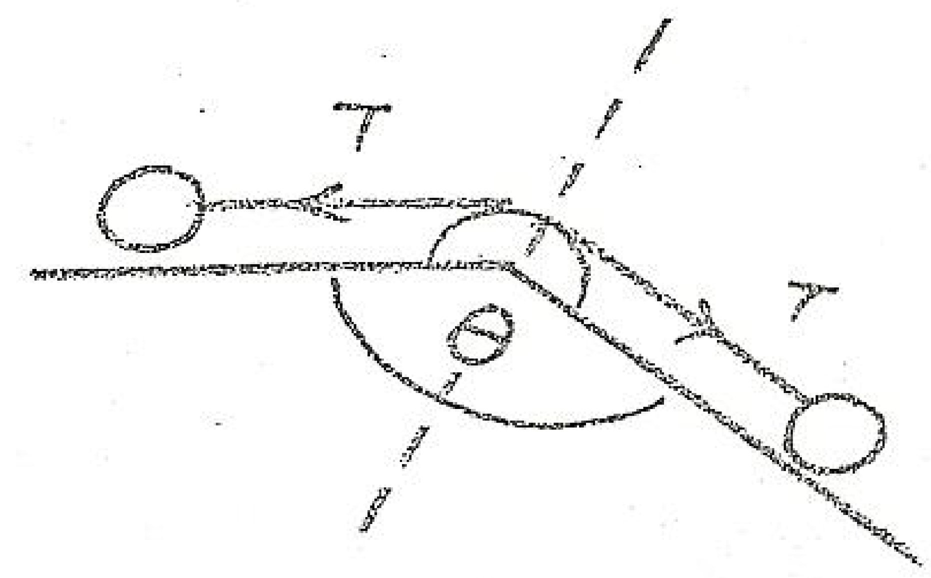
Force on pulley $= 2T$

Acting Down



Force on pulley $= \sqrt{2}\,T$

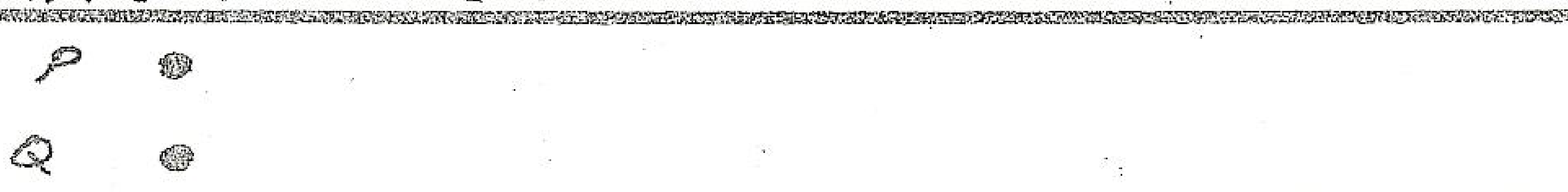
Acting inward along Dotted Line.



Force on pulley $= 2T\cos\left(\frac{1}{2}\theta\right)$

Acting inwards along Dotted line that bisects angle θ

USING THE EQUATIONS OF MOTION WHEN 2 PARTICLES START AT DIFFERENT TIMES. (16)



Suppose P, Q start from the same point but P starts 3 seconds after Q. Suppose P starts from rest, accelerating at $6\,m/s^2$ while Q had a initial velocity of $15\,m/s$ and acc of $2\,m/s^2$. Suppose you need to find where and when will they meet. Q has been in motion longer than P

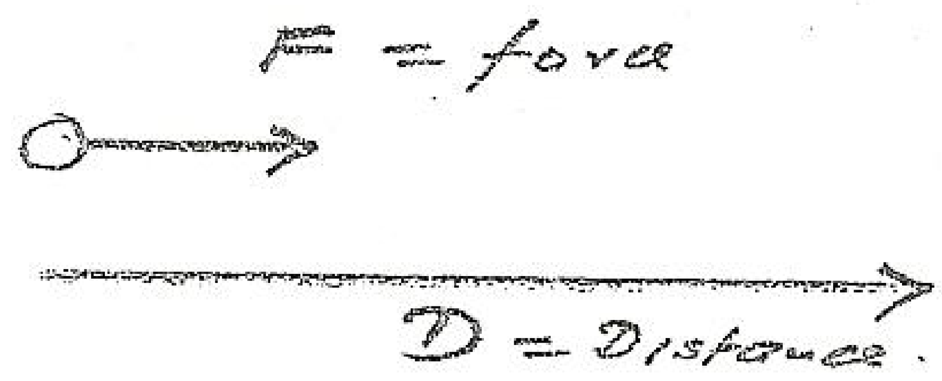
$\therefore$ if t = time for P, $t+3$ = time for Q

or if t = time for Q, $t-3$ = time for P

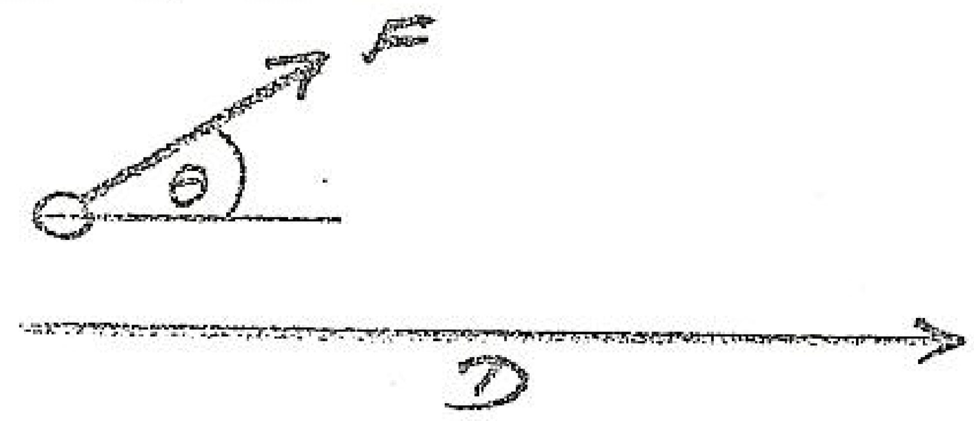
$\therefore$ $S_P = \frac{1}{2}(6)t^2$, $S_Q = 15(t+3) + \frac{1}{2}(2)(t+3)^2$ and putting $S_P = S_Q$ gives you t.

مقصود حمیدی

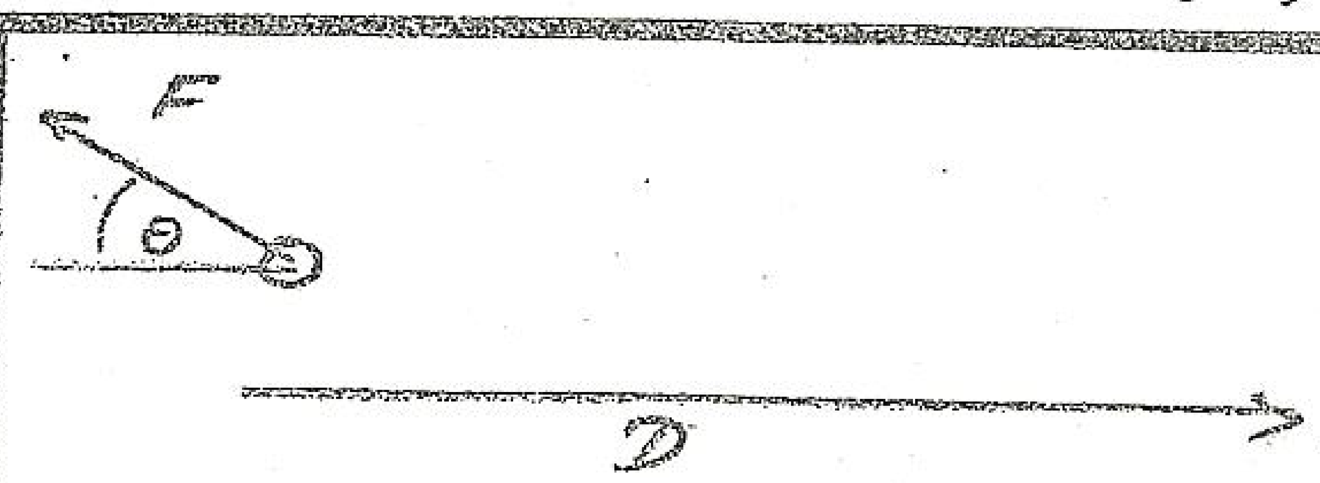
FINDING WORK DONE BY A FORCE (17)



Work = F × D



Work = $F\cos\theta \times d$



Work = $-F\cos\theta \times D$

POWER (18)

$$\text{Power} = \frac{\text{Work Done}}{\text{time taken}}$$

⟹ Work Done = Power × time

This is very important. If the power is constant and time is given, you can use this to find the work Done.

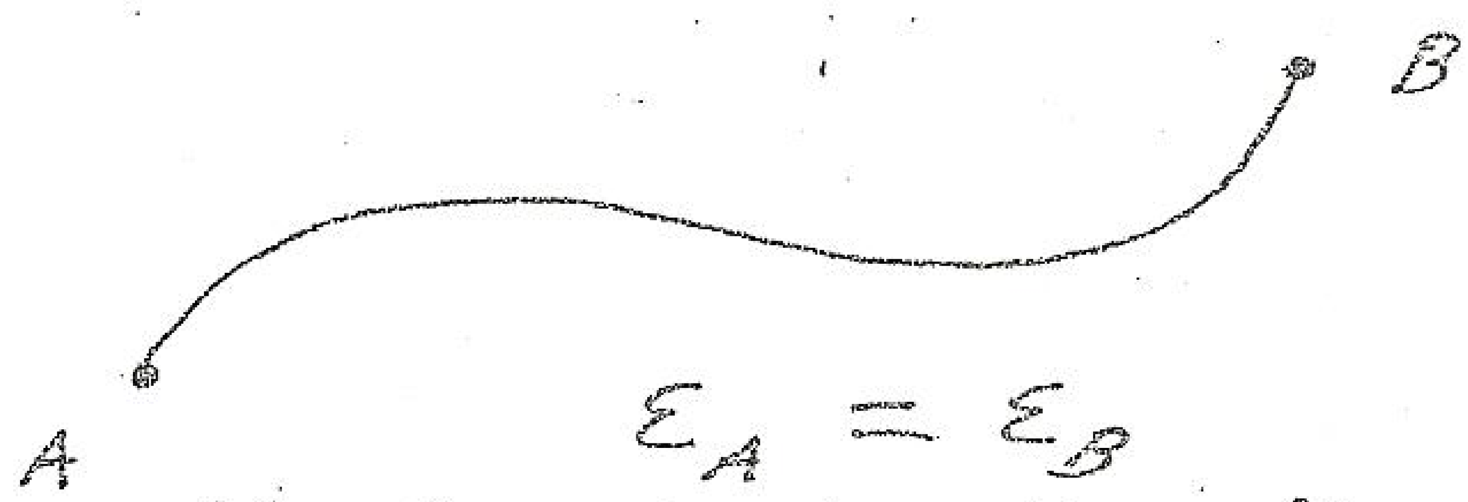
KINETIC, POTENTIAL AND TOTAL ENERGY (19)

$KE = \frac{1}{2}mv^2$

$PE = mgh$, h is the height above the 'ground'. In Any question involving motion from point A to point B you can always assume the lower point as the ground ∴ h = 0 at that point. The total Energy $E = mgh + \frac{1}{2}mv^2$

LAW of CONSERVATION of (Mechanical) ENERGY (20)

If the only force that acts is gravity and there are no frictional or any other external forces, then the initial total energy (E_i) always equals the final total Energy (E_f)



$$E_A = E_B$$

$$\therefore E = mgh + \tfrac{1}{2}mv^2$$

CHANGES in ENERGY WHEN FORCES OTHER THAN GRAVITY ALSO ACT (21)

The Rule now is: $E_f - E_i = (\text{Work})_{\text{engine}} - (\text{Work})_{\text{friction}}$

Staple this to your Brains! This one formula Works in ALL cases. Let me say this again: In any Question where friction or an engine comes in remember

$$\boxed{E_f - E_i = W_{eng} - W_{frict}}$$

you will be given 3 things so you can always solve it for the 4th unknown!

NOTE: W_{eng} refers to the work Done by Engine or any forward pulling force such as tension in a string which is pulling the body or any external force that pushes the body. W_{fric} is the work Done by any resistive force.

احمد مقصود حمیدی

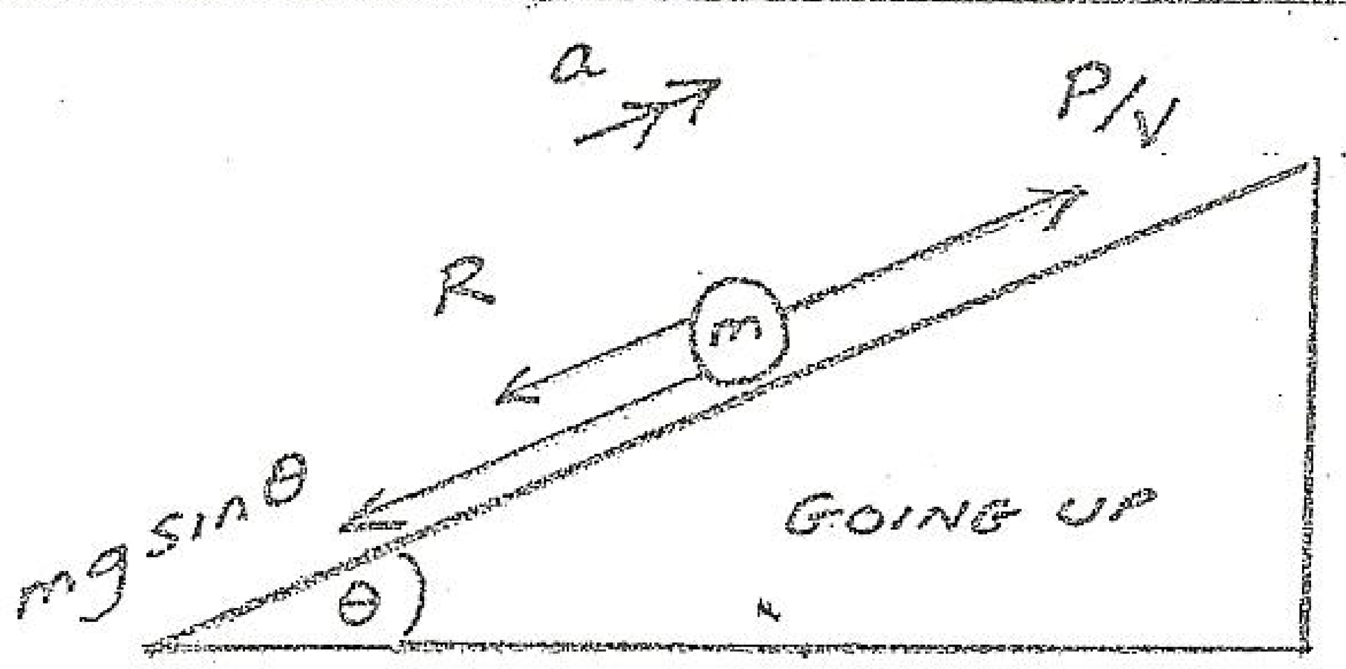
FORCE of A CARS ENGINE (22)

If a cars engine has a power output of P watts, then the force that is gives is

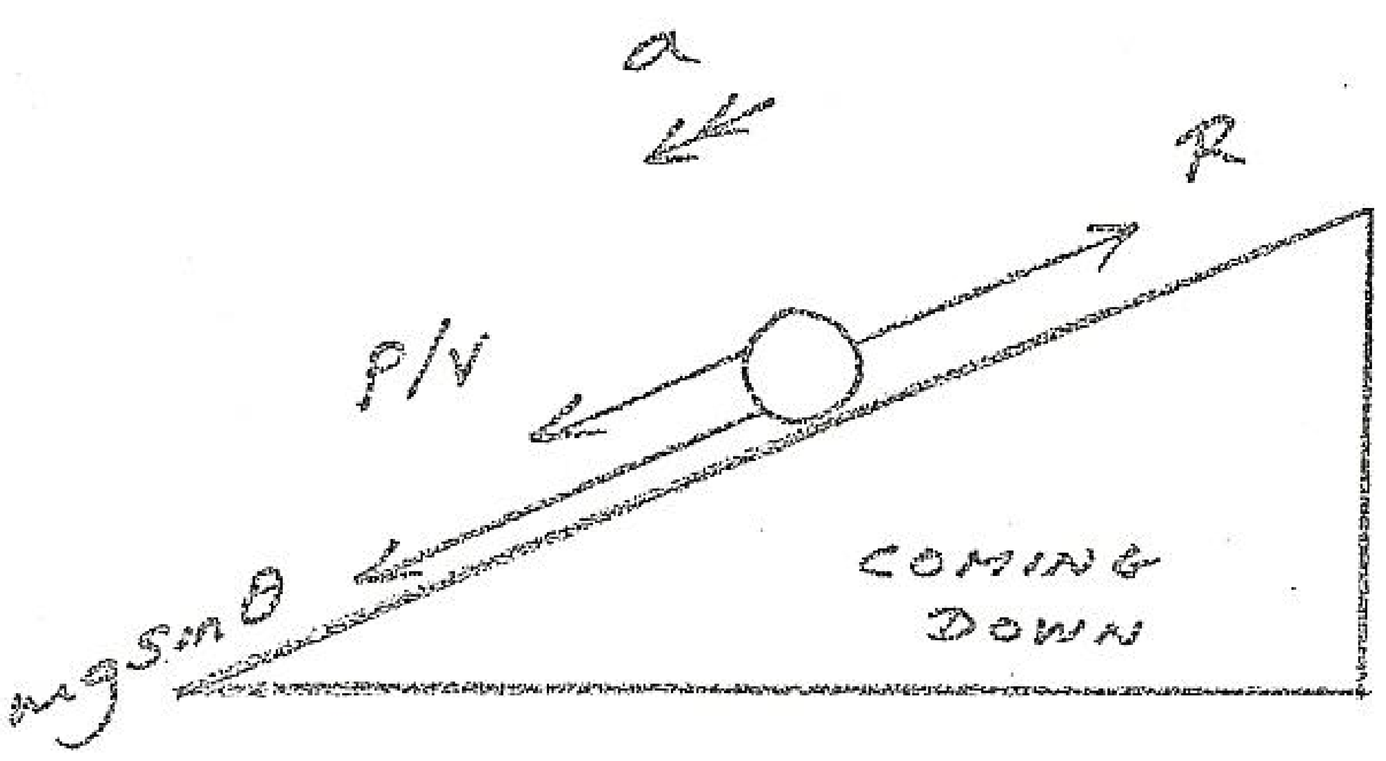
$$F = P/V$$

V = velocity of the car

EQUATION of MOTION of CAR on INCLINED PLANE (24)



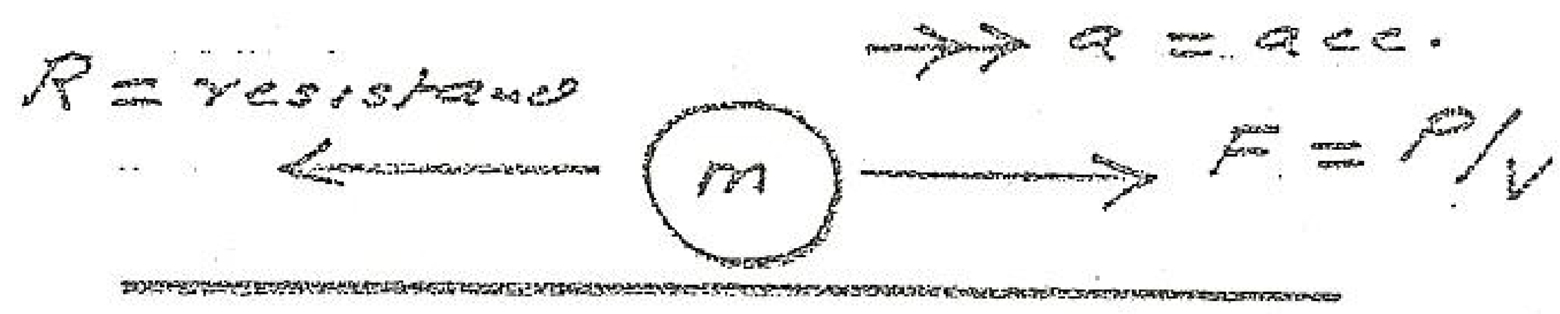
$$\frac{P}{V} - R - mg\sin\theta = ma$$



$$\frac{P}{V} + mg\sin\theta - R = ma$$

NOTE: "freewheels" means the engine is not engaged ∴ Just omit the forward Force.

EQUATION of MOTION of a CAR (23)



$$\frac{P}{V} - R = ma$$

At max speed $a = 0$

∴ at max speed

$$\frac{P}{V} = R$$

(forward force = backward force)

KINEMATICS (25)

Here you will be given S or V or a in terms of t.

RULES:

$$V = \frac{ds}{dt} \quad \therefore \quad s = \int v\,dt$$

$$a = \frac{dv}{dt} \quad \therefore \quad v = \int a\,dt$$

NEVER EVER forget the constant of INTEGRATION! you will need to find its value.

NOTE: the phrase "... where t is the time in seconds after passing O..." means at $t = 0$, $s = 0$ which will help you find c (the constant of integration!) and please Note that $t = 0, s = 0$ Does not always mean $c = 0$!

NOTE: to find S_{max} or S_{min} put $V = 0$, solve for t, plug back in S to get the extreme value

NOTE: to find V_{max} or V_{min}, put $a = 0$, solve for t, plug back in V. IMPORTANT NOTE: suppose you had to find the Distance travelled over the 1st 5 seconds. You must 1st find the time when velocity is 0 suppose that happens at $t = 1$ and $t = 3$. Now you have to find S at 4 values of t:

t	0	1	3	5
s	−3	4	15	6

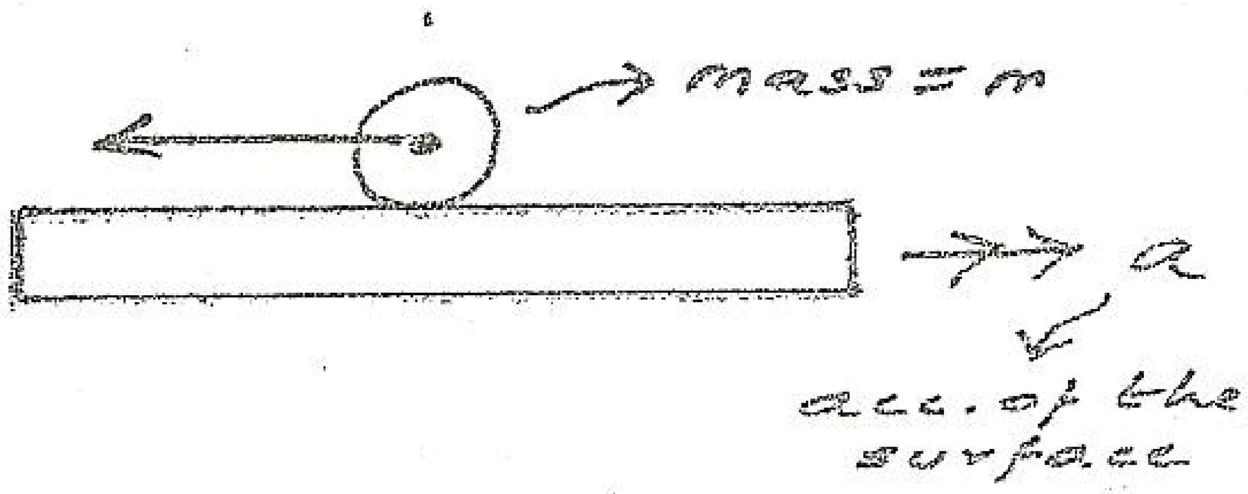
Now subtract each pair of S values to get a positive answer.

you get 7, 11, 9 ∴ Distance travelled = 7 + 11 + 9 = 27.

احمد مقصود حمیدی

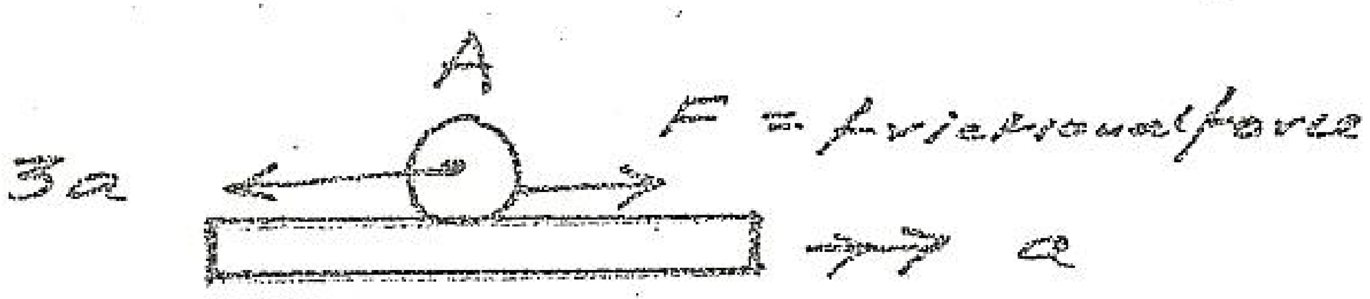
FORCE ON A PARTICLE WHEN THE SURFACE ON WHICH IT IS PLACED IS ACCELERATED

Backward Force $= ma$



So when the surface is accelerated at 'a' m/s^2, the particle sitting on it feels a backward force $= ma$.

Example: A particle of mass 3 kg is placed on a rough surface with coeff of friction 0·3. The surface is moved with an acceleration a m/s^2. Find the max value of a so the particle Does not slide.



as 'a' increases so does the backward force $3a$. When the particle is about to slide $F = \mu R = 3a$ $(R = 3g)$

$\therefore (0 \cdot 3)(3)(10) = 3a$

$\therefore a_{max} = 3 m/s^2$

Now if a was inc. to $5 m/s^2$ how to find the Distance travelled by A in 3 sec?

A Slides to the left. The Net force on the left is $(5)(3) - \mu R$

$= 15 - (0 \cdot 3)(30) = 3a$

$\therefore a = 6 m/s^2$ is the acc. of A on the surface

$\therefore$ In 3 sec. it covers a Distance

$s = ut + \frac{1}{2}at^2$

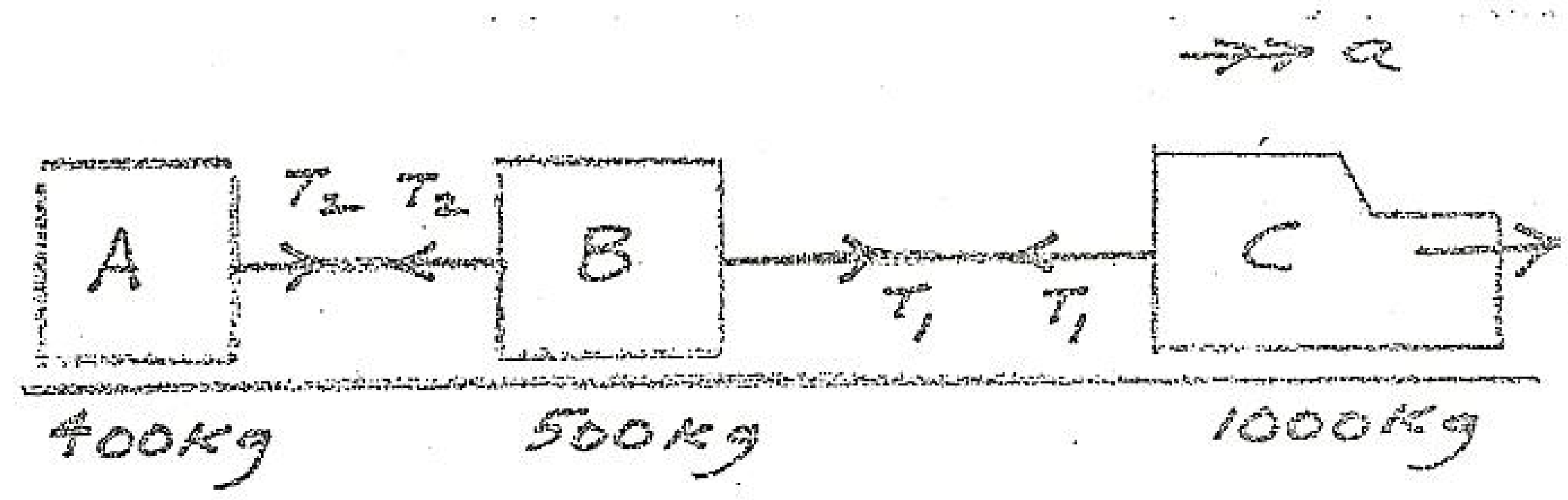
$= \frac{1}{2}(6)(3)^2 = 27 m.$

LAST NOTE:

it helps to pray before your M1 exam.
Good luck.

ALMOST FORGOT! FINAL NOTE ON CONNECTED PARTICLES

A Train pulls two carriages.



The forward force of the engine is $F = 2500 N$. Find the acc. and tension in each coupling. The resistances to motion of A, B, C are 200, 150 and 90 N respectively.

To find acc, regard the system as a single object. The internal T's cancel and the equation of motion becomes:

$2500 - (200 + 150 + 90)$

$= 1900a$

$\therefore a =$

To find T_1, look at C

$F - T_1 - 200 = 1000a$

$\therefore T_1 =$

To find T_2 look at A

$T_2 - 90 = 400a$

$\therefore T_2 =$