Newtonian Mechanics

Work, Energy and Power

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- Energy possessed by a body is the capacity of the body to do work.
- State the principle of *conservation of energy.*

Energy can neither be created nor destroyed, but can be converted from one form to another (or others).

- The total amount of energy in any closed system is constant.
- Energy and work are both scalar quantities, and have the unit Joule.

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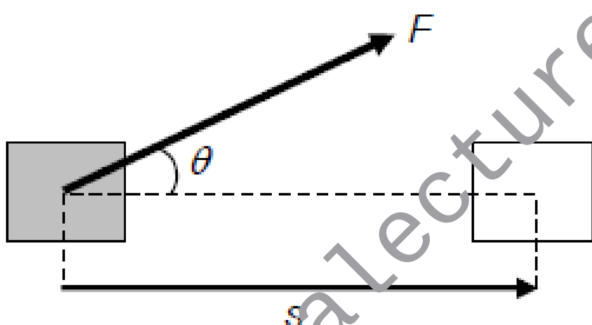
Work in terms of the product of a force and displacement in the direction of the force

In physics, work is done when a force moves its point of application so that some resolved part of the displacement lies along the direction of the force.



1. Define *work*.

The work *W* done on an object by an agent exerting a constant force *F* on the object is defined as the product of the force and the displacement in the direction of the force.

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Define Work

Define *work*.

The work W done on an object by an agent exerting a constant force F on the object is defined as the product of the **force** and the **displacement in the direction of the force**.

$$W = Fs\cos\theta$$

where
W is the work done on the object by the constant force F (joule, J)
F is the constant force acting on the object (Newton, N)
s is the displacement of the object (metre, m)
θ is the angle between F and s (degree, °)

In cases, where no angle is given, you may use the equation
W = Fs

Unit of Work

The S.I. unit for work is the __joule_____.

One joule (1 J) is defined as the work done by a constant force of one newton (1 N) on an object when the object moves one metre (1 m) in the direction of the force.

$$1\ J \equiv (1\ N)\ (1\ m) = 1\ N\ m$$

The unit Newton metre (N m) is usually used for moment, while Joule (J) is usually used for work.

Scenario	Work done on object	Example
Force does not move the object	**Zero**	A man pushing a wall
Force is perpendicular to the object's movement	**Zero**	Weight of a trolley moving along a horizontal surface
Force has a resolved part in the same direction as the object's displacement	**Positive**	A man pushing a trolley
Force has a resolved part in the opposite direction as the object's displacement	**Negative**	Friction opposing the trolley's motion

Note:
Even though work is a scalar, it can be positive or negative.

In the above examples, *positive* work done on an object __**increases**_______ the kinetic energy of the trolley, while *negative* work done ___**decreases**_____ its kinetic energy. *Zero* work done means the kinetic energy _____**remains constant**________.

Work done by a constant force
Sample problem - 1

- The following are examples of work done by a constant force.
  (a) Work done by the man on a box (Fig 6.3).

$$W = Fs\cos\theta$$
$$= (10)(10)\cos 30^\circ$$
$$= 86.6\ \text{J}$$
$$= 87\ \text{J}$$

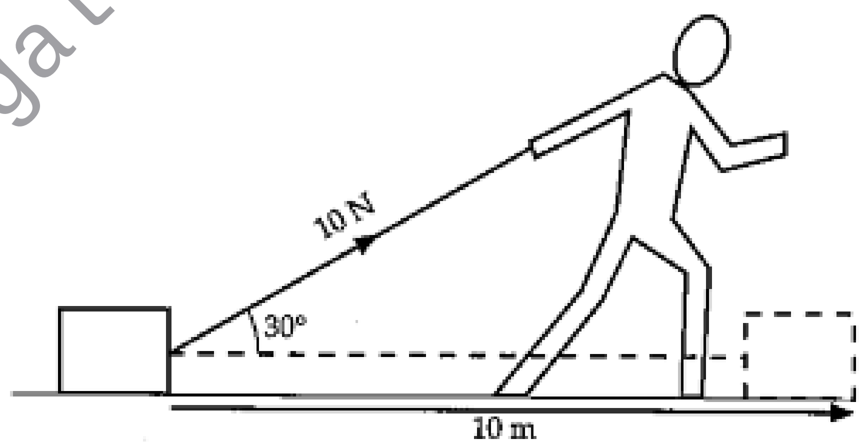


Fig 6.3 Positive work done

Sample problem 2 - Solve it!

(b) Work done by the man in opposing the sliding box (Fig 6.4).

$$W = Fs\cos\theta$$
$$= (10)(10)\cos 120°$$
$$= -50\ \text{J}$$

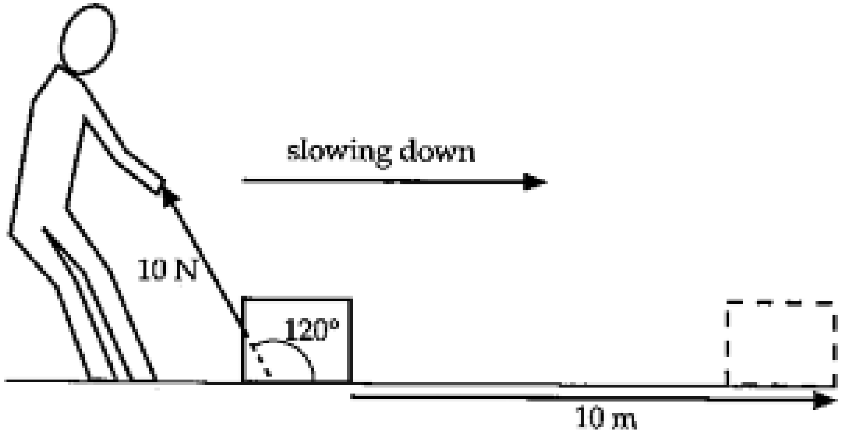


Fig 6.4 Negative work done

Sample problem 3 – Solve it!

(c) Work done by the frictional force in opposing the sliding box (Fig 6.5).

$$W = Fs\cos\theta$$
$$= (2)(10)\cos 180^\circ$$
$$= -20\ \text{J}$$

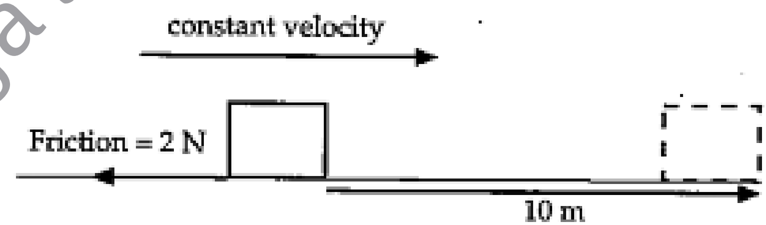


Fig 6.5 Work done by friction

Sample problem 4 – Solve it !

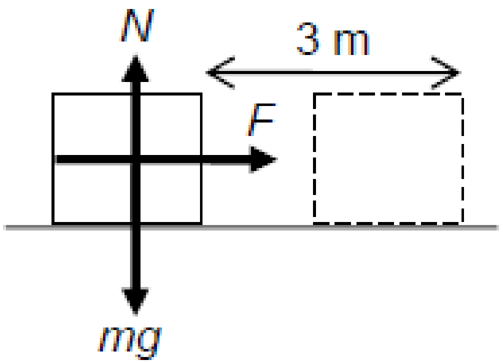
A man pushes a box of mass 5 kg with a force of 2 N along a horizontal frictionless surface. The box is moved through a distance 3 m.

Calculate the work done on the box by
(a) the pushing force;
(b) the normal contact force;
(c) the weight of the box.

(a) $W = Fs\cos\theta = 2 \times 3 = 6$ J

(b) $W = 0$ J
(force perpendicular to displacement)

(c) $W = 0$ J



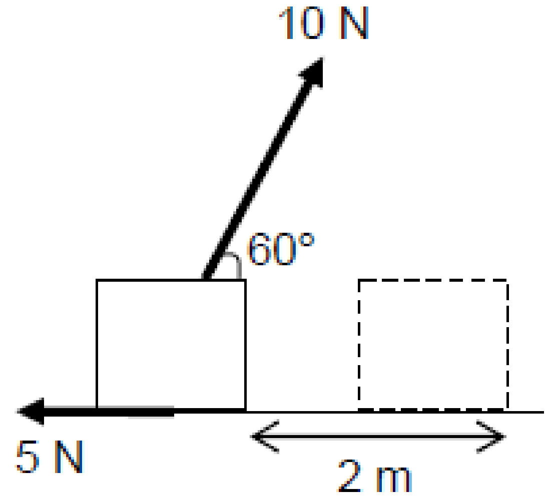
Example 5 – Solve it !

A block is pulled along a rough horizontal surface by a rope pointing at 60° above the horizontal. The tension in the rope is 10 N and the frictional force is 5 N. The block moves a distance of 2 m along the surface.

Calculate the work done on the box by
(a) the tension in the rope;
(b) the frictional force.

(a) $W = Fs\cos\theta = (10\cos 60°)\,2$
$= 10$ J

(b) $W = (-5)\,2 = -10$ J



Example 6 – Solve it !

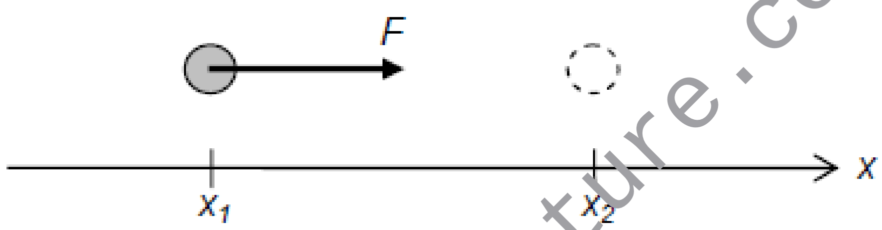
- How much work is done by a person who uses a force of 27.5N to move a trolley 12.3m?

$$W = F \times d = (27.5N)\ (12.3m)$$

$$= 338.25J$$

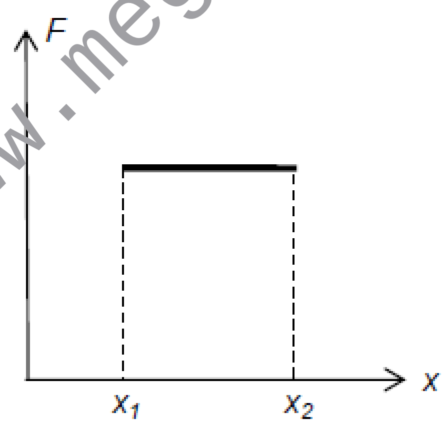
Special case

Suppose a force F is acting on an object along the x-direction and the object moves a distance ($x_2 - x_1$) along the same direction.



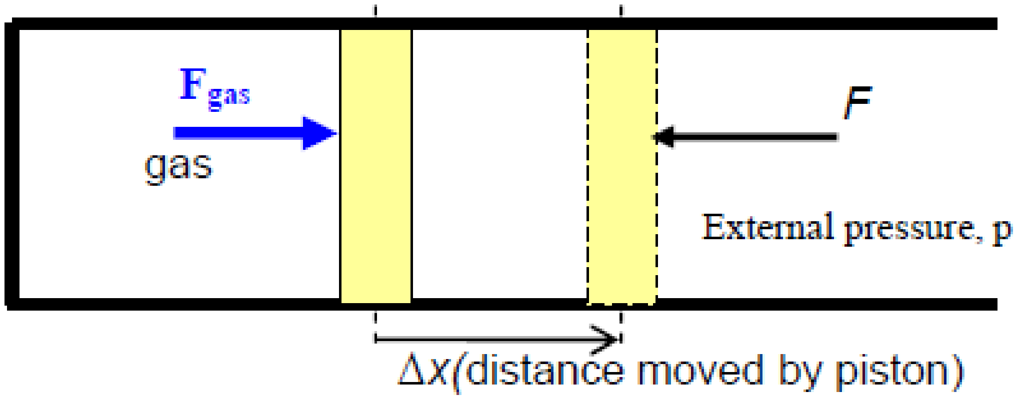
If F is constant,

work done is given by $W = F(x_2 - x_1)$



Work done = area under __force-displacement__ graph

For work done by a gas which is expanding against a constant external pressure



Symbols

F is the force exerted by external pressure on piston
p is the constant external pressure
Δx is the distance moved by piston
A is the cross-sectional area of piston

For a piston with a cross-sectional area *A*, the force *F* acting normal to it is given by $F = pA$.

For a gas with pressure = external pressure p, if the piston is allowed to move outwards **slowly** by a displacement Δx, the gas will expand against the constant external pressure *p*, and $F_{gas} = F$.

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slowly by a displacement Δx, the gas will expand against the constant external pressure *p*, and $F_{gas} = F$.

The work done *W* by the gas against this constant pressure *p* is given by $W = F\,\Delta x = (pA)\,\Delta x = p\,(A\Delta x)$ **=** $p\,\Delta V$ where ΔV is the change in volume.
Hence, when a gas expands with a change in volume ΔV against constant pressure *p*, work done by gas is given by:

1.

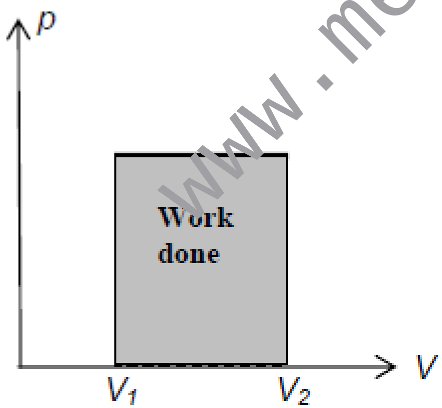
$$W = p\,\Delta V$$

where W is the work done by gas (J)
p is the external pressure (Pa)
ΔV is the change in volume of the gas (m^3) Note: $\Delta V = V_{final} - V_{initial}$

2.

If *p* is constant,

work done is given by $W = p(V_2 - V_1)$



Work done = area under ____pressure-volume________ graph

3.

	When gas expands	When gas is compressed
Piston moves	outwards	inwards
Force exerted by gas on surroundings and displacement moved	in same direction	in opposite direction
Work done by gas on surroundings	**positive**	**negative**
Work done by surroundings on gas	**negative**	**positive**
We say	**Gas does work on surroundings**	**Surroundings do work on gas**

Try to solve it !

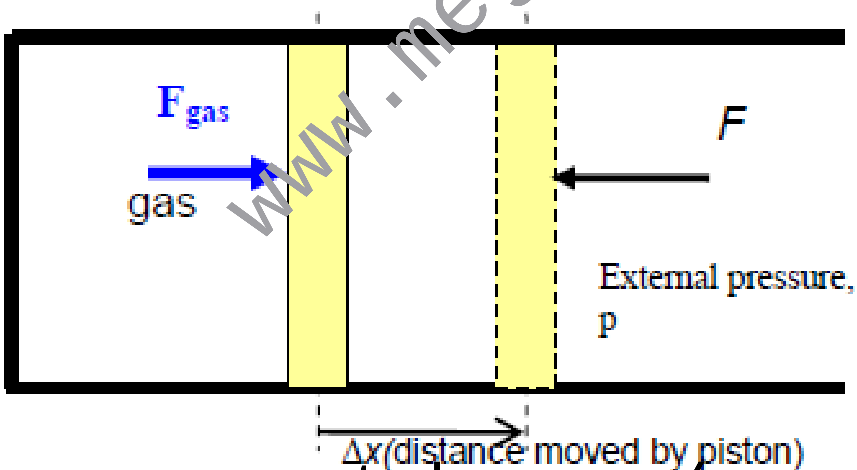
A gas in a cylinder is kept at a constant pressure of 1.1×10^5 Pa. The gas is heated and expands by 25 cm^3.

Calculate the work done **by** the gas.

$1 \text{ cm} = 10^{-2} \text{ m}$

$1 \text{ cm}^3 = 10^{-6} \text{ m}^3$

$W = p\,\Delta V = 1.1 \times 10^5 \times 25 \times 10^{-6} = 2.75 \text{ J}$



Derive from Equations of motion (KE)

1. The kinetic energy (K.E.) is a positive scalar quantity that represents the energy associated with the body due to its __motion__.

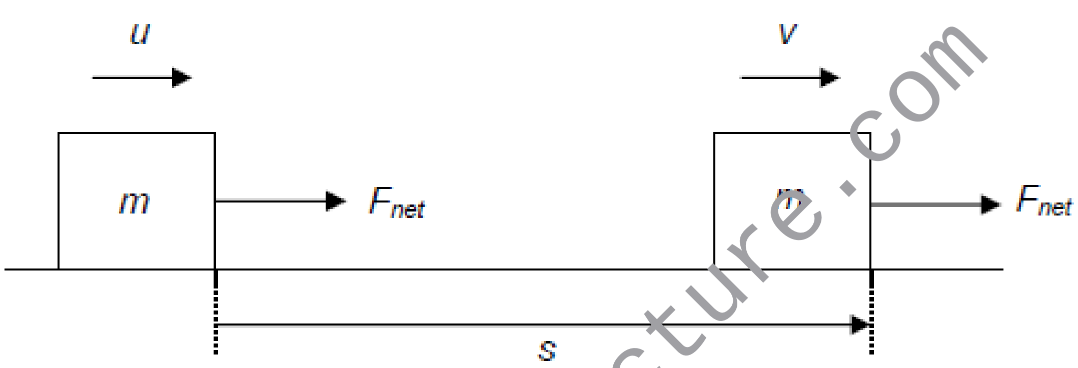
   It is equal to the work done by the resultant force in accelerating the body from rest to an instantaneous speed v.

$$E_k = \frac{1}{2}mv^2$$

where E_k is the kinetic energy of the body moving at speed v (J)
m is the mass of the body (kg)
v is the speed of the body (m s^{-1})

Derive from Equations of motion (KE)

Derive, from the equations of motion, the formula $E_k = ½\, m\, v^2$.



Body initially at rest, $u = 0$:
work done by net force = change in K.E. of body

$$(F_{net})\, s = E_k$$

$$mas = E_k$$

Using $v^2 = u^2 + 2as$: $\quad ½mv^2 - ½m(0)^2 = E_k$

$$\therefore \quad E_k = \frac{1}{2}mv^2$$

Note:

1. E_k is a **scalar** quantity and has the same units as work i.e. joules, J.

2. work done by net force = change in K.E. of body ($W = ½\, mv^2 - ½\, mu^2$)

Sample problem 1 – Solve it !

A car of mass 800 kg and moving at 30 m s^{-1} along a horizontal road is brought to rest by a constant retarding force of 5000 N.
Calculate the distance travelled by the car in coming to rest.

Using work done = change in K.E.:

$$W = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$

$$(-5000)s = 0 - \tfrac{1}{2}(800)(30)^2$$

$$s = 72 \text{ m}$$

Using kinematics equation:

$$v^2 = u^2 + 2as$$

$$0 = 30^2 + 2\left(\frac{-5000}{800}\right)s$$

$$s = 72 \text{ m}$$

Energy in different forms, its conversion and conservation, and apply the principle of energy conservation to simple examples

Types of energy and their sources:

Energy	Source
Mechanical Kinetic energy 1.Gravitational P.E. 2. Elastic P.E.	 Energy possessed by all objects in motion Energy possessed by an object by virtue of its position in a gravitational field. Eg. from waterfall and from raised object. Energy possessed by an object by virtue of its state of deformation. Eg. Compressed or stretched springs, a bent diving board, the stretched elastic band of a catapult.
Electrical	Due to charge or current.
Chemical	Energy possessed by a fuel by virtue of its chemical composition Eg. oils, wood, food, chemicals in electric cells.
Nuclear	Energy in nucleus of atoms due to nuclear composition. Eg. energy released in atomic bombs, produced by nuclear reactors.
Radiant	Energy in the form of electromagnetic waves eg. visible light, radiowaves, ultraviolet, x-rays.
Internal energy	Energy possessed by atoms/ molecules of matter in the form of kinetic energy due to motion of the particles in the matter and potential energy which depends on the separation between atoms / molecules.

Renewable and Non-renewable energy

Renewable sources of energy are those that can be replaced or replenished each day by the Earth's natural processes. Eg. wind power, geothermal, solar energy, tidal energy.

Non-renewable sources of energy are those that are finite or exhaustible because it takes several million years to replace them eg. fossil fuels like coal, oil and natural gas, energy sources that are tapped from minerals eg. nuclear energy from the fission of Uranium nuclei.

Gravitational potential energy, Electric potential energy and Elastic potential energy

1. Potential energy (P.E.) is the energy possessed by a system by virtue of the _relative positions_ of its component parts.

2. Gravitational potential energy (G.P.E.) is associated with the interaction between bodies due to their __masses__.

3. Electric potential energy is associated with the interaction between bodies due to their electric __charges__.

4. Elastic potential energy is possessed by an object due to its state of _deformation__.

Give EXAMPLES of energy in different forms and its conversion

Check your understanding!

- http://www.youtube.com/watch?feature=endscreen&NR=1&v=tdl8wnQOkjM
- http://science.howstuffworks.com/engineering/structural/roller-coaster3.htm

More Examples of Energy conversion

Example diver jumping off a diving board

- The diver uses his gravitational potential energy to do work in bending the diving board.
- The work done is stored as elastic potential energy, which is then converted into kinetic energy of the diver as he is pushed upwards and off the diving board.
- At the same time, some of the elastic potential energy is lost as heat and sound due to dissipative forces in the diving board.

Example burning of fossil fuel

- When fuels such as oil, coal and wood are burnt, the chemical energy stored in these materials is converted into thermal energy (heat) and light energy.

Example hammering a nail into a wooden block

- A person uses the **chemical** energy in his muscles to work against the gravitational pull in order to lift the hammer.
- The work done is converted into the **gravitational potential** energy of the hammer in its raised position.
- As the hammer falls, its gravitational potential energy is converted into **kinetic** energy.
- When the hammer hits the nail, its kinetic energy is used to do work in driving the nail into the wooden block, producing **sound** energy in the air and **thermal** energy in the block, nail and hammer.

Example falling plasticine

- During impact, all kinetic energy is converted into **thermal** and **sound** energies as the plasticine is permanently deformed.

Example bouncing ball

- As the ball falls, its gravitational potential energy is converted into its kinetic energy.
- When the ball hits the ground, the ball is deformed during the collision. Its kinetic energy is converted into **elastic potential** energy. Some kinetic energy may be lost as **thermal** energy or **sound** energy.
- The elastic potential energy is converted back into **kinetic** energy as the ball regains its original shape.
- The kinetic energy is converted into **gravitational potential** energy as the ball bounces upwards, until it reaches its highest position.
- During the flight, presence of air resistance will cause kinetic energy to be dissipated as **thermal energy,** thus reducing the total energy in the ball and its subsequent height after each bounce.

Internal Energy

- Matter consists of atoms and molecules and these are made up of particles having kinetic energy and Potential energy.
- We can define the Internal energy of a system as the sum of the kinetic energy of all its constituent particles plus the sum of all the potential energy of interaction among these particles in the system.

Note : The internal energy does not include potential energy arising out of interaction between the system and it surroundings.

The implications of energy losses in practical devices and use the concept of efficiency to solve problems

1. For practical devices to work, energy input is needed. Most modern practical devices run on electrical energy (e.g. television, computer) or chemical energy (e.g. vehicle).

   When a practical device works, it converts the energy input into both __**useful**__ energy output and __**wasted**__ energy output.

2. Efficiency of a practical device is a measure of how much useful work that device produces from a given amount of energy input.

   Its value does depend on what energy output we consider as useful.

   Efficiency is ____**dimensionless**_____ and can be expressed as a ratio or percentage.

$$\text{Efficiency, } \eta = \frac{\text{useful energy output}}{\text{energy input}} \times 100\%$$

3. We can never make a practical device with 100% efficiency because:

   (a) we have _limited_ control over physical processes (e.g. a filament bulb must heat up before it produces light, but the heat produced becomes wasted energy);

4. Sample Problem

A car has a mass of 800 kg and the efficiency of its engine is rated at 18%. Determine the amount of fuel used to accelerate the car from rest to 60 km h^{-1}, given that the energy supplied by 1 litre of fuel is 1.3×10^8 J.

Note: In this case, the useful energy output is defined to be the change in kinetic energy of the car as it accelerates from rest to 60kmh^{-1}.

$$\text{Useful energy output} = \text{K.E. of car} = \tfrac{1}{2}(800)\left(\frac{60000}{60 \times 60}\right)^2 = 111111 \text{ J}$$

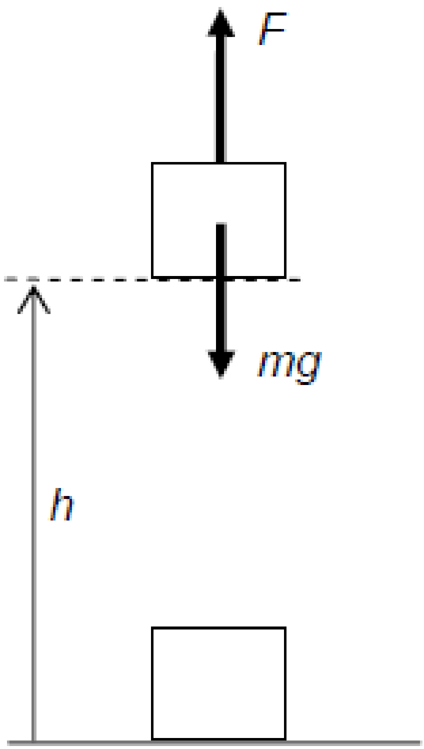
$$\eta = \frac{\text{useful energy output}}{\text{energy input}} \times 100\% \quad \Rightarrow \quad 18\% = \frac{111111}{\text{energy input}} \times 100\%$$

$$\Rightarrow \quad \text{energy input} = 617284 \text{ J}$$

$$\text{Amount of fuel} = \frac{617284}{1.3 \times 10^8} = 0.0047 \text{ litres}$$

Derive, from the defining equation *W = Fs*, the formula *Ep = mgh* for *the potential energy changes near the Earth's surface*

Suppose we want to lift an object of mass *m* to a height *h* above the ground, so that its velocity remains **constant**.



To do so, we must apply a force *F* that is **equal** but **opposite** to the weight *mg* of the object, where *g* is the acceleration of free fall.

The work done *W* by force *F* on the object is:

$$W = Fh = (mg)h$$

Since the object's velocity is constant, its kinetic energy is also **constant**. Hence, by conservation of energy, the work done *W* on the object must be equal to the **gain in G.P.E.** of the object.

Change in G.P.E. near Earth's surface = *mgh*

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Sample problem

- Figure shows a dam with storage of water. The outlet of the dam is 20 m below the surface of the water in the reservoir. Water leaving the dam is moving at 16m/s. Calculate the % of G.P.E that is lost when converted into K.E.

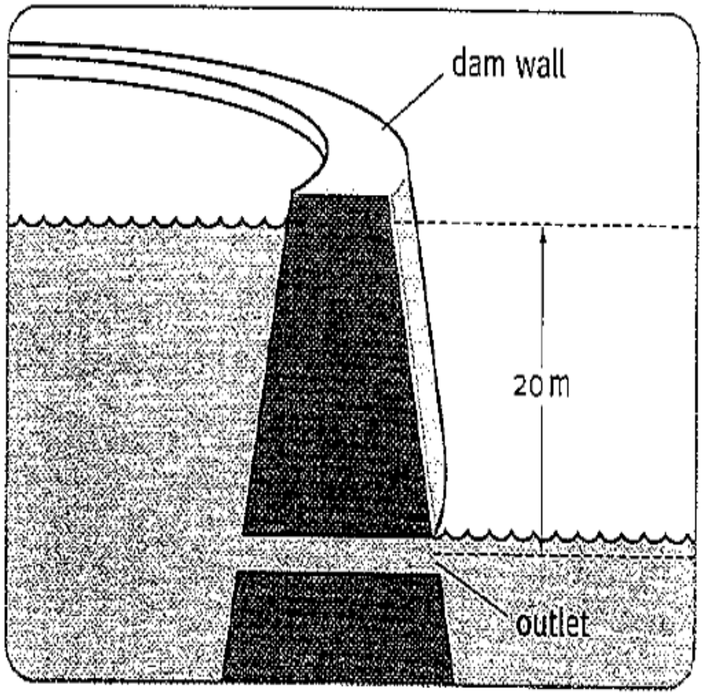

Figure 5.18 Water stored behind the dam has gravitational potential energy; the fast-flowing water leaving the foot of the dam has kinetic energy.

Solution

Step 1 We will picture 1 kg of water, starting at the surface of the lake (where it has g.p.e., but no k.e.) and flowing downwards and out at the foot (where it has k.e., but less g.p.e.). Then:

change in g.p.e. of water between surface and outflow $= mgh = 1 \times 9.81 \times 20 = 196\ \text{J}$

Step 2 Calculate the k.e. of 1 kg of water as it leaves the dam:

$$\text{k.e. of water leaving dam} = \frac{1}{2}mv^2 = \frac{1}{2} \times 1 \times (16)^2 = 128\ \text{J}$$

Step 3 For each kilogram of water flowing out of the dam, the loss of energy is:

$$\text{loss} = 196 - 128 = 68\ \text{J}$$

$$\text{percentage loss} = \frac{68}{196} \times 100\% \approx 35\%$$

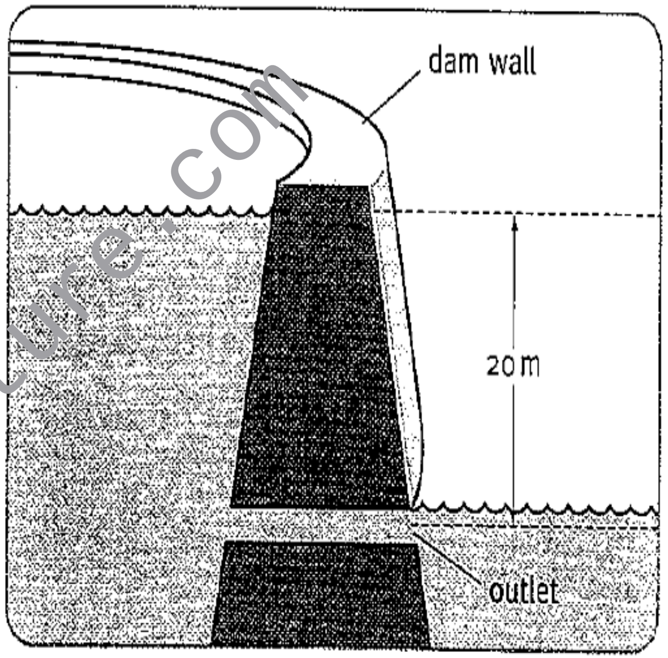

Figure 5.18 Water stored behind the dam has gravitational potential energy; the fast-flowing water leaving the foot of the dam has kinetic energy.

Define power as work done per unit time and derive power as the product of force and velocity.

Define *power*.

Power is defined as:
(i) the work done per unit time, or
(ii) the rate of work done, or
(iii) the rate of energy conversion.

$$P = \frac{W}{t} = \frac{E}{t}$$

where
P is the power (watt, W)
W is the work done (joule, J)
E is the energy converted (joule, J)
t is the time taken (second, s)

The S.I. unit for power is the ___watt___.

One watt is defined as the power when one joule of work is done, or one joule of energy is converted, per second.

1 W = (1 J) / (1 s)

3. Derive power as *the product of force and velocity*.

Suppose a constant force F displaces an object, moving at a constant velocity v, by a distance s over a time interval t, and that F, v and s all point along the same line.

$$P = \frac{W}{t} = \frac{Fs}{t} = F\left(\frac{s}{t}\right) = Fv$$

$$\boxed{P = Fv}$$

4. If force F and/or velocity v is/are not constant, then

(i) the average power $<P>$ is given by $<P> = \frac{\text{total work done}}{\text{total time taken}}$

(ii) the instantaneous power is given by $P = Fv$

where F and v take the values at the instant considered

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Sample problem 1

A car moves along a horizontal road at a constant velocity of 15 m s^{-1}. If the total resistive force experienced by the car is 5000 N, calculate the power output of the car's engine.

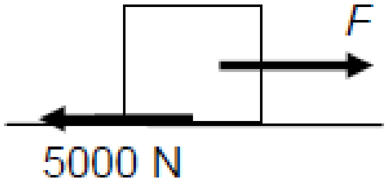
At constant velocity, $F_{net} = 0$

$\therefore$ F = 5000 N

$P = Fv$

= (5000) (15)

= 75000 W



Sample problem 2 – Solve it!

A trolley of mass 7 kg is initially rest at $t = 0$ s.
A cyborg pushes this trolley with a constant force of 95 N along a horizontal floor.
The frictional force acting on the trolley is 11 N.

Calculate
(i) the acceleration of the trolley;
(ii) the speed of the trolley at $t = 4$ s;
(iii) the kinetic energy of the trolley at $t = 4$ s;
(iv) the distance travelled during the first 4 s;
(v) the instantaneous power supplied to the trolley by the cyborg at $t = 4$ s;
(vi) the average power supplied to the trolley by the cyborg during the first 4 s;
(vii) the average power dissipated by friction during the first 4 s;
(viii) the net average power gained by the trolley during the first 4 s, and hence, the total energy gained by the trolley during the first 4 s.

Solution on next slide

Solutions for the question in previous slide

(i) $F_{net} = ma \quad \Rightarrow \quad 95 - 11 = 7a \quad \Rightarrow \quad a = 12 \text{ m s}^{-2}$

(ii) $v = u + at = 0 + 12(4) = 48 \text{ m s}^{-1}$

(iii) $E_k = ½\, mv^2 = ½\,(7)(48)^2 = 8064 \text{ J}$

(iv) $s = ut + ½\, at^2 = 0 + ½\,(12)(4)^2 = 96 \text{ m}$

(v) $P = Fv = (95)(48) = 4560 \text{ W}$

(vi) $\langle P_{supplied} \rangle = ½\,(4560) = 2280 \text{ W}$

(vii) $\langle P_{dissipated} \rangle = \dfrac{W_{friction}}{\Delta t} = \dfrac{F_r s}{\Delta t} = \dfrac{(11)(96)}{4} = 264 \text{ W}$

(viii) $\langle P_{net} \rangle = 2280 - 264 = 2016 \text{ W}$

Check: $\langle P_{net} \rangle \Delta t = (2016)(4) = 8064 \text{ J}$ = K.E. gain by trolley

Relationship between force and potential energy in a uniform field

In a uniform field, a body experiences the same force F at all points.

If this force F moves the body along a distance Δx in its direction, then the work done W by this force F is $W = F\,\Delta x$.

However, by conservation of energy, this work must be compensated by a decrease in potential energy, $-\Delta U$.

$$W = F\,\Delta x = -\Delta U$$

$$F = -\frac{\Delta U}{\Delta x}$$

$$F = -\frac{dU}{dx}$$

where:

- F is the force acting on the point mass/charge placed at that particular point in the force field. (units: N)
- $\frac{dU}{dx}$ is the change in the potential energy of a point mass/charge with a variation of the distance from the source of the force field (units: J m^{-1})

Sample problem – Solve it!

A mass experiences a gravitational force of 24 N in a uniform gravitational field. Calculate the change in its potential energy if it moves a distance of 5.0 m

(a) along the same direction as the gravitational force;

(b) along the opposite direction as the gravitational force;

(c) in a direction perpendicular to the gravitational force.

For each case, indicate whether the change is an increase or decrease.

(a) $\Delta U = -F\,\Delta x = -24 \times 5.0$
$= -120$ J (decrease)

(b) $\Delta U = -F\,\Delta x = -24 \times (-5.0)$
$= +120$ J (increase)

(c) $\Delta U = -F\,\Delta x = -24 \times 0$
$= 0$ J (no change)