Electricity & Magnetism

D.C. Circuits

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Recap........

Types of electric current

- **Direct Current (D.C.)**: Flow of charges in the circuit is in the same direction all the time, from a higher potential to a lower potential (e.g. current from battery).

- **Alternating Current**: Flow of charges in the circuit reverses direction at regular intervals (e.g. current from household mains).

Electric circuits consist of **circuit components** (e.g. batteries, resistors, and switches) connected by **conductors** (e.g. copper cables).

For electric current to flow, the circuit components and conductors must form closed loops. There must also be sources of electrical energy (e.g. batteries) and sinks of electrical energy among the circuit components (e.g. resistors and lamps).

Note : This chapter includes only D.C. In AS syllabus A.C is not included.

Electrical Circuit Symbols

Circuit symbols: Electrical circuits use a lot of components and when circuits are drawn their symbols are used the following are the standered symbols used in circuits:-

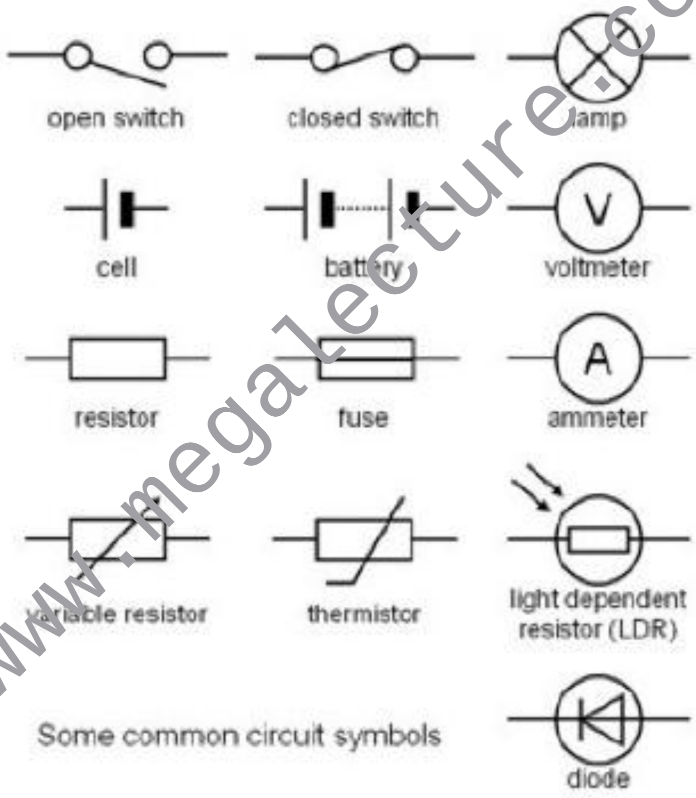
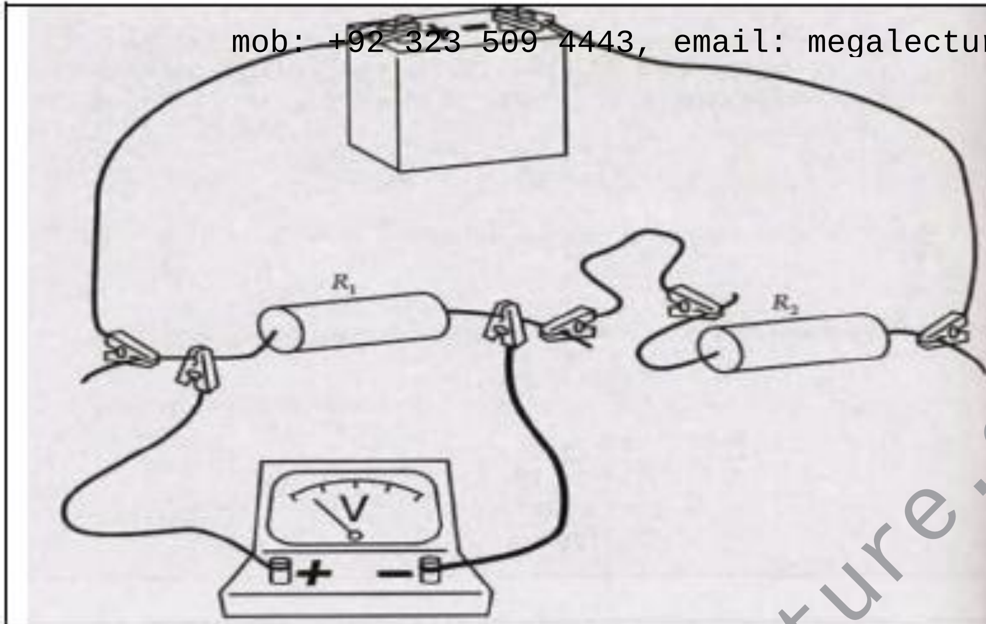
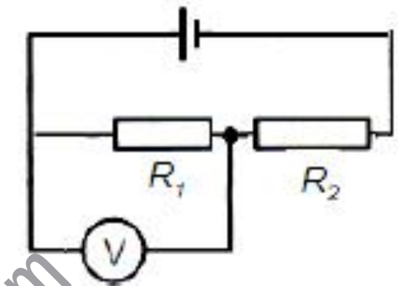


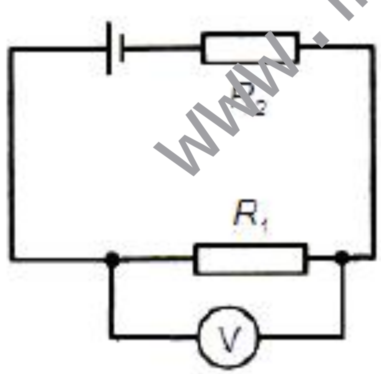
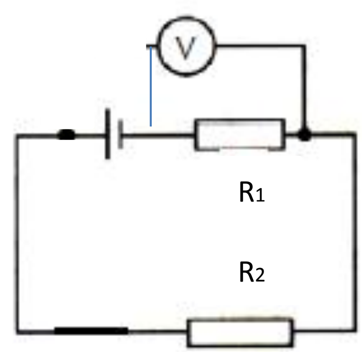
Figure : *Symbols of common circuit elements*

Draw and interpret circuit diagrams containing sources, switches, resistors, ammeters, voltmeters, and/or any other type of component

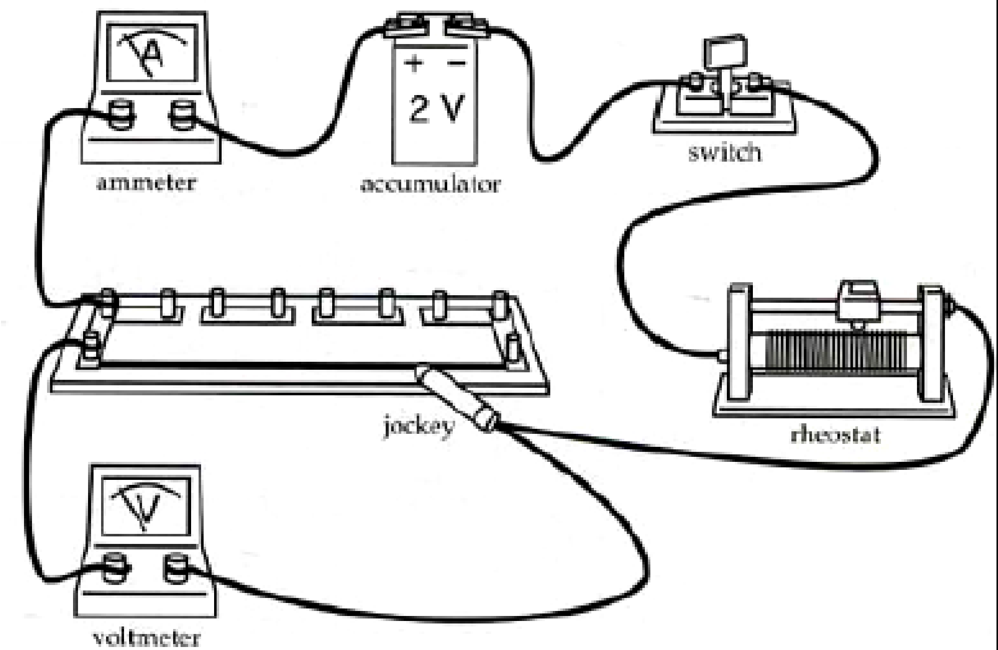
Note that, for a certain electric circuit, there are different ways of drawing its circuit diagram.

	
Actual circuit	Circuit diagram

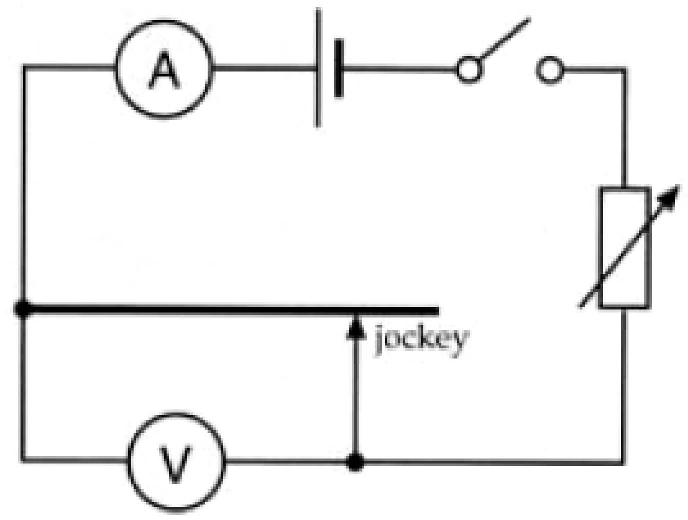
Two other possible circuit diagrams for the above electric circuit are as follows:

Example 1: Draw the circuit diagram for the electric circuit shown below.



Actual circuit



Circuit diagram

COMBINATION OF RESISTORS- RESISTANCE IN SERIES AND PARALLEL

Resistors connected in Series

RESISTORS CONNECTED IN SERIES:-The figure below shows 3 resistors connected in series to an Ideal battery (no internal resistance) Connection in series means the resistors are wired one after the other and the potential difference V is applied across the ends of the whole series.

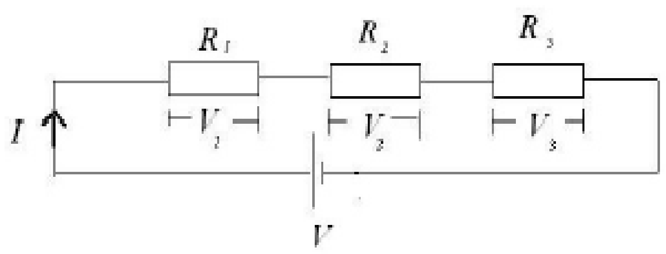


Figure : *Resistors in series-Note the current through all the resistors is the same but voltage is not*

When a potential difference V is applied across the series the current through all the resistors is the same, but the potential difference across each resistor is different and the sum of these individual

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Resistors connected in Series

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potential differences is equal to the applied potential difference V

$$V_1 = IR_1$$
$$V_2 = IR_2$$
$$V_3 = IR_3$$
$$V = V_1 + V_2 + V_3$$
$$V = IR_1 + IR_2 + IR_3$$
$$V = I(R_1 + R_2 + R_3)$$
$$\frac{V}{I} = R_{eq} = R_1 + R_2 + R_3$$

Hence the equivalent resistance of a series combination is given by

$$R_{eq} = R_1 + R_2 + R_3$$

In general if there are n resistors connected in series then

$$R_{eq} = R_1 + R_2 + R_3....R_n$$

IN SERIES

- The resistors connected in series can be replaced by an equivalent resistor R_{eq}
- R_{eq} has the same current I as the individual resistors
- *Two resistors are said to be connected series if current can flow from one resistor to another without branching.*

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Resistors connected in Parallel

RESISTORS CONNECTED IN PARALLEL:- The figure below shows 3 resistors connected in parallel to an Ideal battery (no internal resistance) Connection in parallel means the resistors are wired directly together on one side and directly together on the other side and a potential V applied across the connected sides. When resistors are connected in parallel all resistors have the same potential difference V as that of the source, but the current branches out into I_1,I_2,I_3.(i.e. the P.D. is same but current is not).

The total current in the circuit is the sum of the individual currents hence

$$I = I_1 + I_2 + I_3$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{V}{R_{eq}} = V(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3})$$

Canceling V we get

$$\frac{1}{R_{eq}} = (\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3})$$

In general if there are n resistors connected in parallel then:-

$$\frac{1}{R_{eq}} = (\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} .... \frac{1}{R_n})$$

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Resistors connected in Parallel

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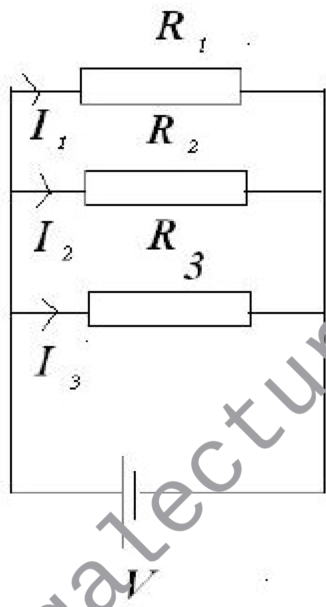

Figure : *Resistance in Parellel-Note that the current branches out but voltage is the same for all 3 resistors*

IN PARALLEL:-

- The resistors connected in Parallel can be replaced by an equivalent resistor R_{eq}
- R_{eq} has the same current V as the individual resistors
- *For resistors connected Parallel current in each of them is different but the voltage is the same*

Solve problems using the formula for the combined resistance of two or more resistors in series.

Example 4

Calculate the effective resistance of a 4 Ω and two 3 Ω resistors connected in series.

Solution:

$$R_{eff} = 4 + (2)(3) = 10\,\Omega$$

Solve problems using the formula for the combined resistance of two or more resistors in parallel.

Example 5

Calculate the effective resistance of a 2 Ω, a 3 Ω and a 4 Ω resistor connected in parallel.

Solution:

$$\frac{1}{R_{eff}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6+4+3}{12} = \frac{13}{12}$$

$$R_{eff} = 0.903\,\Omega$$

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Why the current is the same in all series resistances? And why the voltage is the same in all parallel resistors?

1) In a series circuit of resistors, the same current flows through all the resistors, however potential gets divided according to individual resistance values. Because according to Ohm's law V = I R, and here since I is same, and V is directly proportional to R. Hence the potential will be different across different resistors.

(Note : Theoretically, for resistors that have equal resistance, they have same V).

2) In the case of parallel combination of resistors, the same potential will exist across every resistor, but now current gets divided in the inverse ratio of resistance values. (I = V / R) this is also in accordance with Ohm's law.

Recap……..

Conservation of charge: The net charge of an isolated system remains constant. Charge cannot be created and destroyed, but only in positive-negative pairs.

- It is not possible to destroy or create charge.
- You can cancel out the effect of a charge(or neutralize the charge on the body) on a body by adding an equal and opposite charge to it, but you can't destroy the charge itself.

The following example makes this clear
If a glass rod is rubbed with a silk cloth, due to friction the glass gets charged positively and the silk negatively. The charging is basically due to the transfer of negative charge(electrons) from the glass to the silk. This experiment suggests that:-

- Charge is transfered but not created or destroyed.
- The total charge on the Glass silk cloth system remains the same i.e zero before rubbing and after rubbing.

Gustav Kirchhoff's Junction Rule & Loop Rule

The Junction rule

And

The Loop rule

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Kirchhoff's First Law

- **Kirchhoff's First Law**: At any junction in a circuit, the sum of the currents arriving at the junction is equal to the sum of the currents leaving the junction.
- *This is also known as 'junction rule'.*

Conservation of Charge and the Kirchhoff's First Law

Kirchhoff's first rule is a statement of conservation of electric charge.

- All charges that enter a given point in a circuit must leave that point because charge cannot build up (accumulate) at a point.
- If this does not happen then charges are getting accumulated at a point or charges are created from nowhere! both of which don't happen, In other words - charge is conserved.

The sum of currents meeting at a Junction

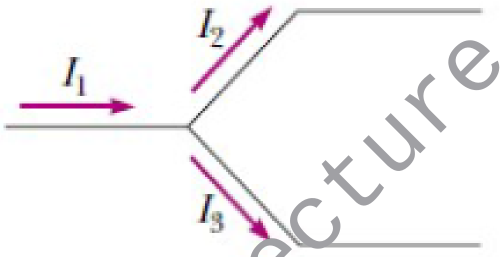


Figure : Current I_1 splits into I_2 and I_3

If we apply this rule to the junction shown in Figure below , we obtain

$$I_1 = I_2 + I_3$$

The law can also be stated as:-The algebraic sum of currents meeting at a junction is zero.

SIGN CONVENTION USED: Currents entering a Junction are taken as positive and currents leaving a junction is taken as negative.

Example:- Consider a junction O in an electrical circuit as show below:- Here the currents I_1; and;

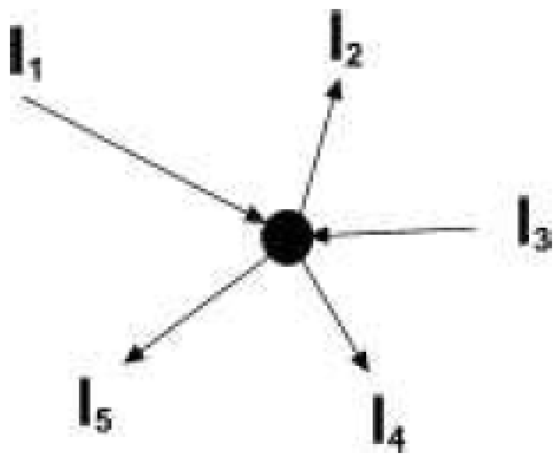


Figure **:** *Currents entering a Junction are taken as positive and currents leaving a junction is taken as negative.*

I_3 are entering the junction O whereas I_2,$I_4$and I_5 leave the junction hence I_1,I_3 are positive and I_2, I_4 and I_5 are negative as the are leaving the junction.

Applying the sign convention we get

$$I_1 + I_3 - I_2 - I_4 - I_5 = 0$$

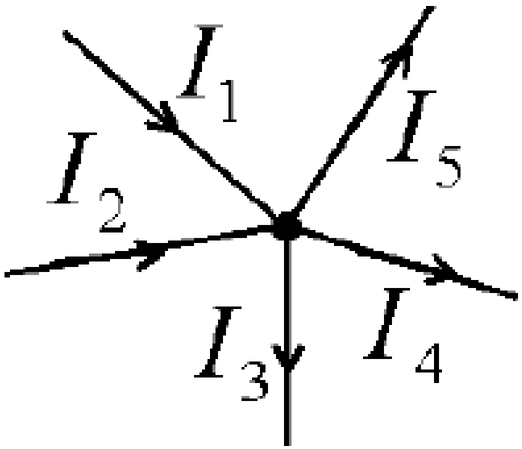
or

$$I_1 + I_3 = I_2 + I_4 + I_5$$

$$(current\ in = current\ out)$$

Conservation of charge (must know in order to solve circuit problems)

Given that we are dealing with steady currents (i.e. no accumulation of charge at circuit junctions), the sum of currents entering a circuit junction is equal to the sum of currents leaving it.



$$I_1 + I_2 = I_3 + I_4 + I_5$$

Taking currents entering circuit junction as positive and currents leaving circuit junction as negative, we have:

$$I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

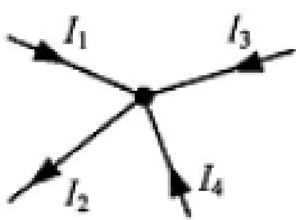
Taking currents leaving circuit junction as positive and currents entering circuit junction as negative, we have:

$$I_3 + I_4 + I_5 - I_1 - I_2 = 0$$

Sample problem 1

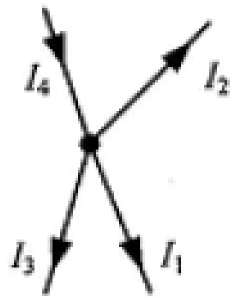
The given diagrams show wires carrying currents I_1, I_2, I_3, and I_4, meeting at a junction. Which of the following diagrams represents the equation $I_1 + I_2 = I_3 + I_4$?

A



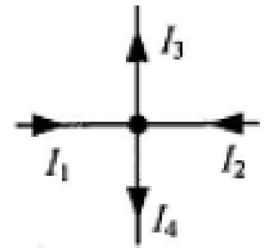
$I_1 + I_3 + I_4 + = I_2$

B



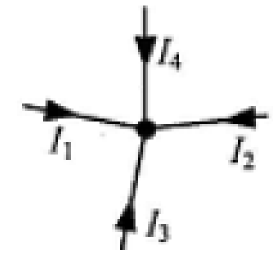
$I_1 + I_2 + I_3 = I_4$

C



$I_1 + I_2 = I_3 + I_4$
(**correct answer**)

D

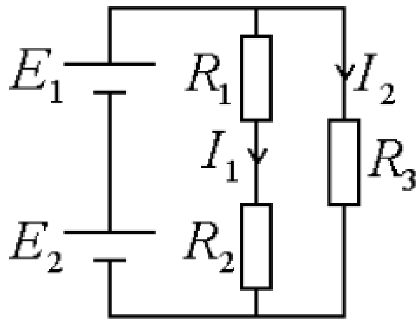


Impossible, all currents are entering and no current leaving.

Conservation of energy (must know in order to solve circuit problems)

The algebraic sum of e.m.f. (i.e. sources of electrical energy) is equal to the algebraic sum of p.d. (i.e. sinks of energy) for any closed loop within the circuit.

Sample problem 2



Find I_1 and I_2 in terms of E_1, E_2, R_1, R_2 and R_3.

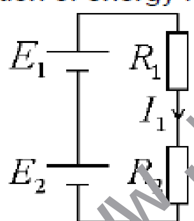
Given that

E_1 = 3.0 V
E_2 = 1.5 V
$R_1 = R_2 = R_3$ = 10 Ω

find the values of I_1 and I_2.

Solution:

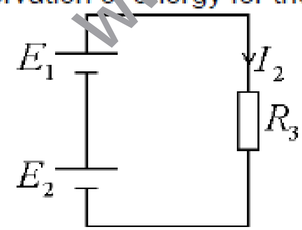
Conservation of energy for the loop:



$$E_1 + E_2 = I_1 R_1 + I_1 R_2 = I_1(R_1 + R_2)$$

$$I_1 = \frac{E_1 + E_2}{R_1 + R_2}$$

Conservation of energy for the loop:



$$E_1 + E_2 = I_2 R_3$$

$$I_2 = \frac{E_1 + E_2}{R_3}$$

Substituting E_1 = 3.0 V, E_2 = 1.5 V and $R_1 = R_2 = R_3$ = 10 Ω,

I_1 = 0.23 A and I_2 = 0.45 A.

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Kirchhoff's Second Law

- **Kirchhoff's Second Law:** The algebraic sum of e.m.f is equal to the algebraic sum p.d for any closed loop within the circuit.
- *This is also known as 'Loop rule'.*

Conservation of Energy and the Kirchhoff's Second Law

Kirchhoff's second rule follows from the law of conservation of energy.

- Let us imagine moving a charge around a closed loop of a circuit. When the charge returns to the starting point, the charge circuit system must have the same total energy as it had before the charge was moved.

- The sum of the increases in energy as the charge passes through some circuit elements must equal the sum of the decreases in energy as it passes through other elements.

- *The potential energy decreases whenever the charge moves through a potential drop -IR across a resistor or whenever it moves in the reverse direction through a source of emf.*
- *The potential energy increases whenever the charge passes through a battery from the negative terminal to the positive terminal.*

Rules for applying Kirchoff's Laws for solving problems

- Currents are labeled with the assumed sense of direction.
- The solution is carried out with the assumed sense of direction and if the actual direction of a particular current is opposite to the assumed direction the value of current will emerge with a negative sign.
- Choose any closed loop in the given network and designate a direction (clockwise or anticlockwise) to traverse the loop for applying the Kirchhoff's II law.
- Go around the loop in the designated direction algebraically adding the potential differences across the resistors (IR Terms) and the source (cells) emf's.
- **SIGNS**: If a resistor is traversed positive first (the end at which the current enters a resistor is positive) then the IR term is taken as negative, similarly for an emf source (say battery) if the positive is encountered first then the emf is taken as negative.
- in order to solve a particular circuit problem, the number of independent equations you need to obtain from the two rules equals the number of unknown currents.

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Find the currents I_1, I_2, and I_3 in the circuit shown in Figure 4 below .

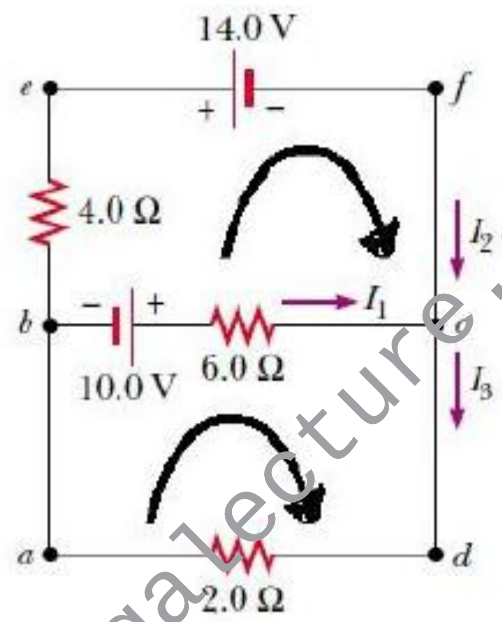


Figure 4: *Example Problem*

- Arbitrarily choose the directions of the currents as labeled in Figure 4
- Applying Kirchhoffs junction rule to junction c gives

$$I_1 + I_2 = I_3 \quad (1)$$

- We now have one equation with three unknowns I_1, I_2, and I_3.

- There are three loops in the circuit *abcda*, *befcb*, and *aefda*.

- Applying Kirchhoffs loop rule to loops *abcda* and *befcb* and traversing these loops clockwise (shown by curved arrows), we obtain the expressions for the closed loop *abcda*

$$10.0V - (6.0\Omega)I_1 - (2.0\Omega)I_3 = 0 \tag{2}$$

for the closed loop *befcb*

$$-14.0V + (6.0\Omega)I_1 - 10.0V - (4.0\Omega)I_2 = 0 \tag{3}$$

which gives

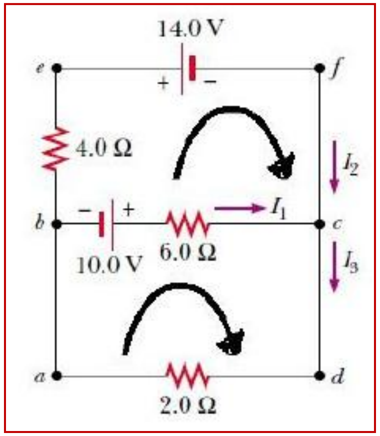
$$(6.0\Omega)I_1 - (4.0\Omega)I_2 = 24.0V \tag{4}$$

- Substituting into Equation $I_1 + I_2 = I_3$ in the equation (2) we get

$$10.0V - (6.0\Omega)I_1 - (2.0\Omega)(I_1 + I_2) = 0$$

which gives

$$10.0V = (8.0\Omega)I_1 + (2.0\Omega)I_2 \tag{5}$$



- solving equations (3) and (4) we get

$$I_1 = 2.0A$$

$$I_2 = -3.0A$$

$$I_3 = -1.0A$$

- To finalize the problem, note that I_2 and I_3 are both negative. This indicates only that the currents are opposite the direction we chose for them. However, the numerical values are correct.

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Potential Divider

The potential divider circuit (fig.) is one of the most useful circuits.The potential divider arrangement can be used to divide the input voltage(V_s) in the ratio that we want. The circuit diagram for a potential divider arrangement is shown below:-

For a potential divider the current through each resistor is the same (why?they are in series,hence).The

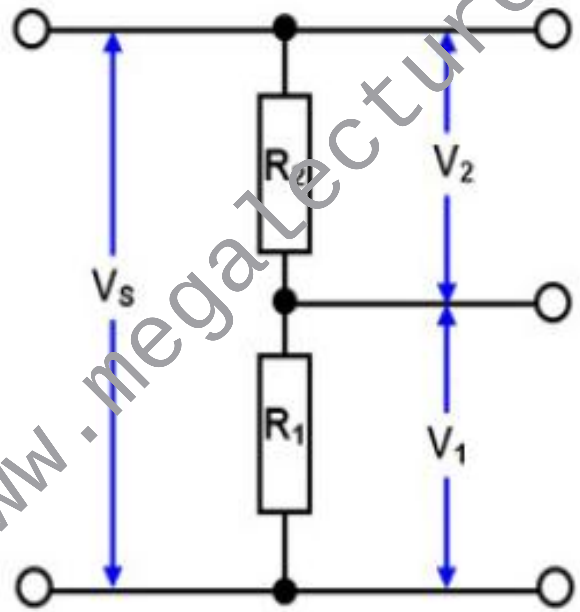

Figure : *A potential Divider circuit*

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Potential Divider

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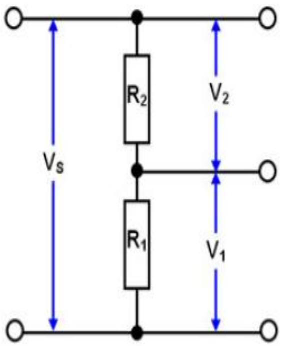
current in the circuit can be found by using the Ohms law and remembering that the total resistance in the circuit is $R_1 + R_2$ we get



$$I = \frac{V_s}{R_1 + R_2} \quad (9)$$

the voltage across the resistor R_1 is given by:-

$$V_1 = IR_1$$

using (9) we get

$$V_1 = IR_1 = \frac{V_s R_1}{R_1 + R_2} \quad (10)$$

Similarly ,the voltage across the resistor R_2 is given by:-

$$V_2 = IR_2$$

using (9) we get

$$V_2 = IR_2 = \frac{V_s R_2}{R_1 + R_2} \quad (11)$$

The ratio V_1 to V_2 can be found as:

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

Application of Potential Divider circuits

Temperature Sensor

A common example of a sensing system is a temperature sensor in a thermostat, which uses a thermistor(A thermistor is a kind of resistor whose resistance decreases as the temperature increases-it is generally made of semiconductors). The thermistor is then used in a potential divider, as in the diagram in fig.8. In this diagram, the potential difference is divided between the resistor and the thermistor. As the temperature rises, the resistance of the thermistor decreases, so the potential difference across it decreases. This means that potential difference across the resistor increases as temperature increases. This is why the voltage is measured across the resistor, not the thermistor. As the source voltage and R are known using the graph below fig.9 the temperature can be found.

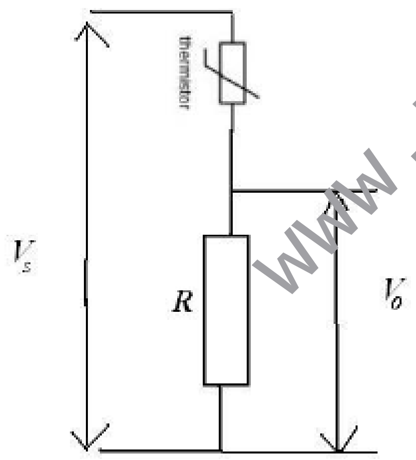


Figure 8: *Thermistor as a temperature sensor*

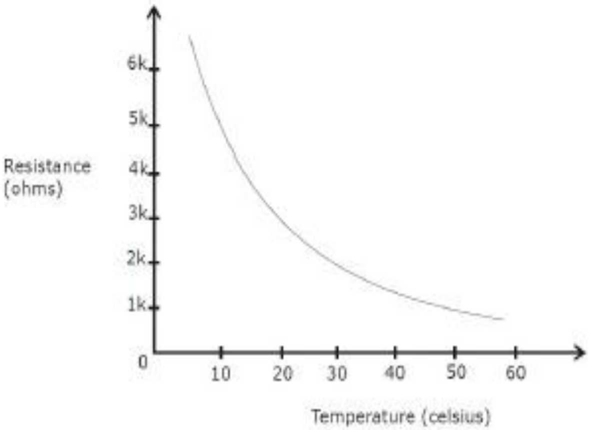


Figure 9: *Temperature Vs resistance for a thermistor*

Application of Potential Divider circuits

Light Dependent Resistors:Light-dependent resistors (***LDRs***) have a resistance which changes in response to changes in light levels, as detected by a photo-sensitive plate on the resistor. Most ***LDRs*** have a negative light coefficient - meaning that their resistance falls as the amount of light falling on them increases. ***LDRs*** are used in light-detection circuits as shown in the figure below using the same technique as in the measurement of temperature the intensity of light can be measured with the circuit shown below .

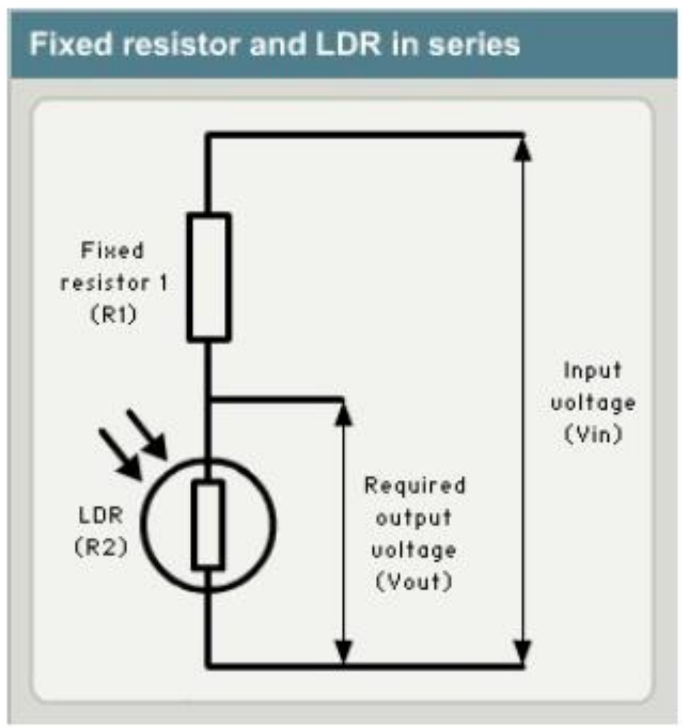


Figure 1[illegible]: LDR used in a potential divider

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Potentiometer

A variable voltage-divider(potentiometer):A variable voltage divider is another form of the potential divider arrangement it is also called the *potentiometer*.Consider a long piece of high resistance wire AB connected to a battery as shown below:-

Between A and J a voltmeter is connected.The point J is a movable contact as the point j is moved

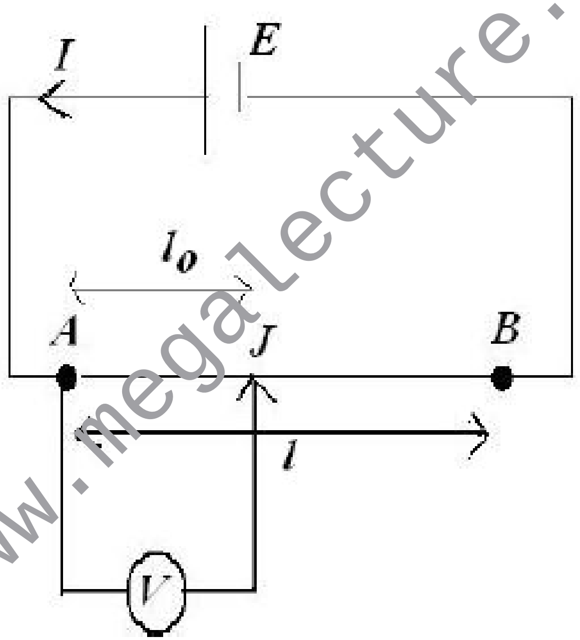

Figure 11: A variable voltage potential divider

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Potentiometer

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the potential drop increases,hence by varying the length of AJ the potential can be varied.Let ρ be the resistivity of the wire AB and let E be Emf of the battery,let l_o be the length of wire AJ and let l be the length AB then

$$E = IR = I\frac{\rho l}{A}$$

The drop V across the length AJ whose resistance is R_o is given by

$$V = IR_o = I\frac{\rho l_o}{A}$$

Dividing the above two equations we get

$$\frac{V}{E} = \frac{l_o}{l}$$

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Potentiometer and its application

Principle of Potentiometer

POTENTIOMETER:

Construction: The potentiometer consists of a long uniform wire usually made of manganin or constantan (high Resistivity low temp coeff of resistance).The ends of the wire are connected to binding screws A and B. A meter scale is fixed on the board. The potentiometer has jockey J with the help of which contact can be made with the potentiometer wire.

Principle: The fall of potential across any portion of the wire is directly proportional to the length of that portion, if the area of cross section is uniform and the current constant. If I is the constant current through the potentiometer wire and if R is the resistance between the wire between A and J Then

$$V = IR$$

$$V = I\frac{\rho}{A}l$$

Note : This equation is going to be used for next application.

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Potentiometer and its application

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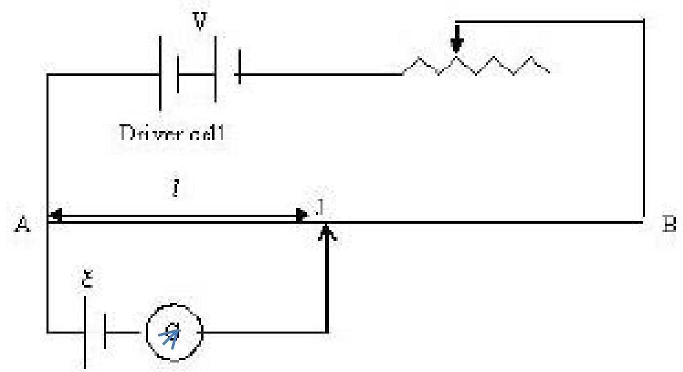

$$V = \phi l$$

where $\phi = I\frac{\rho}{A}$,and ϕ is called the potential gradient.
Hence the potential drop across the length of the potentiometer wire is directly proportional to the length. This is the principle of the potentiometer.

Note : The above equation is used for next application.

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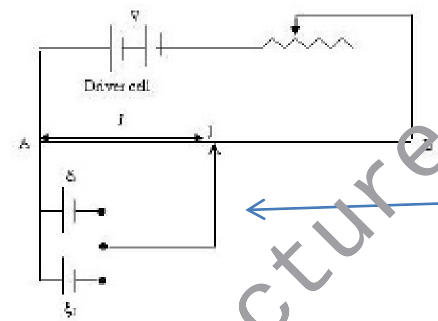
Potentiometer and its application

Comparison of EMF

Comparison of EMF: The potentiometer can used to compare the emf of two cells with the help of the circuit diagram shown below. Let the emf's of the cells to be compared be ϵ_1 and ϵ_2 let the driver cell potential be V. The cells are connected to the potentiometer through a two way key. First the key



Note : Galvanometer is connected along with Jockey

put in the two way key such that the cell with emf ϵ_1 is connected to the circuit.The jockey(movable contact) is moved along the wire till at a point there is no deflection in the galvanometer , at a distance l_1 from A.

The EMF of the cell is proportional to the balancing length

$$\epsilon_1 = \phi l_1$$

similarly when the second cell is connected

$$\epsilon_2 = \phi l_2$$

dividing the equations

$$\frac{\epsilon_1}{\epsilon_2} = \frac{l_1}{l_2}$$

Note:(i)The Driver cell potential should always be greater than the potential of the cells whose emf's are compared.(ii)If one cells emf is accurately known the other cells emf can be determined.