Matter

Deformation of Solids

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Deformation of Solids

Definitions:

- **Stress**: is a measure of the force required to cause a particular deformation.
- **Strain**: is a measure of the degree of deformation.
- **Elastic Modulus**: the ratio of stress to strain

$$\text{Elastic Modulus} = \frac{\text{stress}}{\text{strain}}$$

The elastic modulus determines the amount of force required per unit deformation. A material with large elastic modulus is difficult to deform, while one with small elastic modulus is easier to deform.

Deformation of Solids : Changes in Length

Changes in Length

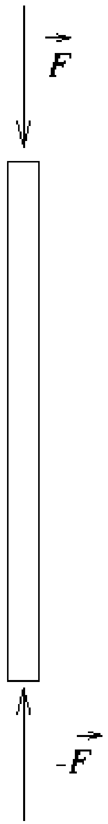
To stretch or compress something you must exert a force on it at either end.

Tensile Stress is the force per unit cross-sectional area exerted on the ends.

(Note the surface whose area we wish to measure is perpendicular to the force.)

Changes in Length (continued)

Tensile Stress



- **Tensile Strain** is the fractional change in original length.
- **Young's Modulus** (*Y*) is the ratio of tensile stress to tensile strain:

$$Y = \frac{\text{tensilestress}}{\text{tensilestrain}} = \frac{F/A}{\Delta L/L_O} = \frac{FL_O}{A\Delta L}$$

where *F* is the applied force, L_O is the original length of the object, *A* is the cross-sectional area of the object, and ΔL is the change in the length of the object. Notice that *Y* has S.I. units of N/m^2.

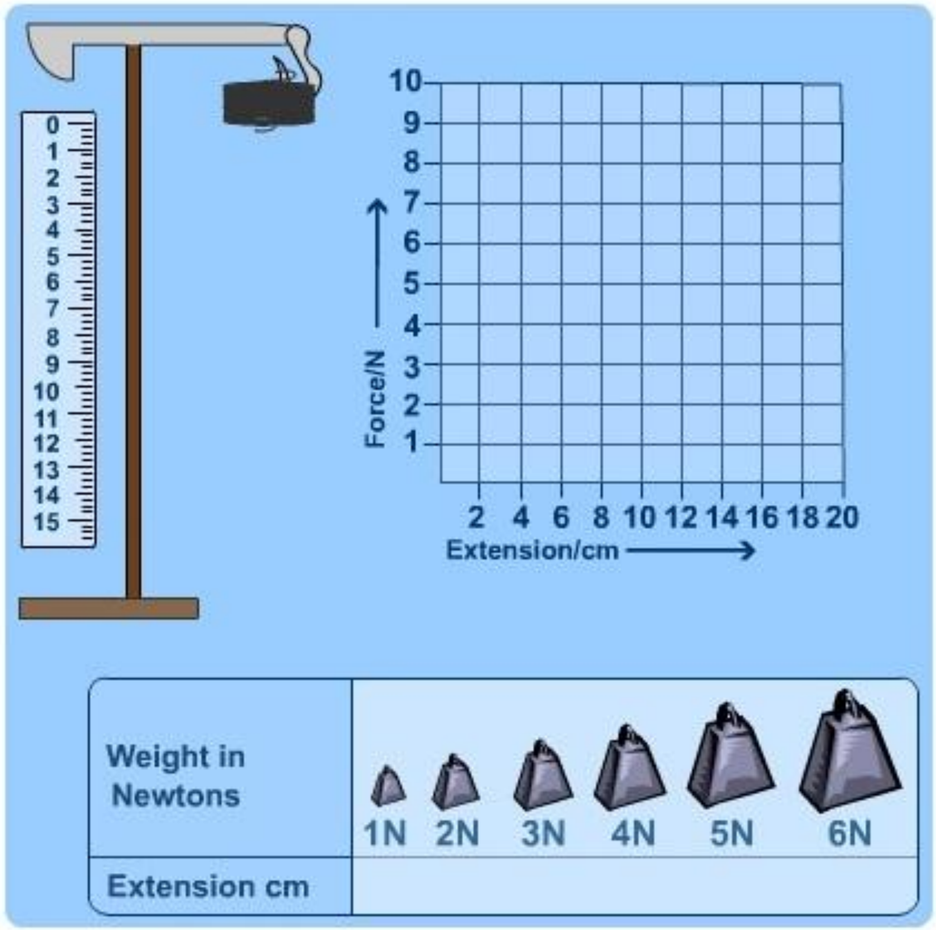
Hooke's Law

- Hooke's Law states that, for relatively small deformations of an object, the displacement of the deformation is directly proportional to the deforming force or load.

- Forces can cause objects to **deform**.

- The way in which an object deforms depends on its dimensions, the material it is made of, the size of the force and direction of the force.

If you measure how a spring stretches (extends its length) as you apply increasing force and plot extension (e) against force (F);



the graph will be a straight line.

Note: Because the force acting on the spring (or any object), causes stretching; it is sometimes called tension or tensile force.

This shows that **Force** is proportional to **extension. This is Hooke's law. It can be written as:**

F = ke

Where:.

F = tension acting on the spring.

e is extension = $(l - l_o)$; l is the stretched length and l_o is original length, and:

k is the gradient of the graph above. It is known as the spring constant.

The above equation can be rearranged as

$$k = \frac{F}{e}$$

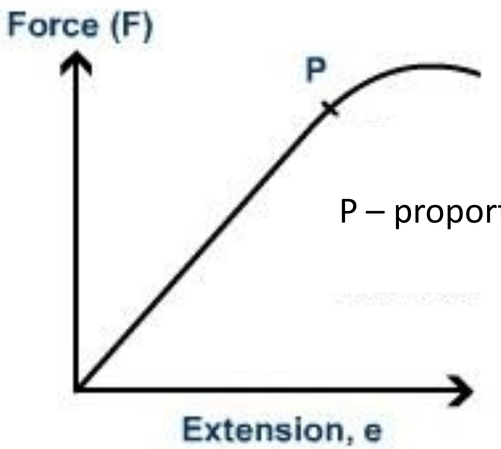
Spring constant = Applied force/extension

The **spring constant** k is measured in $\mathbf{Nm^{-1}}$ because it is the **force per unit extension.**

The value of *k* does not change unless you change the shape of the spring or the material that the spring is made of.

A stiffer spring has a greater value for the spring constant

In fact, a vast majority of materials obey Hooke's law for at least a part of the range of their deformation behaviour. (e.g. glass rods, metal wires).



P – proportionality limit

In the diagram above, if you extend the spring beyond point P, and then unload it completely; it won't return to its original shape. It has been permanently deformed. We call this point the **elastic limit** - the limit of **elastic behaviour.**

If a material returns to its original size and shape when you remove the forces stretching or deforming it (reversible deformation), we say that the material is demonstrating **elastic behaviour.**

If deformation remains (irreversible deformation) after the forces are removed then it is a sign of **plastic behaviour.**

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Calculating stress

- **Stress**

➢ Stress is a measure of how strong a material is. This is defined as how much force the material can stand without undergoing some sort of physical change.

➢ Hence, the formula for calculating stress is the same as the formula for calculating pressure: $\sigma = \frac{F}{A}$

➢ where σ is stress (in Newtons per square metre but usually Pascals, commonly abbreviated Pa).

Calculating strain

Stress causes strain.

- Applying force on an object causes it to stretch. Strain is a measure of how much an object is being stretched. Strain is the ratio of extension to the original length.
- The formula for strain is: $\epsilon = \frac{\Delta l}{l_0} = \frac{l - l_0}{l_0} = \frac{l}{l_0} - 1$
- Where l_0 is the original length of some bar being stretched, and l is its length after it has been stretched. Δl is the extension of the bar, the difference between these two lengths.

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Calculating Young's Modulus

- Young's Modulus is a measure of the stiffness of a material. It is defined as the ratio of stress to strain. It states how much a material will stretch (i.e., how much strain it will undergo) as a result of a given amount of stress.
- The formula for calculating it is: $E = \frac{\sigma}{\epsilon}$
- Strain is unit less so Young's Modulus has the same units as stress, i.e. N/m^2 or Pa.

Quantity	Equation	Symbol	Units
Stress	tension/cross sectional area $= F / A$	(sigma) σ	$N\ m^{-2} = Pa$
Strain	extension per **original** length $= \Delta L/L$	(epsilon) ε	no units (because it's a ratio of two lengths)
Young Modulus	stress/strain	E	$N\ m^{-2} = Pa$

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Tensile strength & Yield strength

➢ Tensile Strength

Tensile strength which is also known as **Ultimate tensile strength** or **ultimate strength** is the maximum stress that a material can withstand while being stretched or pulled before failing or breaking. Tensile strength is the opposite of compressive strength and the values can be quite different.

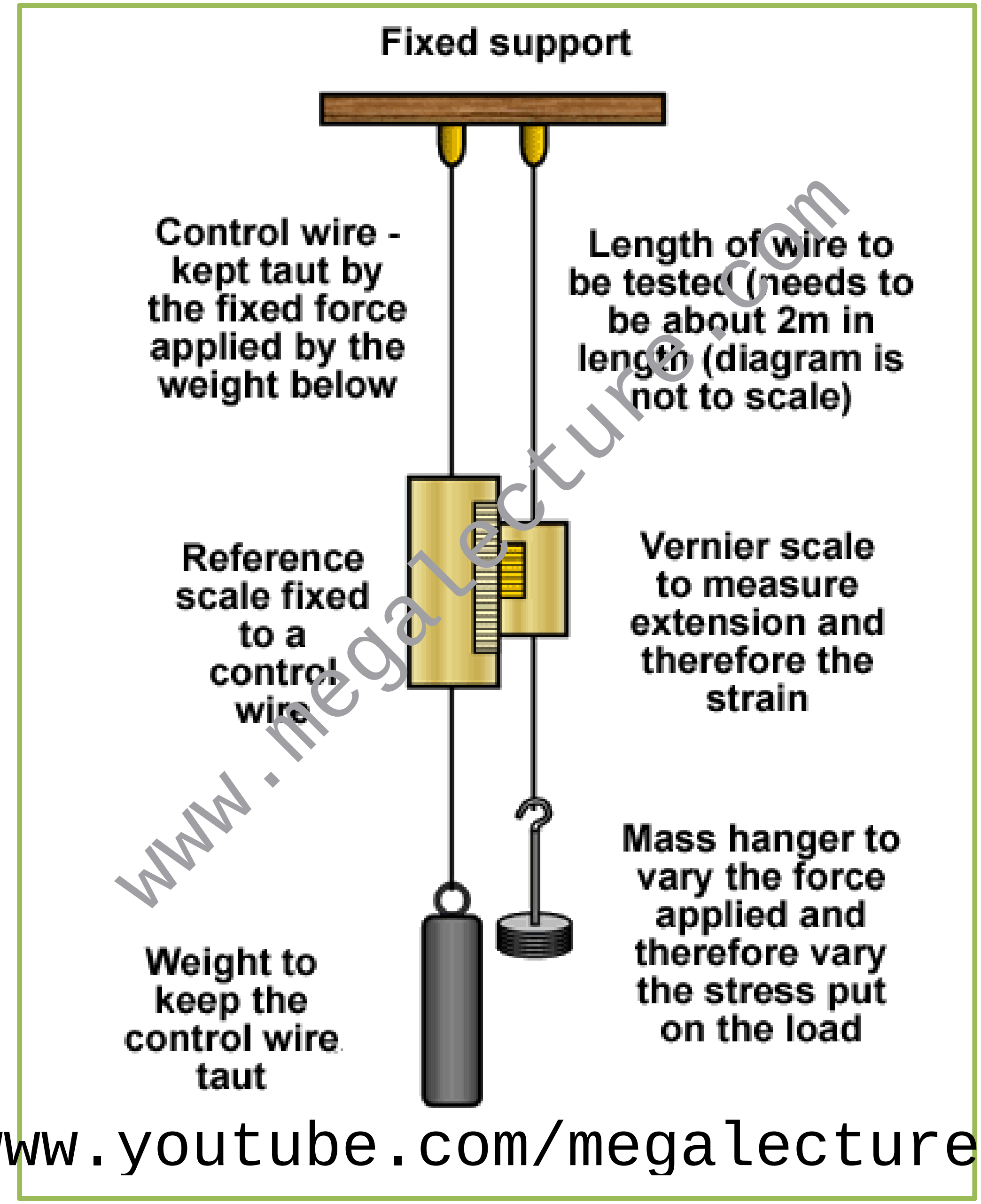
➢ Yield Stress or Yield strength or Yield point

The yield stress is the level of stress at which a material will deform permanently. This is also known as Yield strength or Yield point. Prior to the yield point the material will deform elastically and will return to its original shape when the applied stress is removed.

It can be experimentally determined from the slope of a stress-strain curve created during tensile tests conducted on a sample of the material.

- The value of the Young's Modulus is quoted for various materials but the value is only approximate.
- This is because Young's Modulus can vary considerably depending on the exact composition of the material.
- For example, the value for most metals can vary by 5% or more, depending on the precise composition of the alloy and any heat treatment applied during manufacture.
- If a big force only produces a small extension then the material is 'stiff' and E is a big value. If a force produces a big extension then the material is not very stiff - it is easier to stretch and the value of E will be smaller.

An experiment to measure the Young's Modulus



An experiment to measure the Young's Modulus (continued)

- To minimize errors the control wire is the same length, diameter and material as the test wire. This means that **errors due to expansion (from the surroundings)** during the experiment are avoided as the test wire and control wire would both expand by the same amount and the scale would adjust position and eliminate the error.
- The wire must have **no kinks** in it otherwise there will be big extensions due to the wire straightening out rather than just stretching.
- Care must be taken that the **limit of proportionality** is **not exceeded**. This can be checked by removing the load after each addition of the weight. If the limit has not been exceeded the wire should return to the length it was before the weight was added.
- The wire is as **long** as possible (usually about 2m long) and it is as **thin** as possible so that **as big an extension as possible** can be recorded. (A typical extension for a 5N loading will be 1mm).

An experiment to measure the Young's Modulus (continued)

- The test wire is loaded with the weight hanger so that it is taut before readings are taken.
- The vernier scale is read and the result recorded as addition of 0N.
- Weights - usually starting at 0N and increasing in 5N increments to 100N - are then added and a reading of the vernier scale is taken at each addition.
- The experiment should be repeated twice and any anomalous results repeated and checked.

An experiment to measure the Young's Modulus (continued)

- A graph of load against extension is plotted. It should be a straight line through the origin (provided measurements are accurate).
- The gradient of that graph will be F/e. Using that value we can find the value of Young's Modulus for the wire.

$$E = \frac{\text{stress}}{\text{strain}}$$

$$= \frac{\frac{F}{A}}{\frac{e}{\ell}} = \frac{F\ell}{Ae}$$

$$= \ell/A \times \text{Gradient}$$

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Proportionality limit and Yield strength

- **Proportionality limit and Elastic limit**

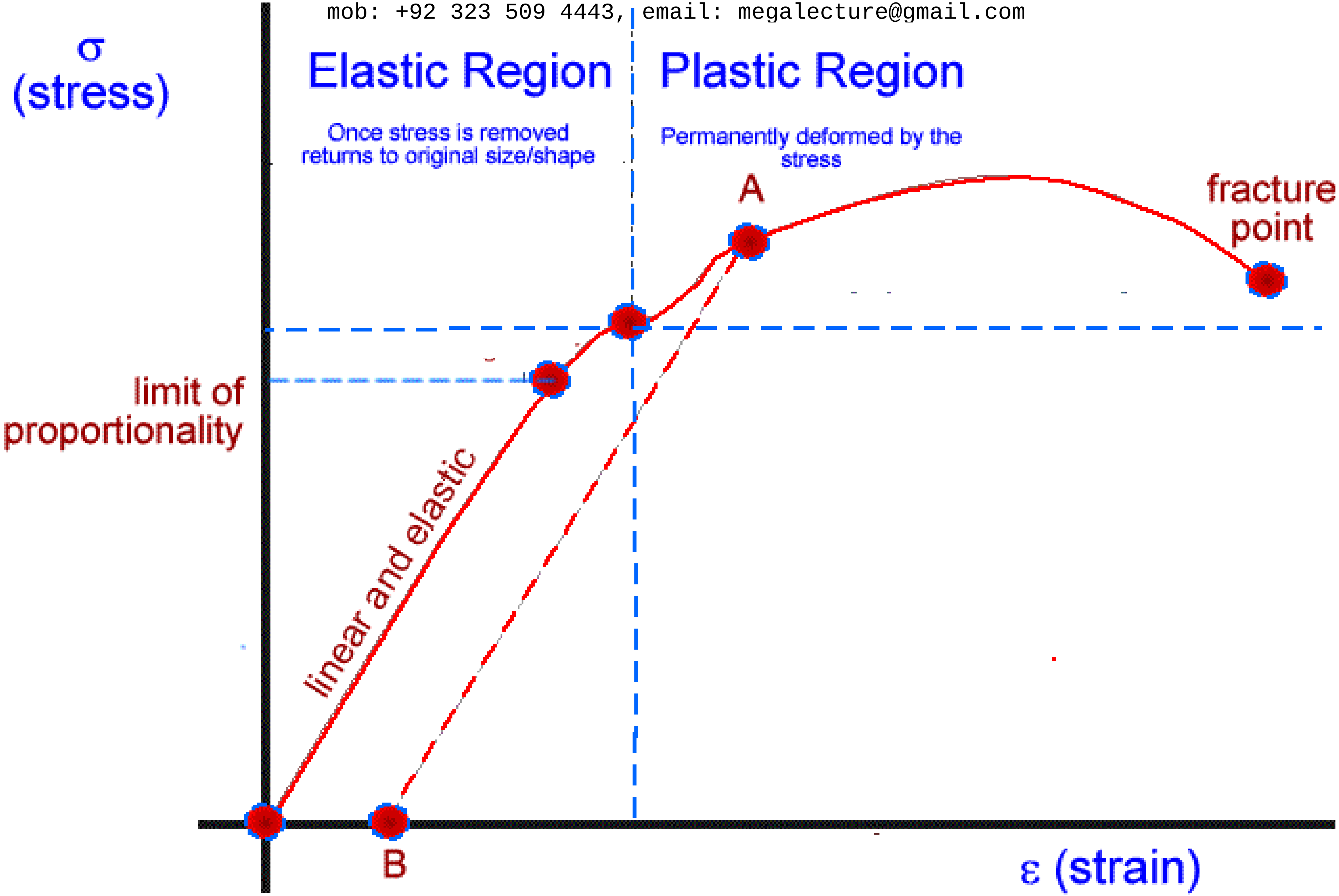
  Maximum amount a material can be stretched by a force and still (or may) return to its original shape depends on the material.

- **Yield point or Yield strength**

  The point where there is a large permanent change in length with no extra load force.

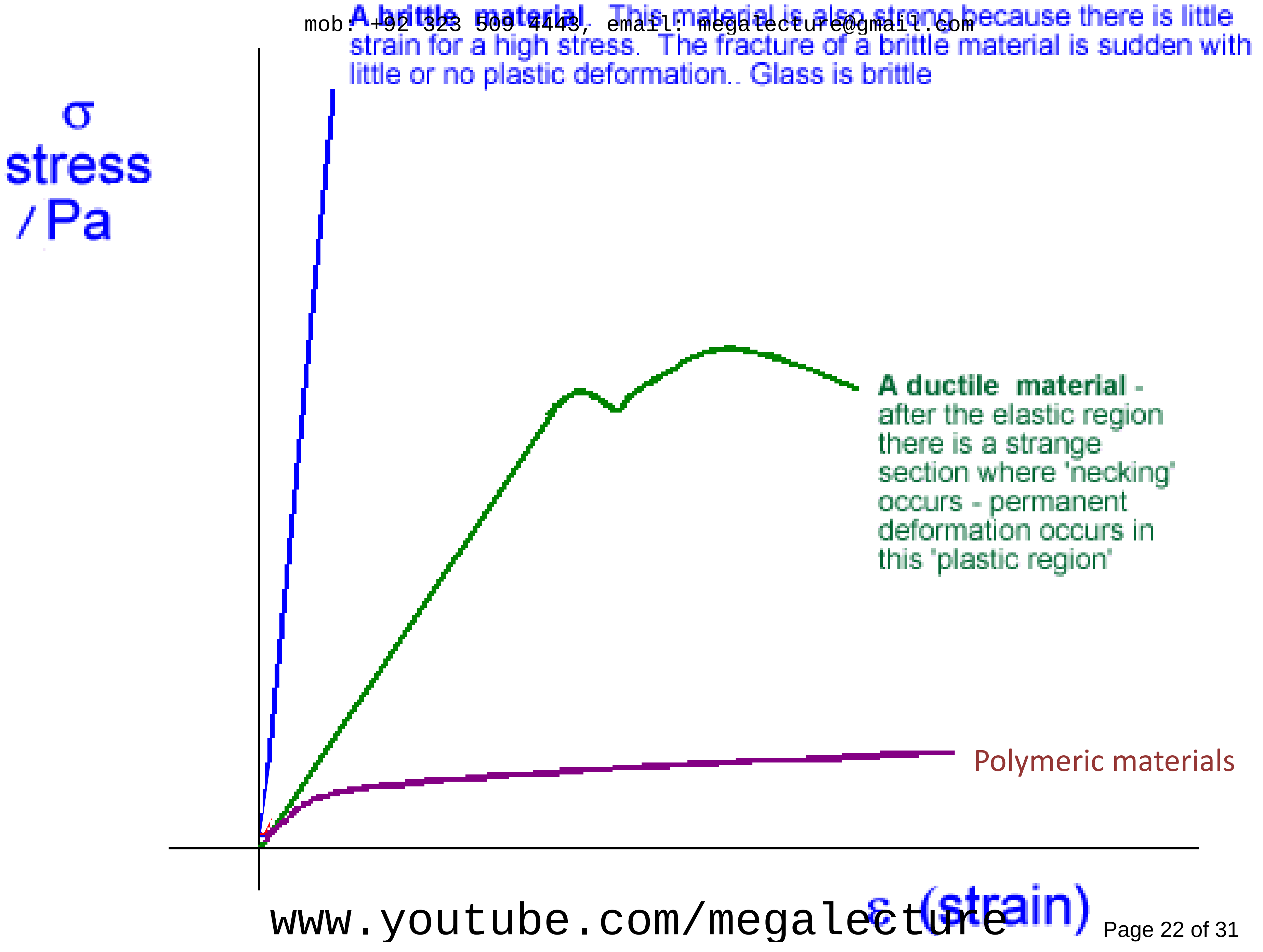
  yield point :- interface between elasticity and plasticity

- Elastic limit - up to which material can sustain the load and return back to its original position.
- Although these two points are so close to each other it can be treated as one, on a case to case basis.
- It depends upon material whether it's brittle or ductile.


σ
(stress)
Elastic Region
Plastic Region
Once stress is removed
returns to original size/shape
Permanently deformed by the
stress
A
fracture
point
limit of
proportionality
linear and elastic
B
ε (strain)

Explaining Graph in the previous slide.....

- The stretching behavior is summarized in a stress-strain graph in the previous slide. As the stress is increased initially Hooke's Law is obeyed - the stress-strain relationship for the wire is linear & elastic.
- Just before the plastic region is reached we get the limit of proportionality - beyond this for a small section we see non-linear behaviour but the stretching is still elastic.
- After the *yield strength*, the material enters the plastic deformation region, which means that the stretch of the wire is permanent. (For example, if the wire is stressed to point A on the graph and the stress is slowly decreased, the stress-strain curve follows the dotted line instead of the original curve to point **B** and there is a permanent extention when all stress is removed.) At the facture point the wire snaps.
- Differences in the shape and limits of the stress-strain diagram determines whether a material is considered ductile or brittle, elastic or plastic.


A brittle material. This material is also strong because there is little strain for a high stress. The fracture of a brittle material is sudden with little or no plastic deformation.. Glass is brittle
σ stress / Pa
A ductile material - after the elastic region there is a strange section where 'necking' occurs - permanent deformation occurs in this 'plastic region'
Polymeric materials
ε (strain)

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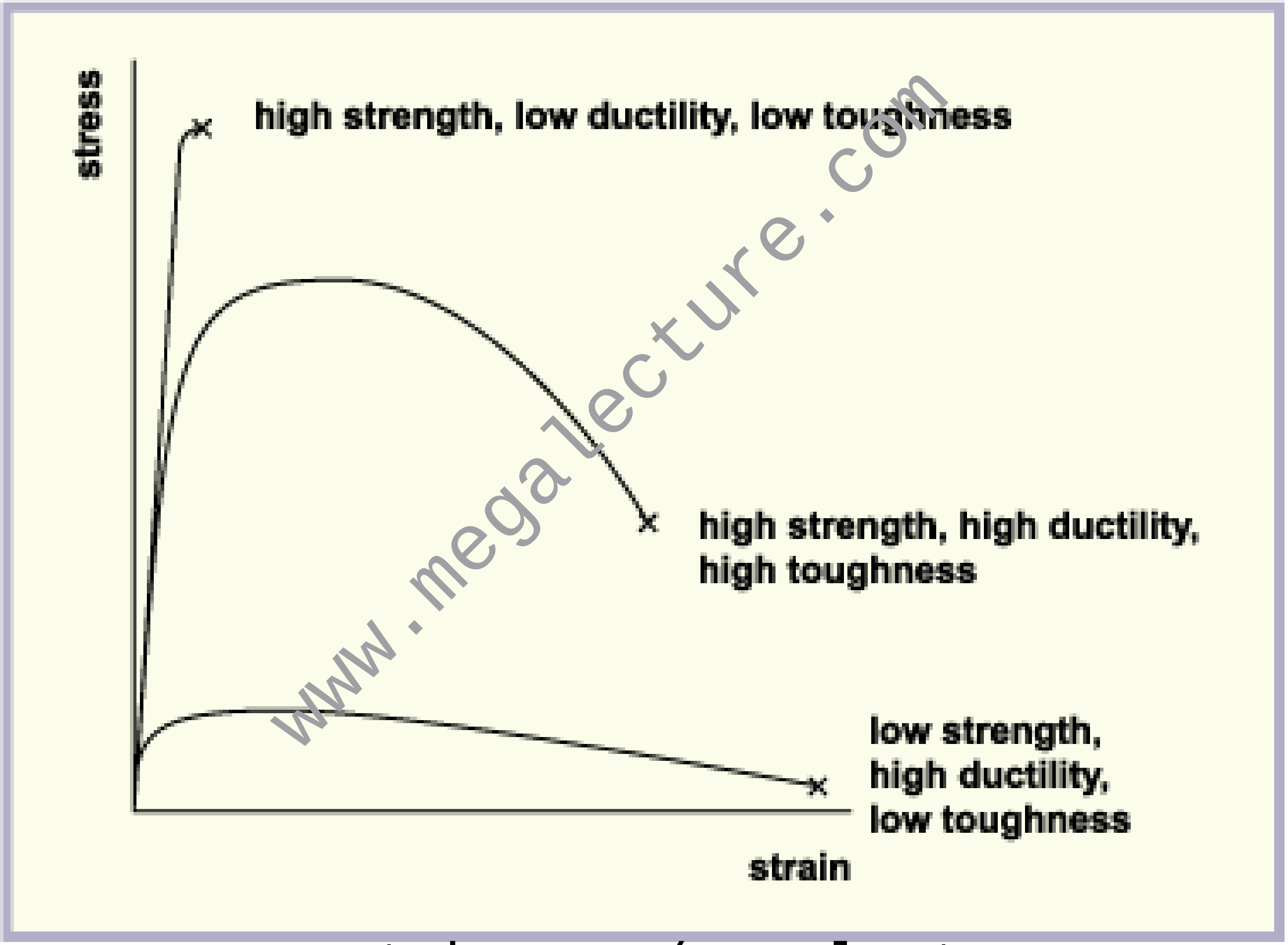
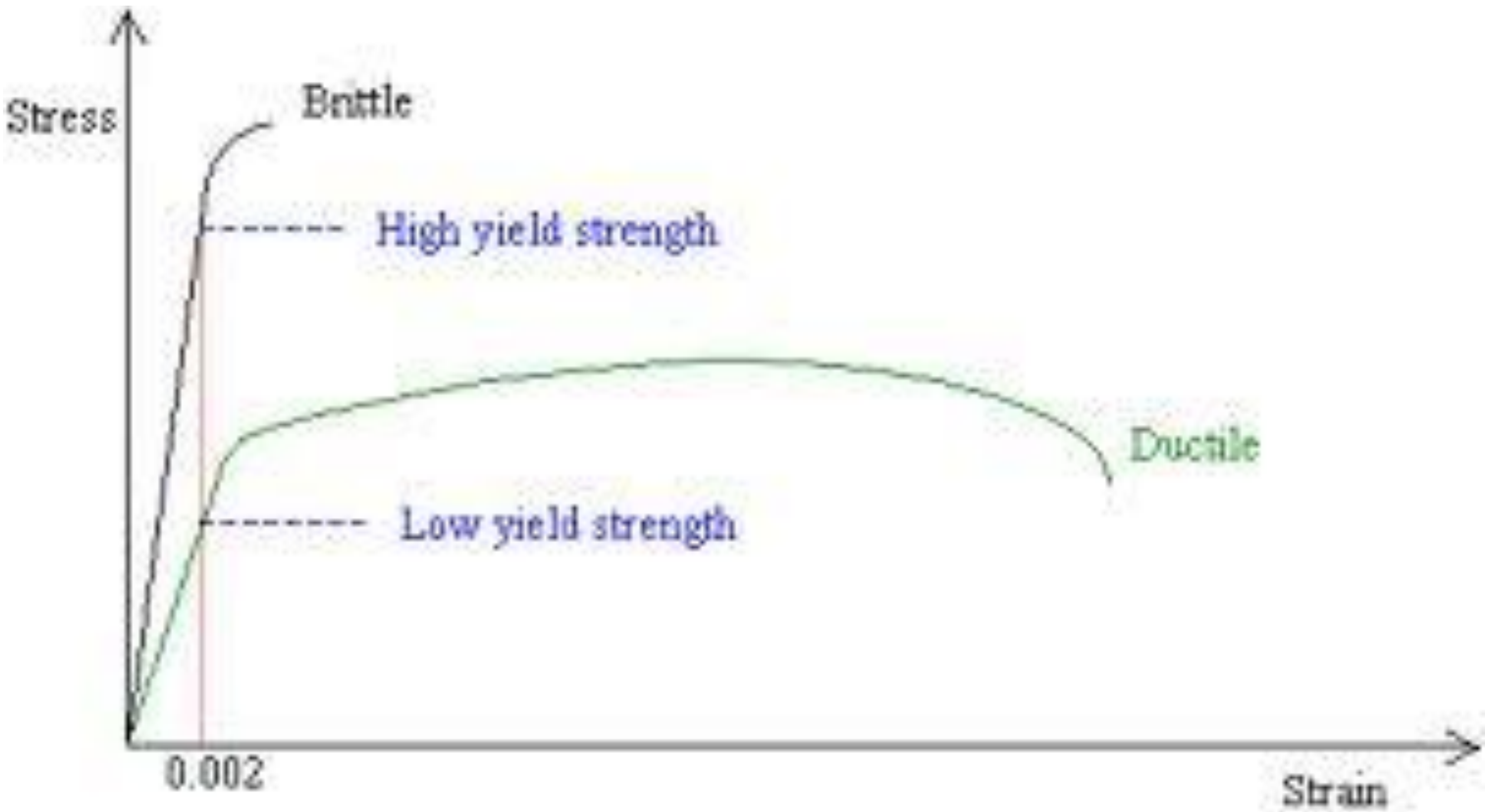
Strength, Ductility & Toughness



High Yield strength & Low yield strength



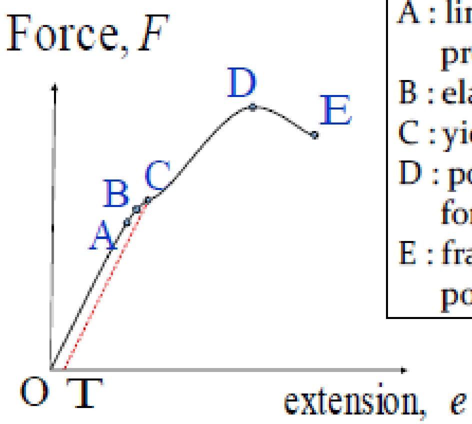
Energy in deformations

- Whenever we apply force to an object, it will cause deformation. If the deformation caused is within the elastic limit, the work done in deforming the object is stored within it as potential energy. We call this (elastic) **'strain energy'**. It can be released from the object by removing the applied force.

- The strain energy then performs work in **un-deforming** the object and returns to its original state.

Force-extension graphs for typical ductile, brittle and polymeric materials, including an understanding of ultimate tensile stress.

Force-Extension Graphs and Stress-Strain Graph



A : limit of proportionality
B : elastic limit
C : yield point
D : point of maximum force (stress)
E : fracture (breaking) point

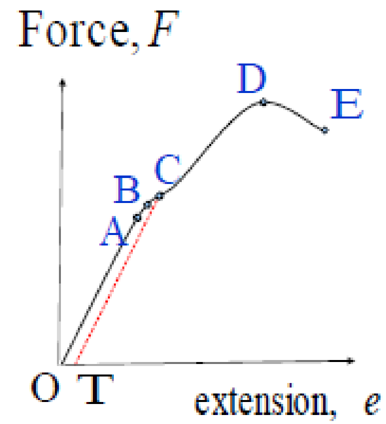
Force-Extension Graphs

k : force(Hooke) constant

OA

- The force (stress) increases linearly with the elongation (strain) until point A. Point A is the limit of proportionality.

A : limit of proportionality



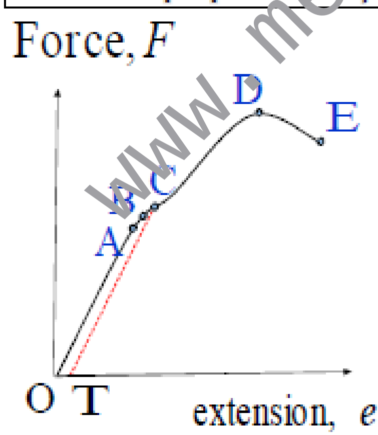
Force-Extension Graphs

k : force(Hooke) constant

OA

- The straight line graph (OA) obeys Hooke's law which states that "*Below the limit of proportionality, the restoring force, F_s is directly proportional to the elongation, e.*"

A : limit of proportionality



Force-Extension Graphs

k : force(Hooke) constant

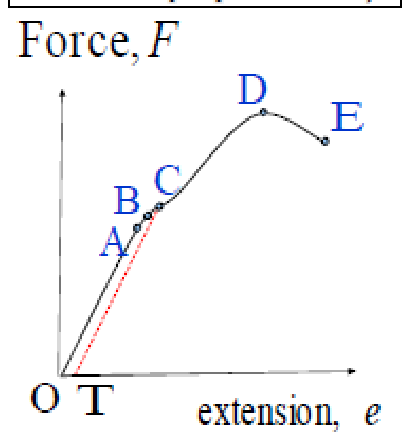
OA

$F_s = -ke$

Where

k = force (Hooke) constant

A : limit of proportionality



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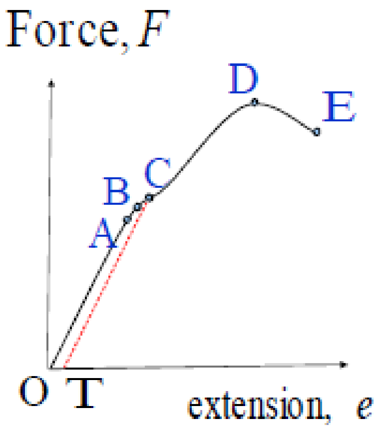
k : force(Hooke) constant

Force-Extension Graphs

OA

- $F_s = -ke$
  Where
  k = force (Hooke) constant
- The negative sign indicates that the restoring force is the opposite direction to increasing elongation.

A : limit of proportionality



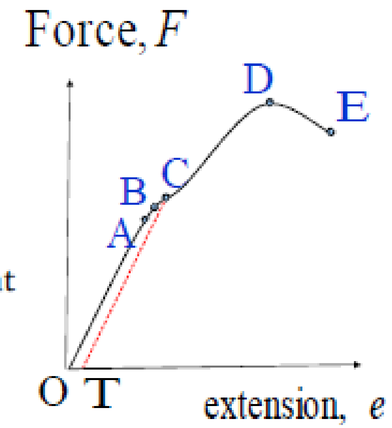
k : force(Hooke) constant

Force-Extension Graphs

B: This is the elastic limit of the material.

- Beyond this point, the material is permanently stretched and will never regain its original shape and length. If the force (stress) is removed, the material has a permanent elongation of OT.

B : elastic limit



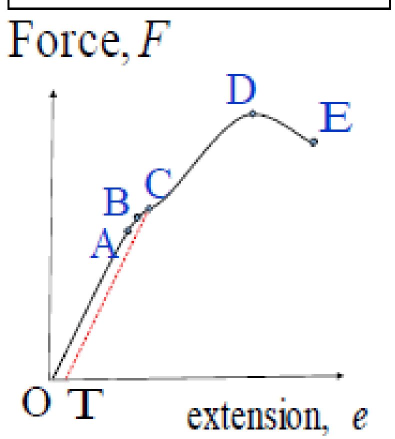
k : force(Hooke) constant

Force-Extension Graphs

OB

- The area between the two parallel line (AO and CT) represents the work done to produce the permanent elongation OT.
- OB region is known as elastic deformation.

B : elastic limit



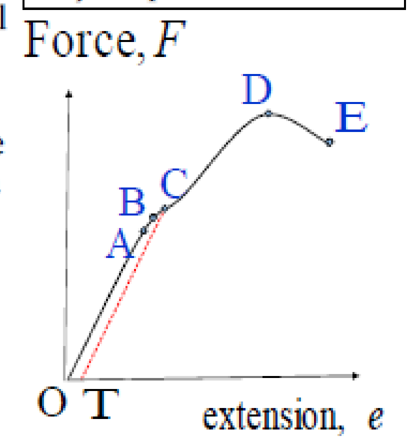
k : force(Hooke) constant

Force-Extension Graphs

C -The yield point marked a change in the internal structure of the material.

- The plane (layer) of the atoms slide across each other resulting in a sudden increase in elongation and the material thins uniformly.

C : yield point



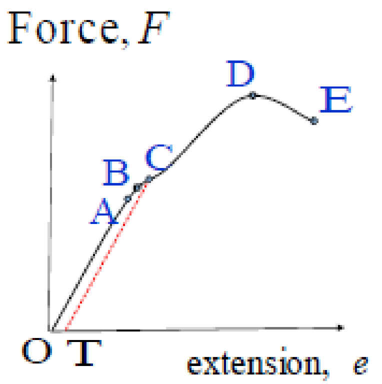
k : force(Hooke) constant

Force-Extension Graphs

D

- The force (stress) on the material is maximum and is known as the breaking force (stress). This is sometimes called the Ultimate Tensile Strength (UTS).

D : point of maximum force (stress)



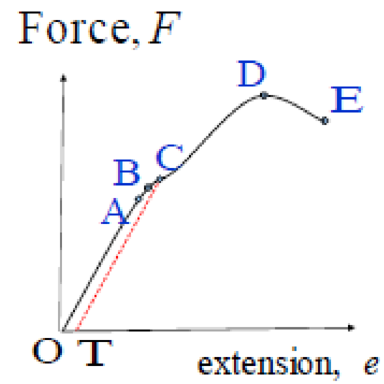
k : force(Hooke) constant

Force-Extension Graphs

E

- This is the point where the material breaks or fractures.

E : fracture (breaking) point



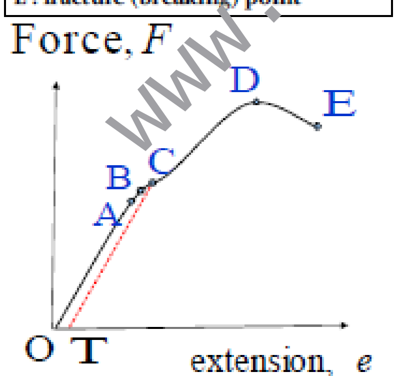
k : force(Hooke) constant

Force-Extension Graphs

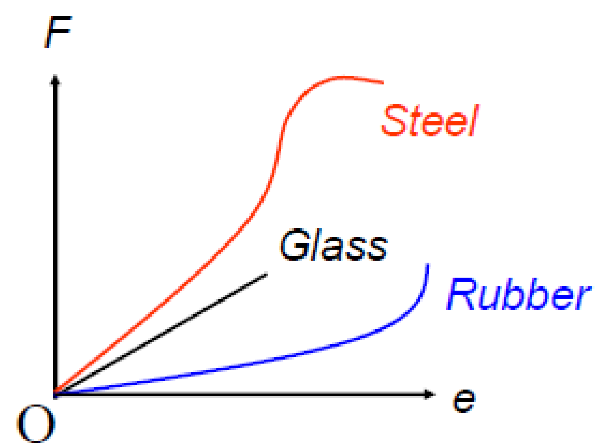
CDE

- This region is known as plastic deformation.
- - When the force (stress) increases, the elongation (strain) increases rapidly.

C : yield point
D : point of maximum force (stress)
E : fracture (breaking) point



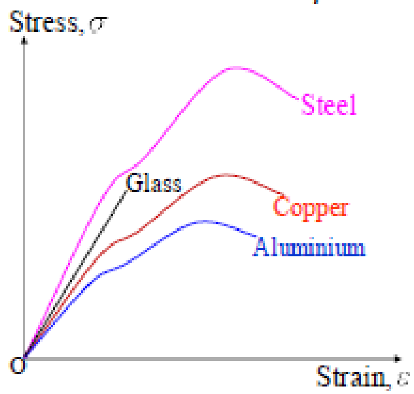
Extension-Force Graphs



Types of materials

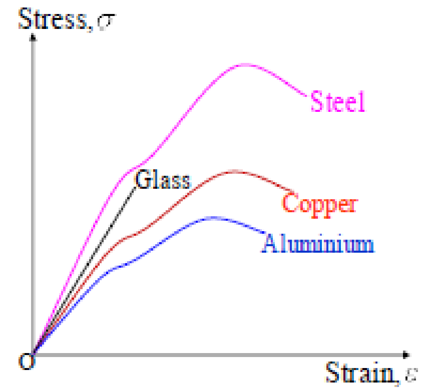
- Ductile materials - undergo plastic deformation before breaking.
- such as steel, copper, aluminium.

Stress-Strain Graph



Types of materials

- Brittle materials - do not show plastic behaviour (deformation).
- such as glass.



Young's Modulus (Y @ E)

- Definition – is defined as *the ratio of the tensile stress to the tensile strain if the limit of proportionality has not been exceeded.*

$$Y = \frac{\text{Tensile stress}}{\text{Tensile strain}} \qquad Y = \frac{\left(\frac{F_\perp}{A}\right)}{\left(\frac{e}{l_0}\right)} \Rightarrow Y = \frac{F_\perp l_0}{Ae}$$

Young's Modulus (Y @ E)

- Its dimension is given by

$$[Y] = \frac{[F_\perp][l_0]}{[A][e]} = ML^{-1}T^{-2}$$

- The unit of Young's modulus is $\text{kg m}^{-1}\text{ s}^{-2}$ @ N m^{-2} @ Pa.