

Oscillations & Waves

Superposition

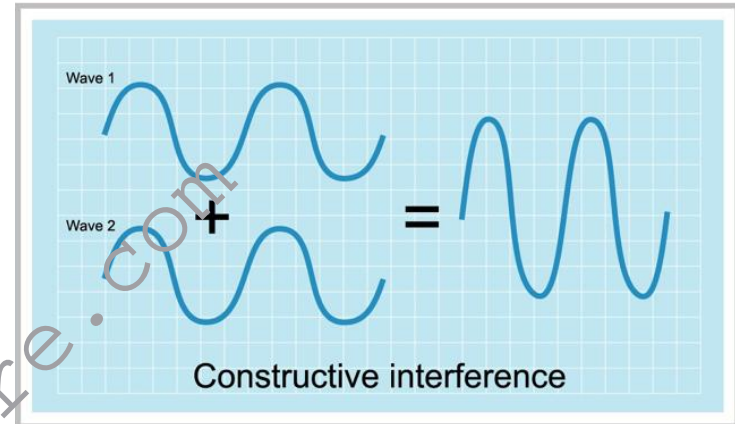
Marline Kurishingal

Recap....

- Refer Ripple tank experiment
- Demonstration of Ripple tank experiment to show wave properties.
- <http://www.youtube.com/watch?v=-8a61G8Hvi0>

- Refer the figure on right with two waves arriving at a point at the same time in opposite directions.

- If they **arrive in Phase** – that is, if their crests arrive at exactly the same time – they will interfere constructively.



- A resultant wave will be produced which has crests much higher than either of the two individual waves and troughs which are much deeper.

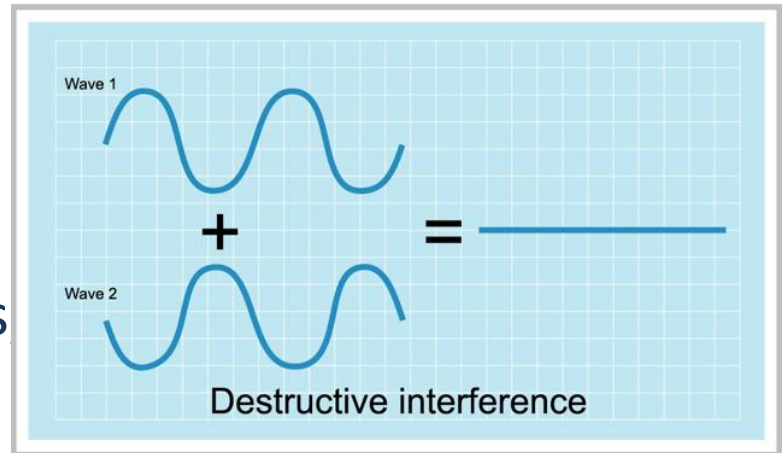
- If the 2 incoming waves have the same frequency and equal amplitude A , the resultant wave produced by constructive interference has an amplitude of $2A$.

- The frequency of the resultant is the same as that of incoming waves.

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Destructive Interference

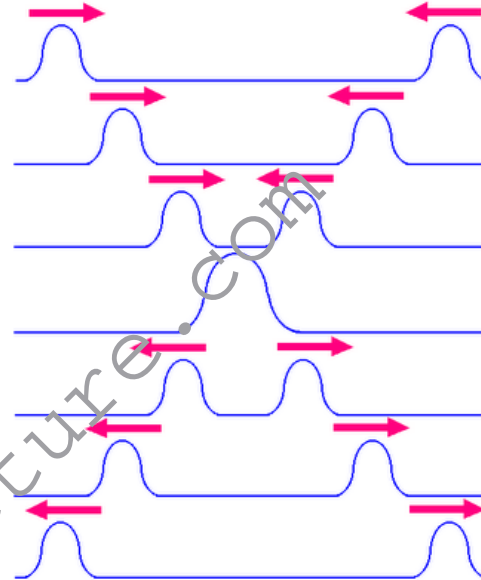
- Refer the figure on right with two waves arriving at a point at the same time.
- If they **arrive out of Phase** – that is if the crests of one wave arrive at same time as the troughs from the other – they will interfere destructively.



- A resultant wave will have a smaller amplitude. (based on case to case)
- In the case shown in figure where the incoming waves have equal amplitude, the resultant wave has zero amplitude.

Interference and Superposition of Waves

- When two waves meet they will interfere and superpose. After they have passed they return to their original forms. This is true if they are coherent or not.



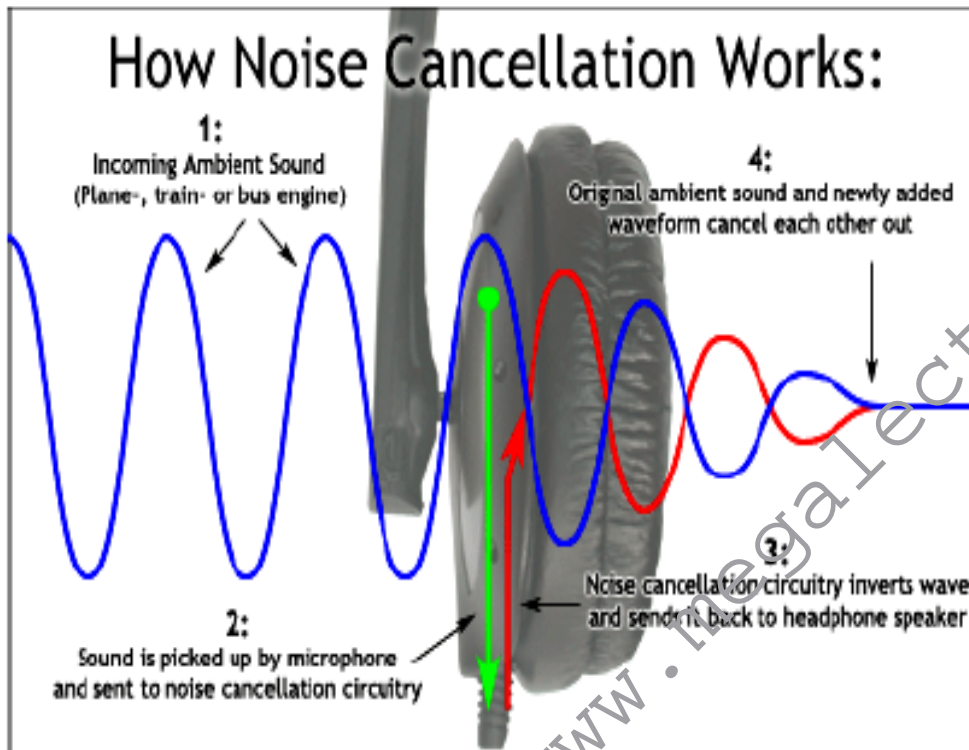
- At the point they meet, the two waves will combine to give a resultant wave whose amplitude (or intensity) may be greater or less than the original two waves.
- The resultant displacement can be found by adding the two displacements together. This phenomenon leads to the **Principle of Superposition**.

The principle of Superposition

- The Principle of Superposition states that when two or more waves meet at a point, the resultant displacement at that point is equal to the sum of the displacements of the individual waves at that point.

Note : Displacement is a vector, so remember to add the individual displacements taking account of their directions.

Application of the Principle of Superposition



Active Noise Cancellation

The muffling of ambient noise using insulating material in the headphones is called *passive* noise cancellation.

Active noise cancellation utilizes the principle of superposition to pick up the ambient noise, inverts the wave and generates this sound wave within the headphone. This inverted wave cancels the ambient noise, preserving only the sound waves that the listener wants to hear.

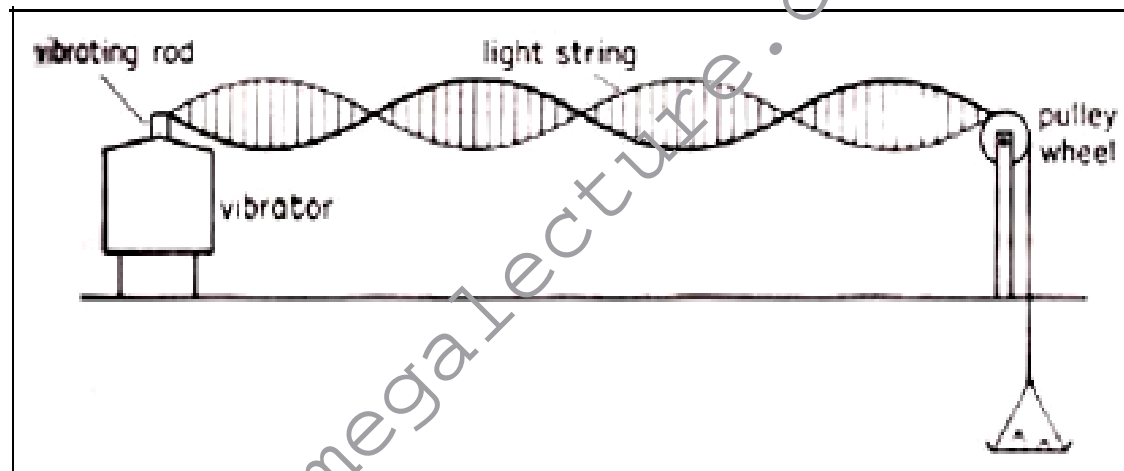
Note : The transverse shown in above figure is for the demo of cancellation of noise only. Remember, sound waves are to be represented in longitudinal form.

Stationary Wave

- A stationary wave is set up by the **superposition of two progressive waves** of the same type, amplitude and frequency travelling in opposite directions.
- A stationary (or standing) wave is one in which some points are permanently at rest (**nodes**), others between these nodes are vibrating with varying amplitude, and those points with the maximum amplitude (**antinodes**) are midway between the nodes.

Melde's apparatus to illustrate stationary waves in stretched string

By adjusting the weight of the scale pan, stationary waves are set up as shown below.



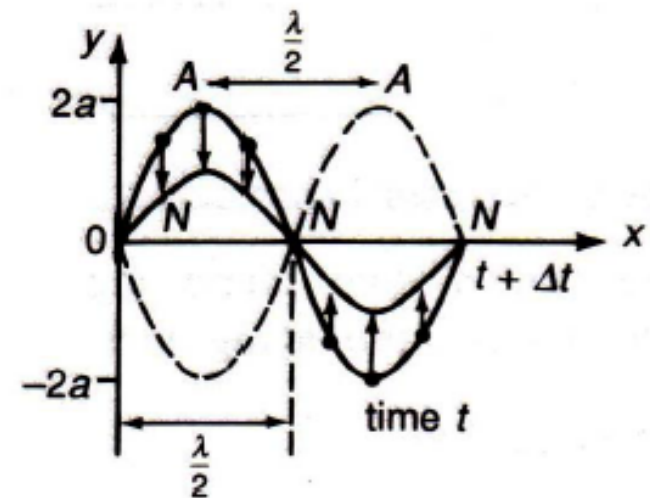
<http://www.youtube.com/watch?v=4BoeATJk7dg>

Uses and application of Melde's experiment (for your information only)

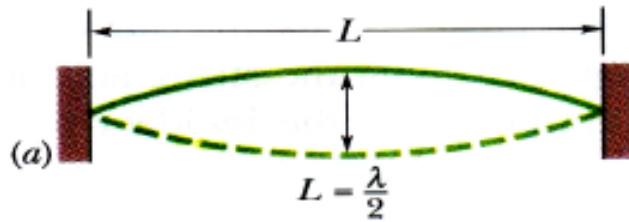
- Melde's experiment teaches us creation of standing waves.
- One can create a great product of neutralising the sounds by creating sounds with same wave length and frequency as the source.
- For example : If we know exactly the frequency of any machine (say an aeroplane flying over your building every day during take off and landing) and if we can measure the wave length of sounds that machine creates..create a product that can create similar waves in opposite direction, so that they undergo mechanical interference and the machine sound is neutralised.

Characteristics of Stationary Waves

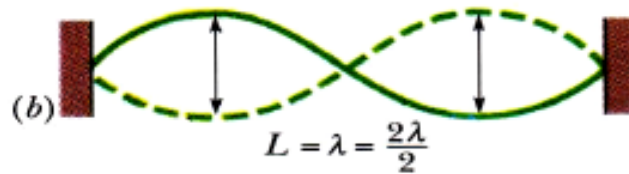
- i. There is no sign of any progressive wave in either direction.
- ii. Individual particles are oscillating with the same frequency, except at the nodes.
- iii. The amplitudes of oscillation of the particles vary from a maximum at the antinodes (A) to zero at the nodes (N).
- iv. All particles in the same segment or loop (region between 2 adjacent nodes) are vibrating **in phase**. Adjacent segments are **anti-phase**.
- v. Adjacent nodes or adjacent antinodes are half a wavelength apart, i.e. $NN = AA = \frac{\lambda}{2}$
- vi. A node and the next antinode are $\frac{\lambda}{4}$ apart.
- vii. Energy is trapped (stored) in stationary waves, since there is no energy is transferring away.



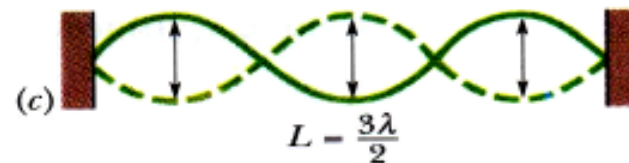
Formation of Stationary Waves in Strings



Fundamental mode
or 1st harmonic



2nd harmonic



3rd harmonic

Let a string be stretched between two clamps separated by a fixed distance L .

When the string is plucked, struck or bowed, it can vibrate in several modes simultaneously.

The simplest possible pattern of the stationary wave consists of one loop (**fundamental mode**), with the nodes at the 2 ends of the string - diagram (a).

The next simplest pattern has 2 loops, and the next has 3 loops...etc.

In short, just add nodes between the 2 ends, for subsequent modes of vibration.

Formation of Stationary Sound Waves in air in Pipes

1. Stationary sound waves in air can be formed in both closed and open pipes.
2. In a **closed** pipe, when a sound wave is originated from the open end, the sound wave propagated into the pipe is reflected by the cylindrical wall and from the closed end. A stationary wave is then formed.

At **resonance**,

a) node N is always formed at the closed end of the pipe

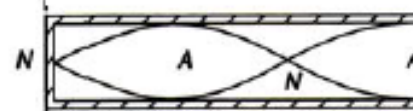
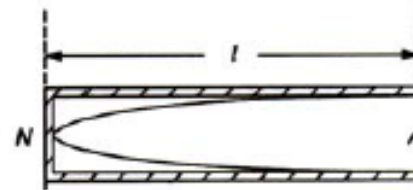
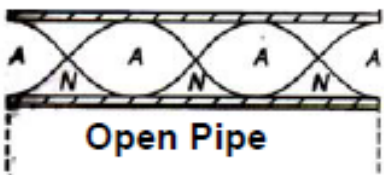
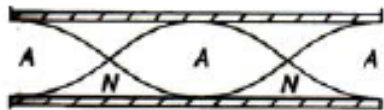
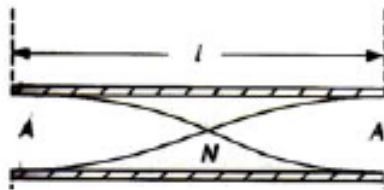
- the air layer at this end is permanently at rest

b) antinode A is formed at the open end of the pipe

- the air layer at this end is free to vibrate

3. In an **open** pipe, when a sound wave is originated from the open end, the sound wave propagated into the pipe to the other open end where it is reflected by the walls and on encountering air at the other open end. A stationary wave is set up.

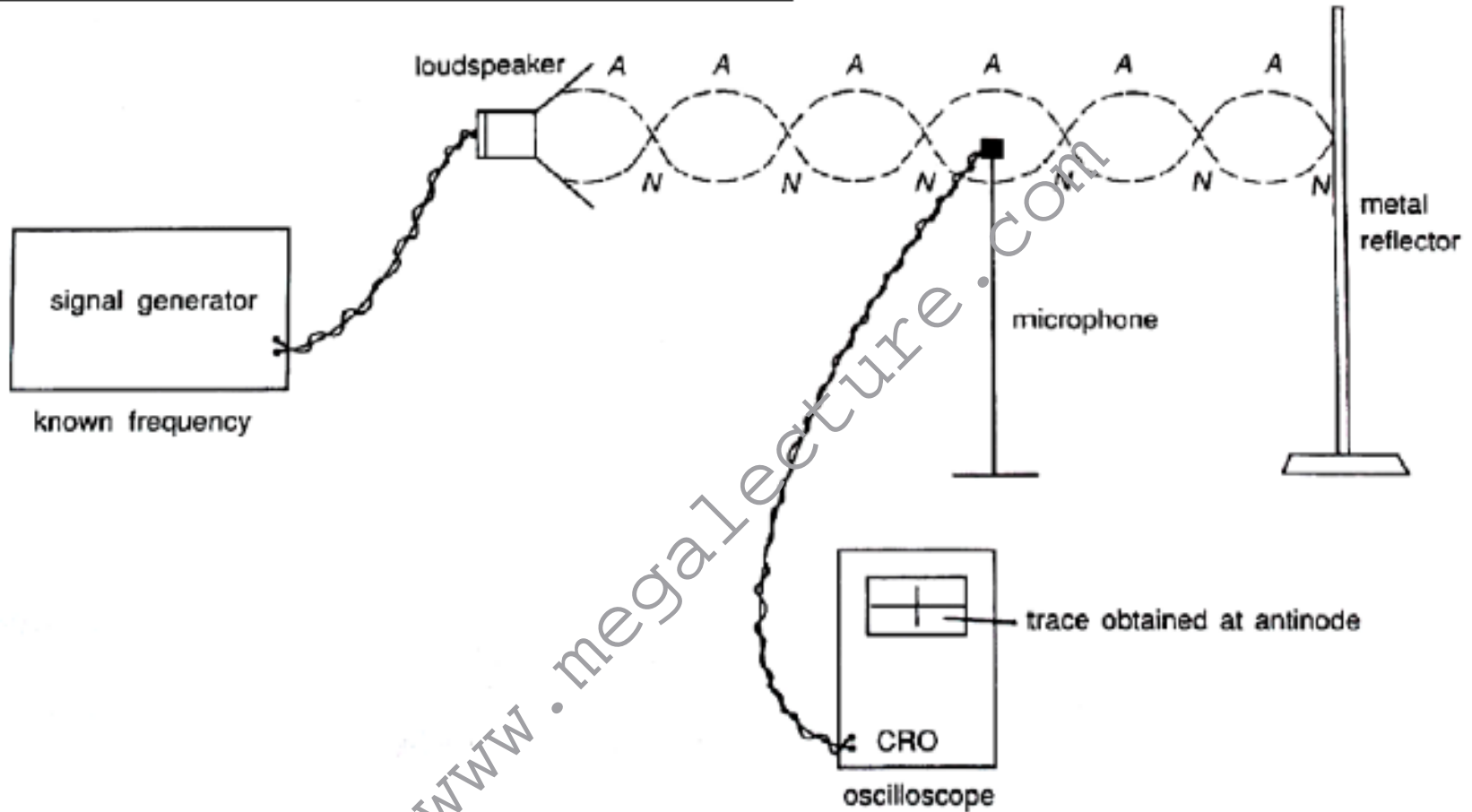
At **resonance**, since the ends of the pipe are open, **both ends are antinodes**.



Closed Pipe
(at one end)

The **resonance** of sound in pipe may be **varied** by changing either the **frequency** of the sound wave, or **length** of the pipe.

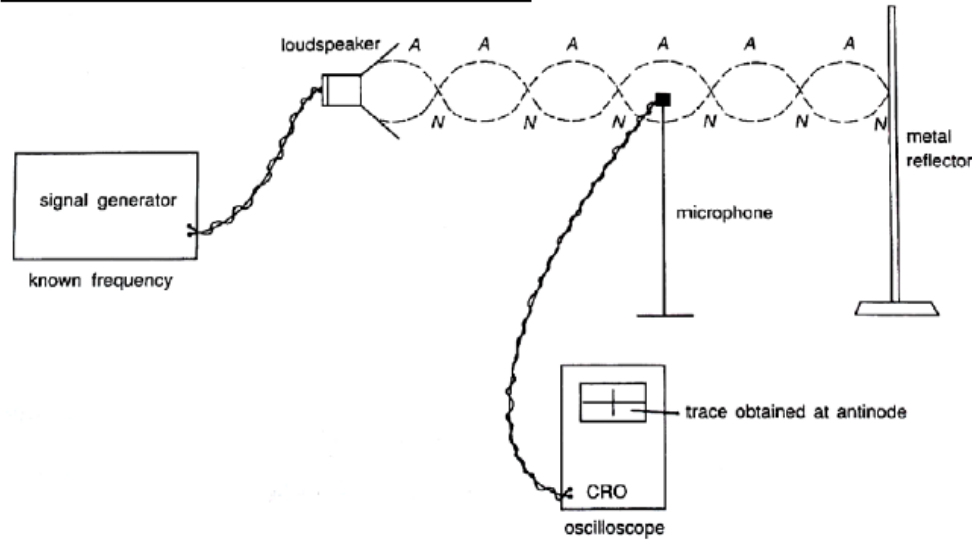
Formation of Stationary Sound Waves in air



Stationary sound waves in air can be demonstrated using the set-up as shown above.

Note : In A level Physics syllabus, this part appears in previous chapter 'waves'.

Formation of Stationary Sound Waves in air



Stationary sound waves in air can be demonstrated using the set-up as shown above.

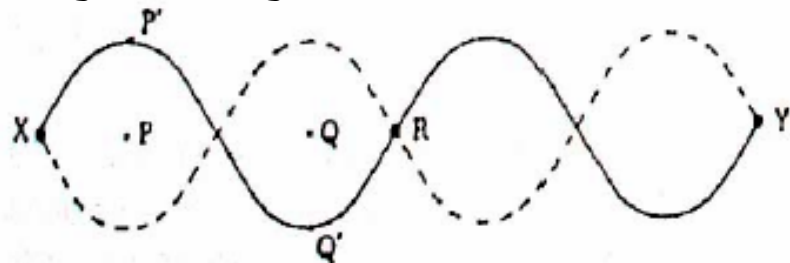
- i. The incident sound wave is generated by the loudspeaker attached to a signal generator.
- ii. The incident wave is reflected by the metal reflector (which must be appropriately positioned at a node). The reflected wave and the incident wave superpose to form a stationary wave.
- iii. The detection of nodes and antinodes is done using a microphone attached to a CRO.
- iv. By moving the microphone slowly forward and backward, the vertical trace (or amplitude) on the CRO screen is seen to vary from minimum to maximum, indicating the positions of nodes and antinodes.

- A] To measure the **wavelength** of the sound wave:
Measure the distance moved by the microphone, d , between 2 successive maxima or minima (e.g. $d = 33$ cm).
Since this corresponds to $\frac{\lambda}{2}$, the wavelength $\lambda = 2d = 2(33 \text{ cm}) = \mathbf{66 \text{ cm}}$
- B] To measure the **frequency** of the sound wave:
By measuring the period, T , of the sound wave, the frequency can be determined.
Set the time base of the CRO to a suitable value (e.g. 0.5 ms cm^{-1}). Place the microphone where the CRO shows a sinusoidal trace. The period, T , is determined by measuring the distance between 2 crests or 2 troughs, (e.g. 4 cm).
 \therefore the period, $T = 4 \text{ cm} \times 0.5 \text{ ms cm}^{-1}$
 $= 2 \text{ ms} = 2 \times 10^{-3} \text{ s}$
Hence the frequency, $f = \frac{1}{T} = \frac{1}{2 \times 10^{-3}} = \mathbf{500 \text{ Hz}}$
- C] Hence, **speed** of sound can be calculated from $\mathbf{v = f\lambda = (500)(66 \times 10^{-2}) = 330 \text{ m s}^{-1}}$

Sample problem 1

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A string is stretched under constant tension between fixed points X and Y. The solid line shows a stationary wave at an instant of greatest displacement. The broken line shows the other extreme displacement.



Which one of the following statements is correct?

- A The distance between P and Q is one wavelength.
- B A short time later, the string at R will be displaced.
- C The string at P' and the string at Q' will next move in opposite directions to one another.
- D At the moment shown, the energy of the standing wave is all in the form of kinetic energy.
- E The standing wave shown has the lowest possible frequency for this string stretched between X and Y under this tension.

Solution:

- A Incorrect. For a stationary wave, one wavelength contains two complete “loops” (i.e. distance XR). Hence, PQ is only $\frac{1}{2}$ wavelength.
- B Incorrect. R is a node, hence it will permanently be at rest.
- C **Correct!** In a stationary wave, particles in adjacent loops will always be moving opposite to each other (i.e. P' will move down, and Q' will move up).
- D Incorrect. The energy is alternating between KE and PE, depending on the location of each particle in the wave.
- E Incorrect. The lowest possible frequency is the fundamental frequency, in which there

Sample problem 2

A boy blows gently across the top of a piece of glass tubing the lower end of which is closed by his finger so that the tube gives its fundamental note of frequency, f . While blowing, he removes his finger from the lower end. The note he then hears will have a frequency of approximately

- A** $f/4$ **B** $f/2$ **C** f **D** $2f$ **E** $4f$

Solution

At resonance, for a closed tube (with one end closed), the stationary wave (at fundamental frequency) formed is shown in (1). When the lower end is removed (2 open ends), the stationary wave (in fundamental mode) formed looks like (2).

In (1) : $L = \frac{1}{4} \lambda \rightarrow \lambda = 4L$

Given that the frequency = f

Using $v = f\lambda \rightarrow f = \frac{v}{\lambda} = \frac{v}{4L}$

(Ans: D)

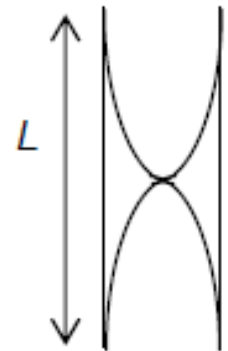


(1)

In (2) : $L = \frac{1}{2} \lambda \rightarrow \lambda = 2L$

If the frequency = f_2

Using $v = f\lambda \rightarrow f_2 = \frac{v}{\lambda} = \frac{v}{2L} = 2f$



(2)

Sample problem 3

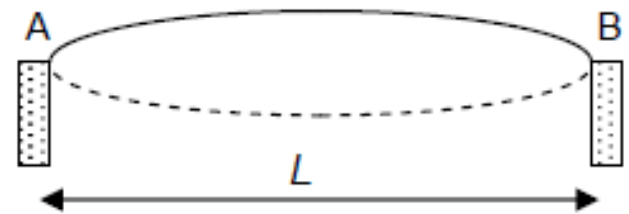
A suspension bridge is to be built across a valley where it is known that the wind can gust at 5 s intervals. It is estimated that the speed of transverse waves along the span of the bridge would be 400 m s^{-1} .

The danger of resonant motions in the bridge at its fundamental frequency would be greatest if the span had a length of _____ m.

Solution:

In fundamental mode, the stationary wave has a single “loop” at resonance, with nodes at the 2 ends.

Hence, if the length of the bridge, L , is equal to $\frac{1}{2} \lambda$, the bridge will resonate with the fundamental frequency.



Given that the wind gusts at 5 s intervals $\rightarrow f_{\text{driver}} = \frac{1}{T} = \frac{1}{5} = 0.2 \text{ Hz}$.

Using $v = f\lambda \rightarrow \lambda = \frac{v}{f} = \frac{400}{0.2} = 2000 \text{ m}$

Hence, the bridge has the danger of resonating (fundamental mode) if $L = \frac{1}{2} \lambda = \underline{\underline{1000 \text{ m}}}$

Sample problem 4

An organ pipe of effective length 0.6 m is closed at one end.

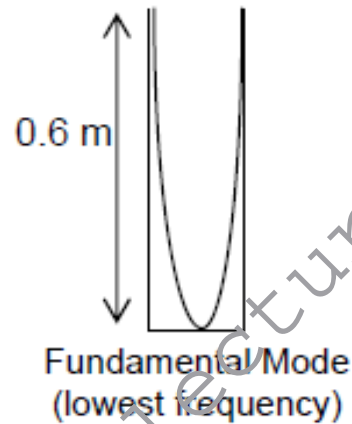
Given that the speed of sound in air is 300 m s^{-1} , the two lowest resonant frequencies are

Solution:

For fundamental mode:

$$\frac{1}{4} \lambda = 0.6 \text{ m} \rightarrow \lambda = 2.4 \text{ m}$$

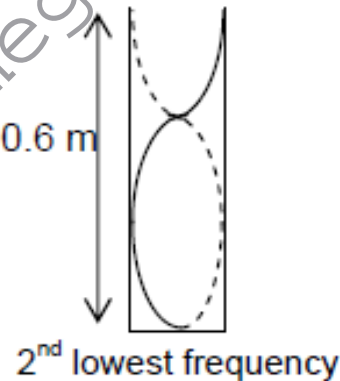
$$f_{\text{fundamental}} = \frac{v}{\lambda} = \frac{300}{2.4} = \mathbf{125 \text{ Hz}}$$



For 2nd lowest frequency:

$$\frac{3}{4} \lambda = 0.6 \text{ m} \rightarrow \lambda = 0.8 \text{ m}$$

$$f_{\text{2nd lowest}} = \frac{v}{\lambda} = \frac{300}{0.8} = \mathbf{375 \text{ Hz}}$$



Hence the two lowest resonant frequencies are **125 Hz** & **375 Hz**.

Sample problem 5

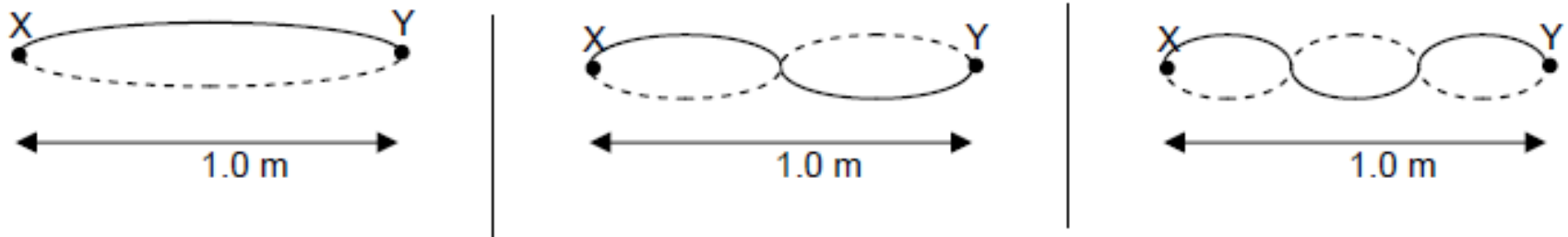
A taut wire is clamped at two points 1.0 m apart. It is plucked near one end.

What are the 3 longest wavelengths present on the vibrating wire?

Solution

Step 1: Draw the 3 modes corresponding to the 3 longest wavelengths (between X & Y).

Step 2: Since the wire is clamped at the 2 points (X & Y), they must be nodes.



Step 3: Relate the length of wire to the corresponding wavelength, λ , in each case.

$$\begin{aligned} \frac{1}{2} \lambda &= 1.0 \text{ m} \\ \rightarrow \lambda &= \underline{\underline{2.0 \text{ m}}} \end{aligned}$$

$$\lambda = \underline{\underline{1.0 \text{ m}}}$$

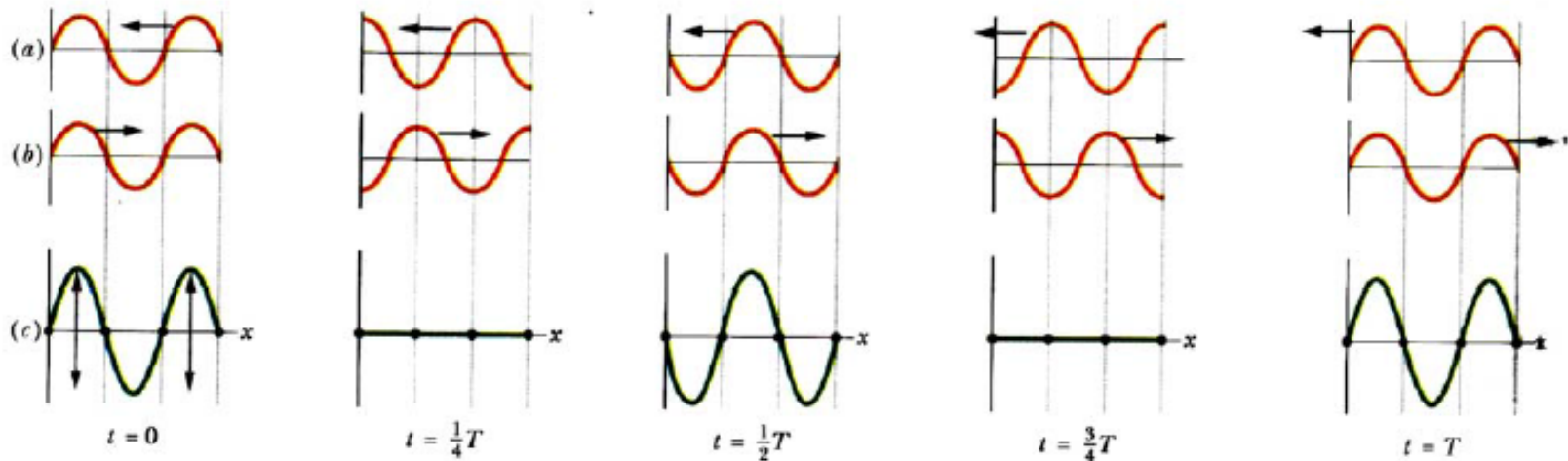
$$\begin{aligned} (1.5)\lambda &= 1.0 \text{ m} \\ \rightarrow \lambda &= \underline{\underline{0.67 \text{ m}}} \end{aligned}$$

The formation of a stationary wave using a graphical method

Nodes and Antinodes

Formation of a stationary (standing) wave

A stationary wave is formed when **two progressive waves** of the same type, wavelength and amplitude travel in **opposite directions** superpose in the same medium.



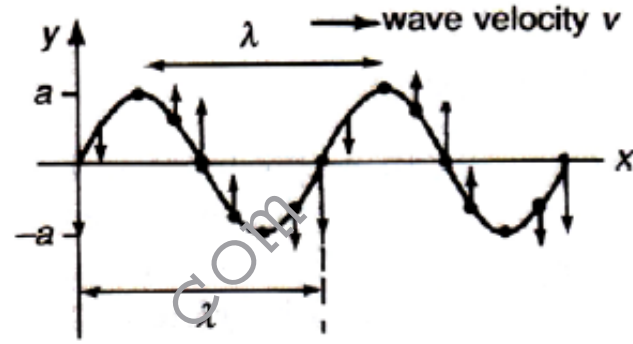
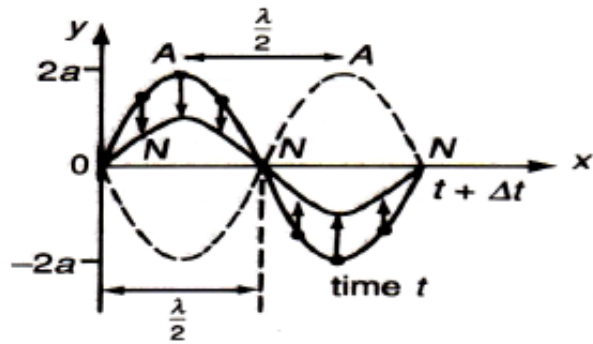
In the diagrams above, of the two progressive waves in a string, one is travelling to the left (a), and the other to the right (b).

The resultant wave (c) is a stationary wave, obtained by applying the superposition principle.

Note :

- there are **positions along the string which do not move** – called **nodes** (marked by dots)
- halfway between successive nodes are **antinodes**, where the **amplitude** of the **resultant wave is maximum** (double the amplitude of the individual waves).
- wave patterns shown in diagram (c) are those of a **stationary** or **standing wave** because the wave patterns do not move left or right (i.e. the positions of the nodes and antinodes do not change).

Comparison between Stationary and Progressive Wave Motions



	Stationary Wave	Progressive Wave
Amplitude	Varies according to position, from zero at the nodes (permanently at rest) to a maximum of $2a$ at the antinodes.	Is the same for all particles in the path of the wave (amplitude = a).
Frequency	All particles vibrate in SHM with the same frequency as the wave (except for those at the nodes which are at rest).	All particles vibrate in SHM with the frequency of the wave.
Wavelength	$2 \times$ (distance between a pair of adjacent nodes or antinodes) = $2NN = 2AA$.	Distance between adjacent particles which have the same phase.
Phase	Phase of all particles between 2 adjacent nodes is the same.	All particles within one wavelength have different phases.
Waveform	Does not advance. (The curved string becomes straight twice in each period.)	Advances with the velocity of the wave.
Energy	No transfer away of energy, but there is energy associated with the wave.	Energy is transferred in the direction of travel of the wave.

Sample problem 6

A standing wave is set up on a stretched string XY as shown in the diagram. At which point(s) will oscillation be exactly in phase with that at point P?

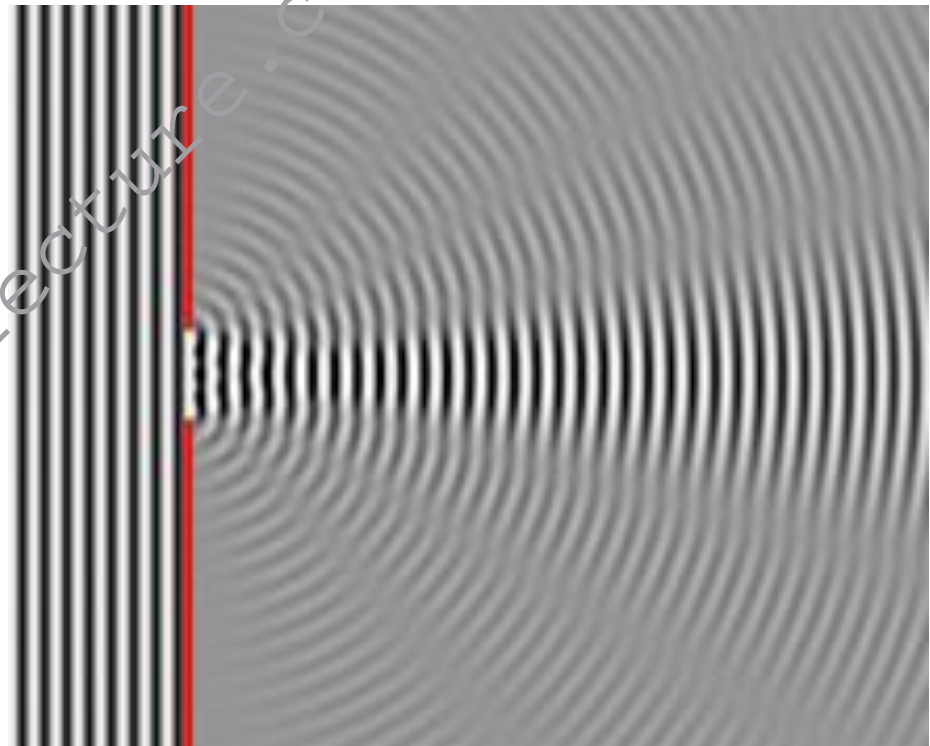


Solution:

In a stationary wave, all points between 2 successive nodes are in phase. In this case, 1 & 2 are in phase with each other, but are in anti-phase with P. Hence, only 3 is in phase with P.

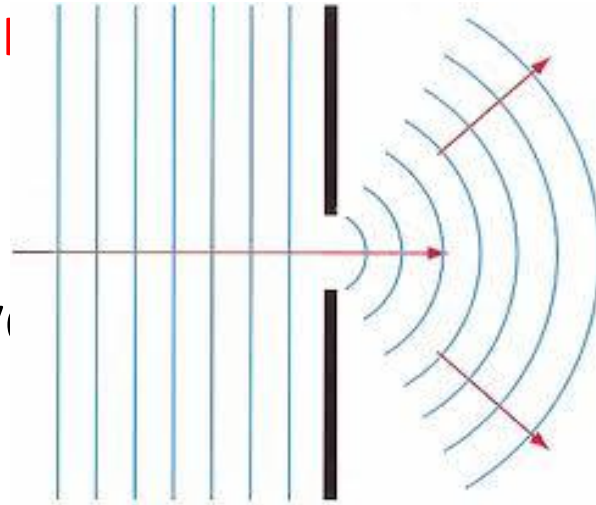
Diffraction

- **Diffraction** is the spreading of waves through an aperture or round an obstacle.
- It is observable when the width of the aperture is of the same order of magnitude as the wavelength of the waves.



Diffraction (continued)

- The **extent of the diffraction effect** is dependent on the relative sizes of the aperture to the wavelength of the wave



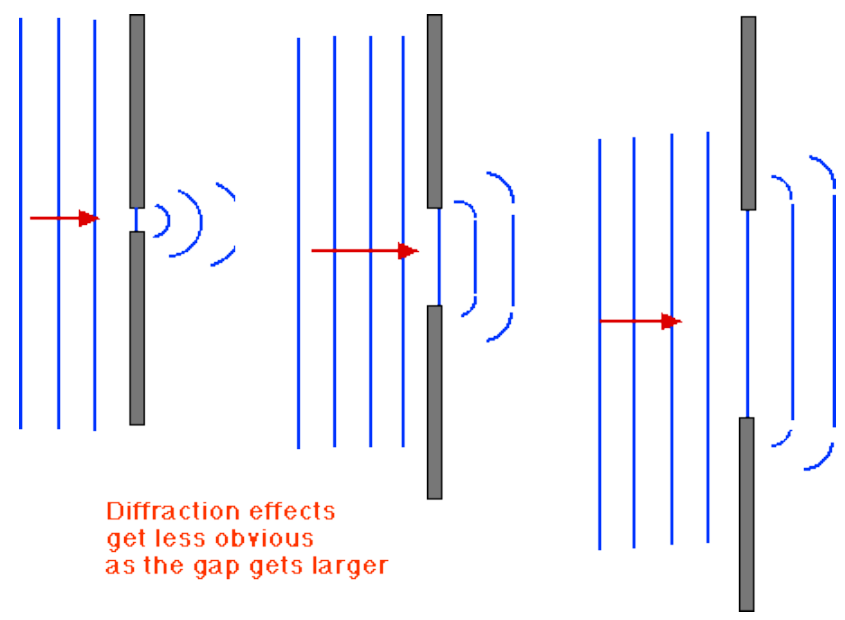
- The smaller the size of the aperture, the greater the spreading of the waves (if the width of the aperture is about the same size as the wavelength, λ , the diffraction effect is very considerable).
- *Size of the aperture refers to the width of the slit or gap.*

Experiments which demonstrate diffraction including the diffraction of water waves in a ripple tank with both a wide gap and a narrow gap

Note : Huygens' explanation of Diffraction is not mentioned in syllabus.

Generally, the bigger the wavelength in relation to the width of the aperture, the greater is the spreading or diffraction of the waves.

- The diagrams below show the plan view of diffraction of plane water waves through gaps of different width, in a ripple tank. Note that the wavelengths **do not change after passing through the gap**.



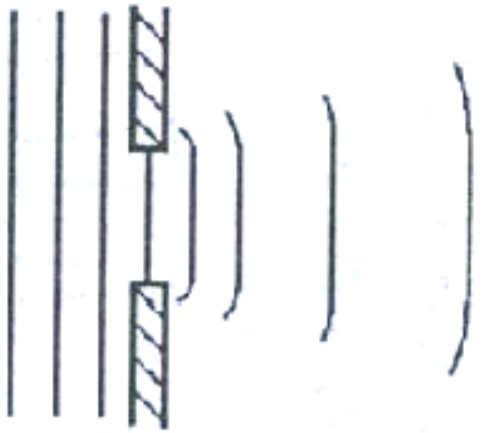
- It is the **relative sizes** of the aperture to the **wavelength** that is important.
- <http://www.acoustics.salford.ac.uk/feschools/waves/diffract.htm>

Application of Diffraction

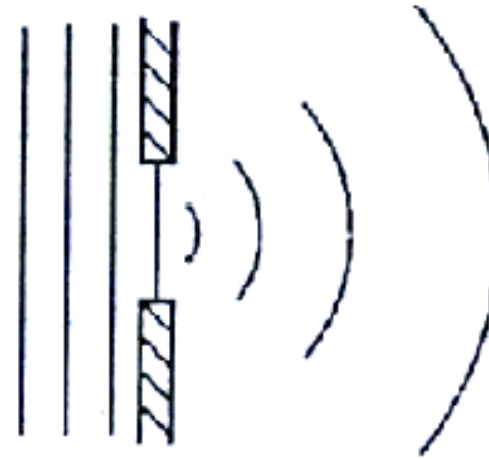
- The forms of jetties are used for directing currents and they are constructed sometimes of high or low solid projections.



The diagrams below are **INCORRECT!** Why?



(c)



(d)

In (c), diffraction effect is right, but wavelength increases, which is incorrect.

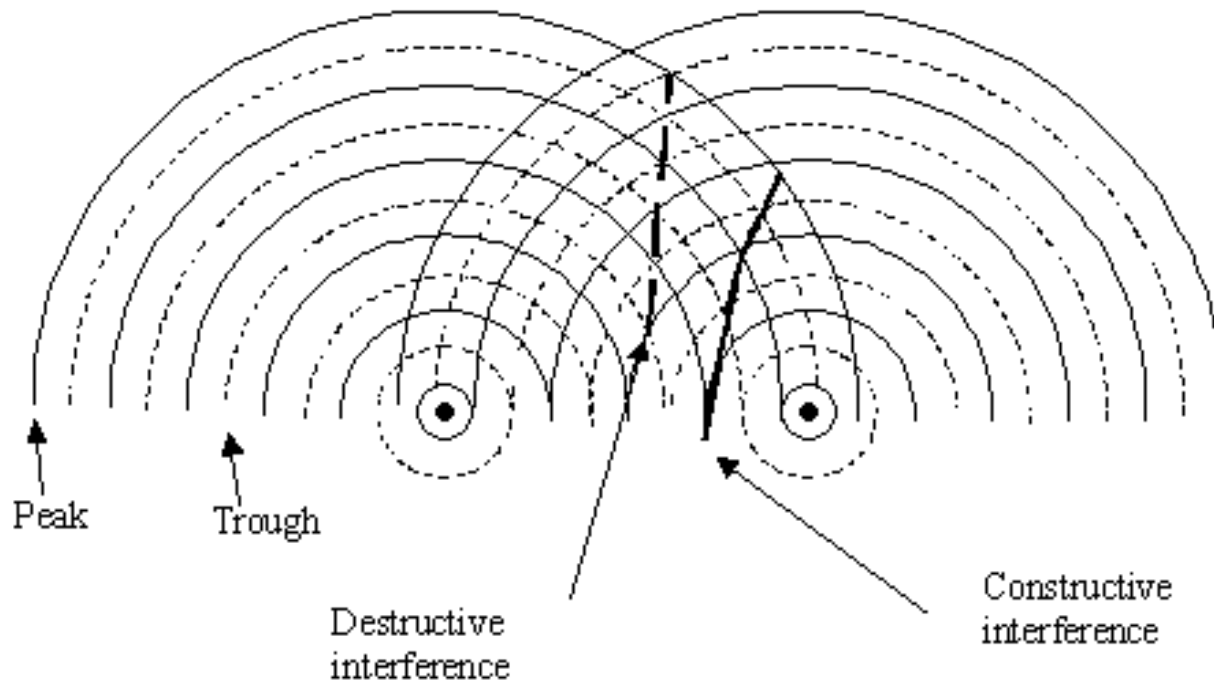
In (d), diffraction effect is too much for the given large slit size and the wavelength should not be increasing.

Note : Huygen's explanation of diffraction is not mentioned in syllabus

Interference

Interference

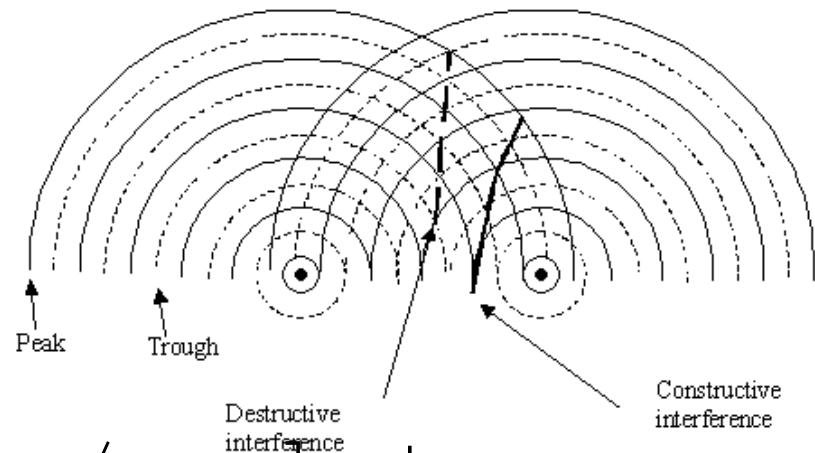
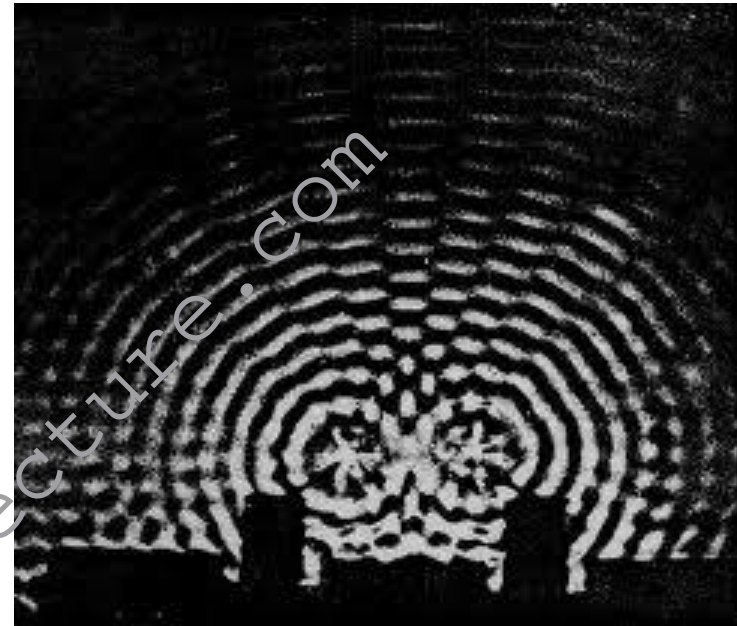
- **Interference** is the superposing of two or more waves to give a resultant wave whose displacement is given by the Principle of Superposition.



- Watch Demo (Name is Double slit coherent wave interference patterns)
- <http://www.youtube.com/watch?v=dNx70orCPnA>

Interference (continued)

- At regions of maxima, constructive interference occurs (i.e. the waves arrive at these points in phase), resulting in maxima amplitude, hence high intensity.
- At regions of minima, destructive interference occurs (i.e. the waves arrive at these points in anti-phase), resulting in minima or zero amplitude, hence low or zero intensity.



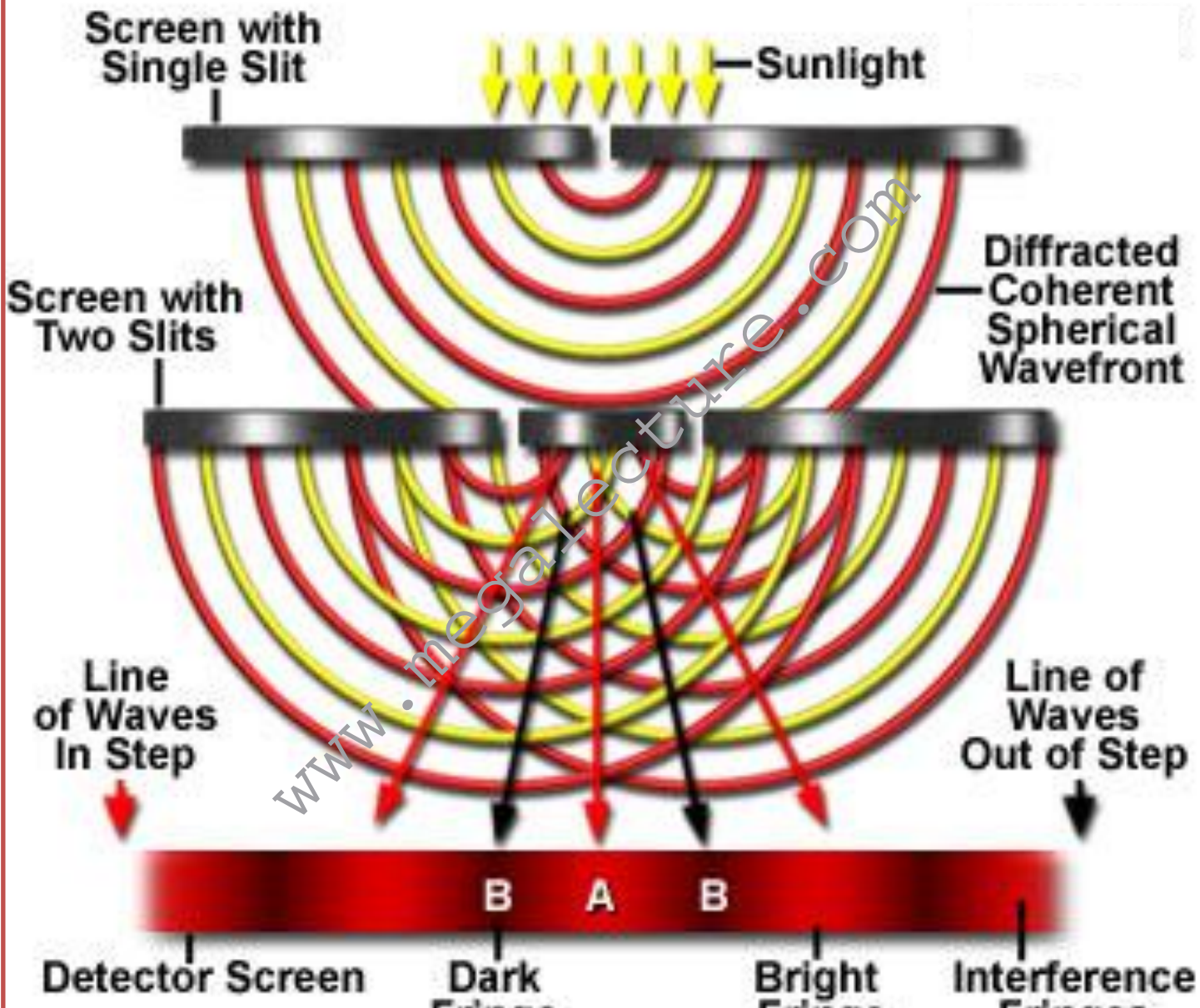
Experiments that demonstrate Two-source Interference

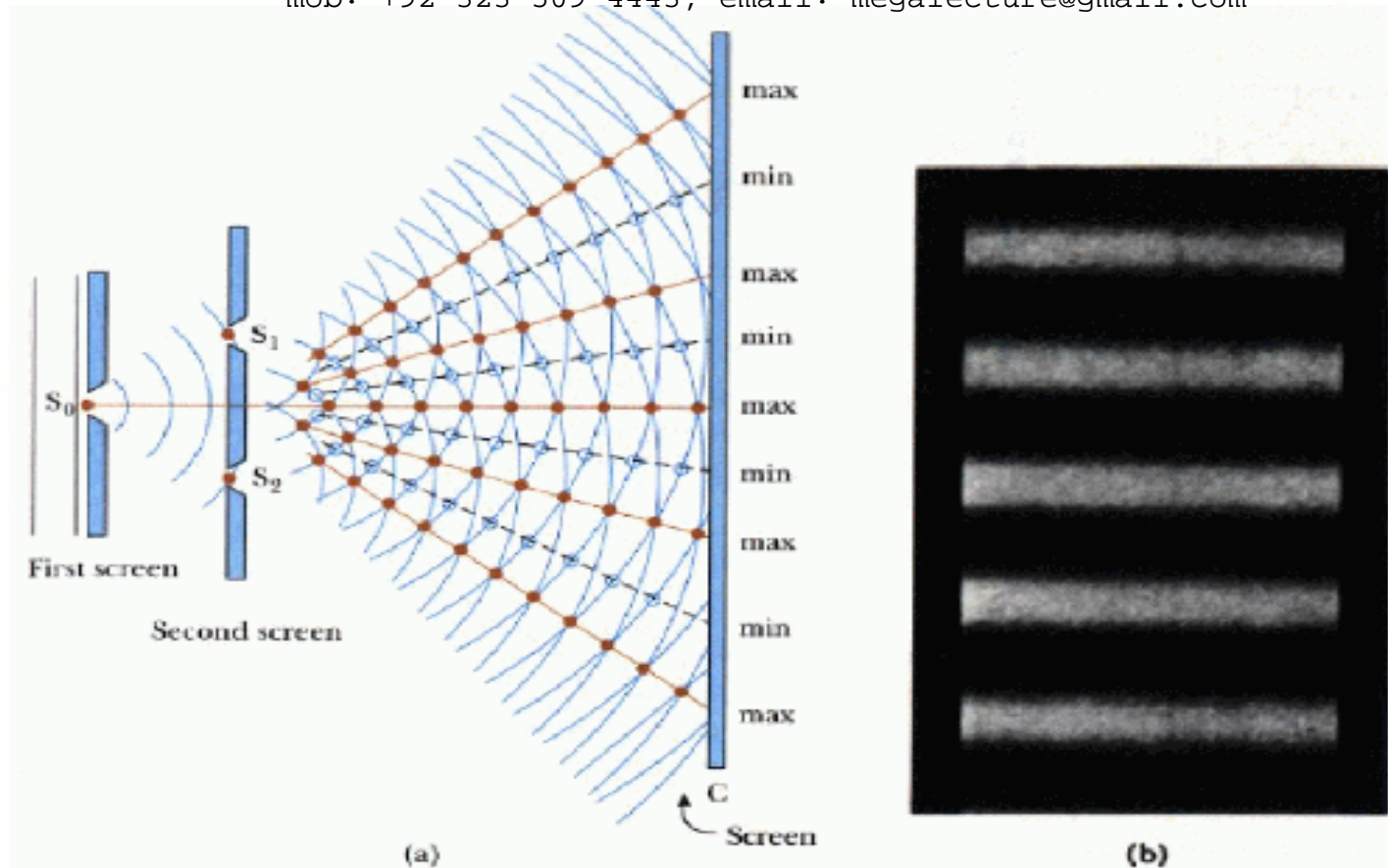
Reference :

<http://www.youtube.com/watch?v=9UkkKM1IkKg>

Thomas Young's Double Slit Experiment

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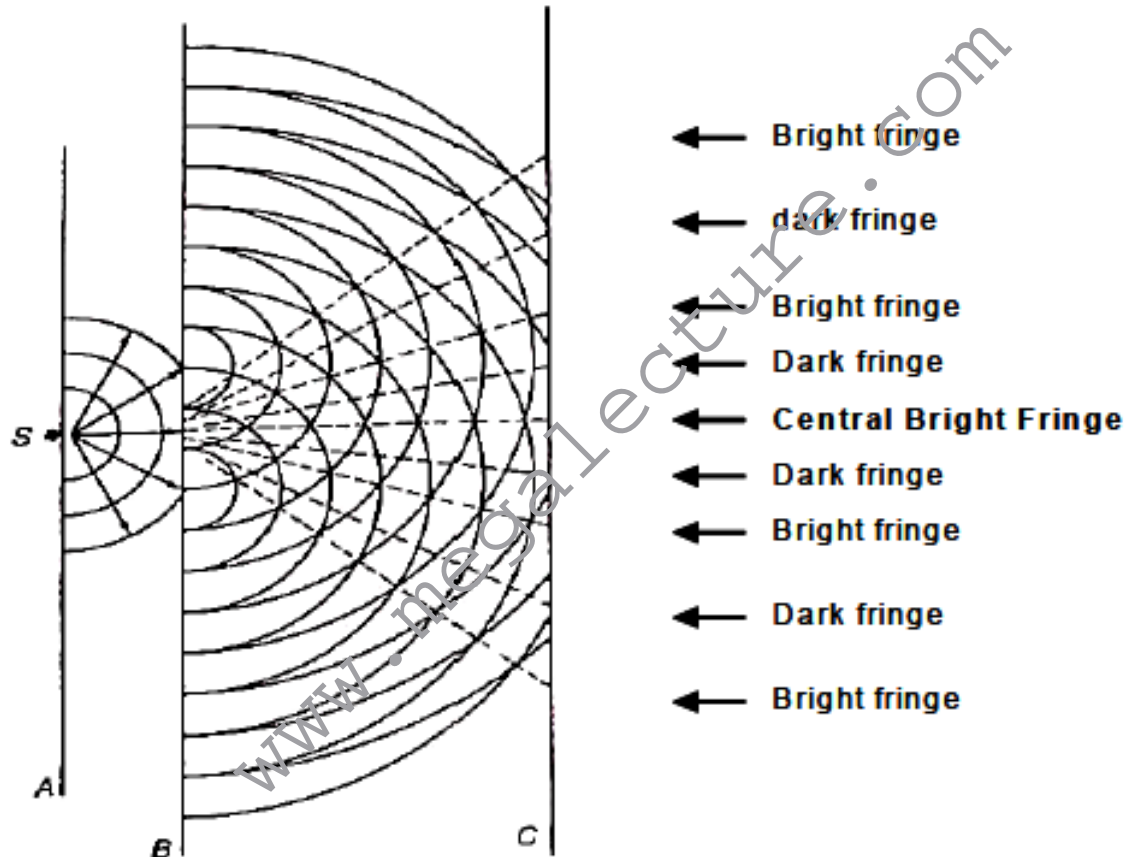




In diagram (a) above, the narrow double slits act as wave sources. Slit S_1 and S_2 behave as coherent sources (waves coming from them are always at a constant phase difference) that produce an interference pattern on screen C.

This interference pattern (fringe pattern) is shown in diagram (b). Separation between successive bright fringes (centre to centre) is the **fringe spacing**.

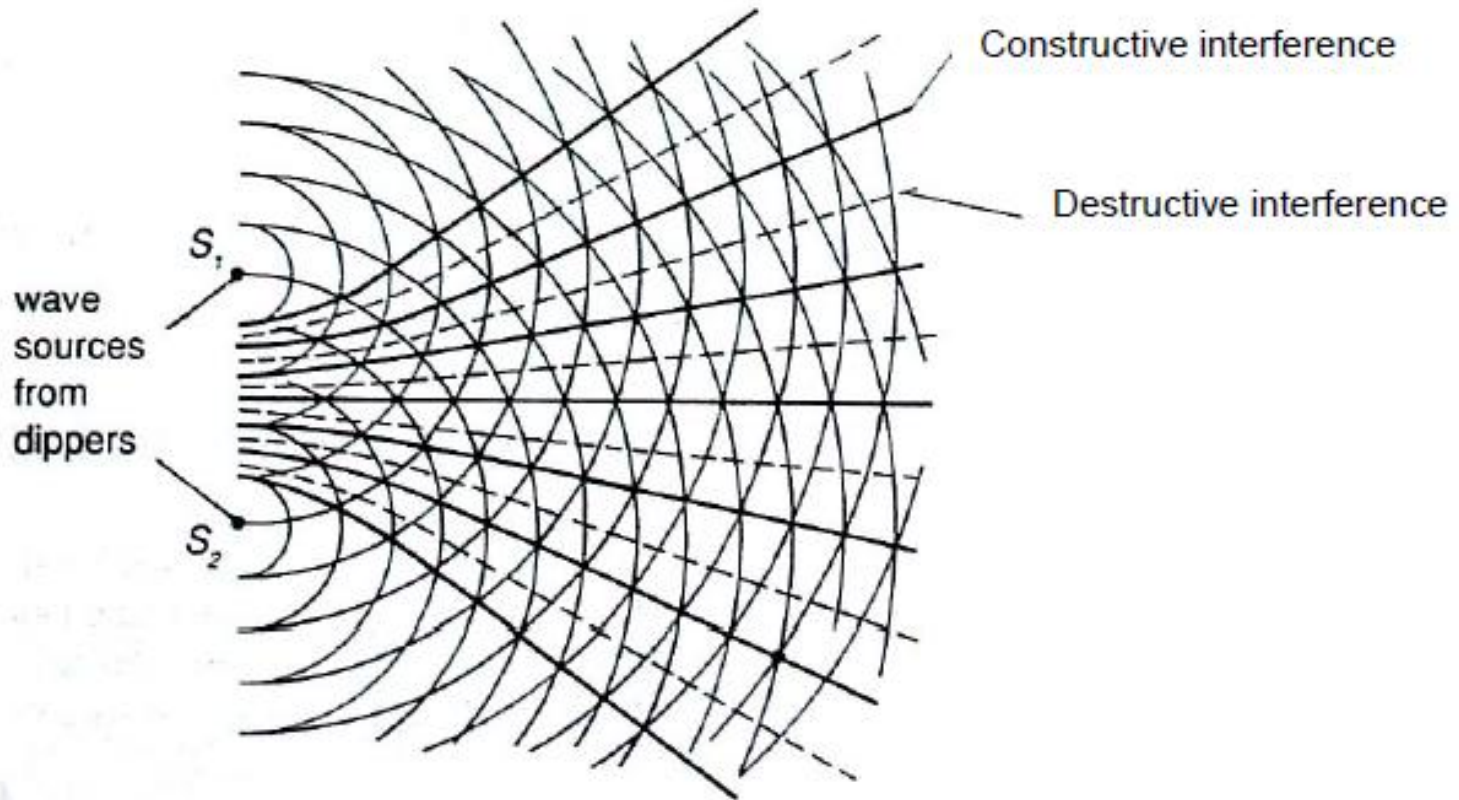
The bright fringes are formed due to constructive interference (i.e. the waves arrive at these points in phase), while the dark fringes are due to destructive interference (i.e. the waves arrive at these points in anti-phase - hence no resultant amplitude, which then appears dark).



Schematic diagram of Young's double-slit experiment is shown above.

Two-source interference with water

An interference pattern involving water waves is produced by two vibrating sources at the water surface. The lines represent crests, and the spaces between the lines represent troughs. The regions where the lines intersect (spaces also intersect) have constructive interference. The regions where lines intersecting spaces have destructive interference.



Conditions required for two-source interference fringes to be observed

For interference fringes to be **observable**:

- The sources must be **coherent**; that is, they must maintain a constant phase difference.
- The sources must have the same frequency (for light waves, this means that they must be monochromatic).
- The principle of superposition must apply (the sources must produce the same type of waves).
- The sources must have (approximately) the same amplitude .

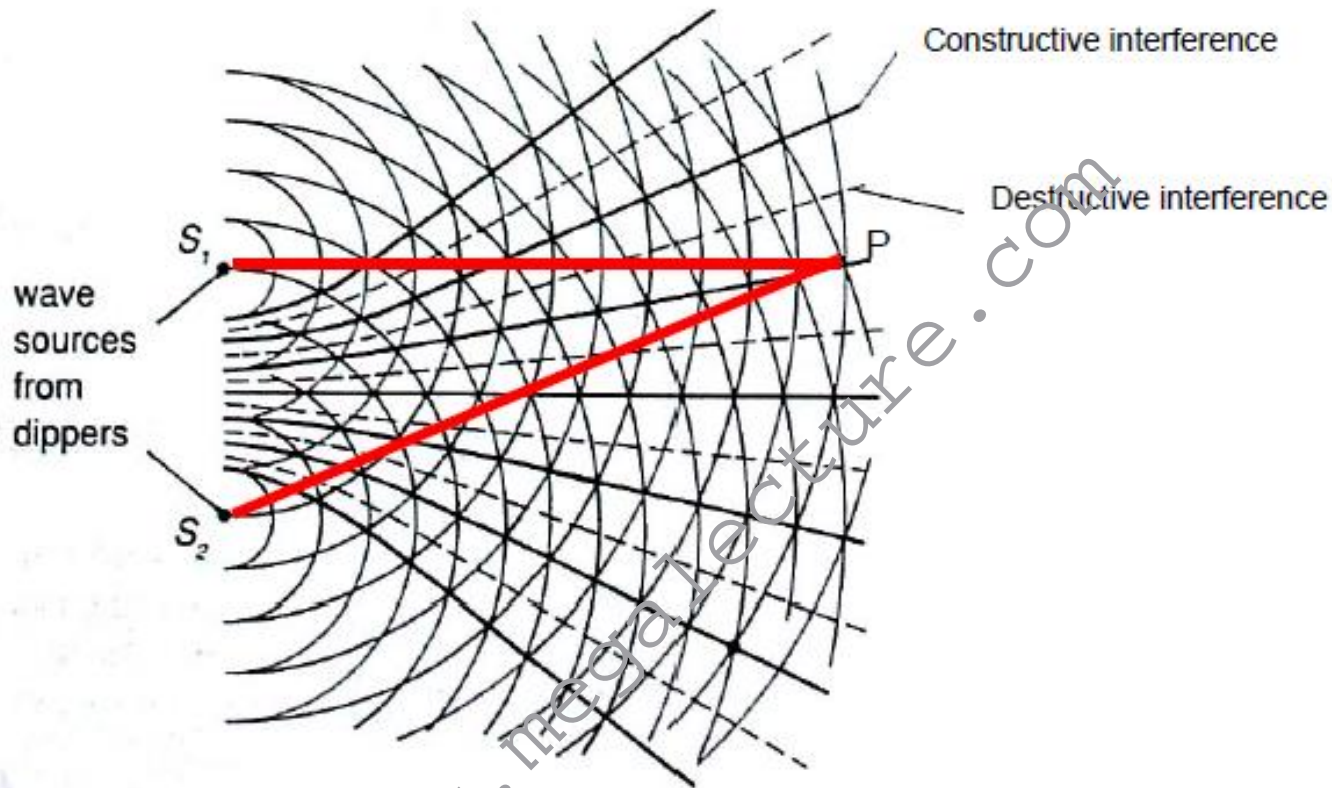
- **For light waves,**
 - ❖ the wavelengths used should be in the visible range (400 nm to 700 nm).
 - ❖ the source (slit) separation (d) is around the order of 10^{-4} m.
 - ❖ the screen-slits distance D is around 1 – 2 m.

Just for your understanding only : (not in syllabus)

What is the meaning of 'a constant phase difference' between two coherent waves' ?

- The primary source of light is transition of electrons. This happens for every source be it the fluorescent tubes or the sun .
- As an electron jumps to its higher level it reaches an unstable excited state. It stays there for about 10 ns and comes back to the ground state. Thus every 10ns a new stream of light is produced.
- If we have two sources of light then the phase difference between any two waves would be random. In fact it would change every 10ns or so. *This is why we don't find interference in practical life.*
- To have a constant phase difference between two waves (i.e. to be coherent.) the waves should be from a common source, so that there is no ab-nitio phase difference to get a constant phase difference and two waves (from the same source) can be made to have some path difference.

Conditions for constructive interference

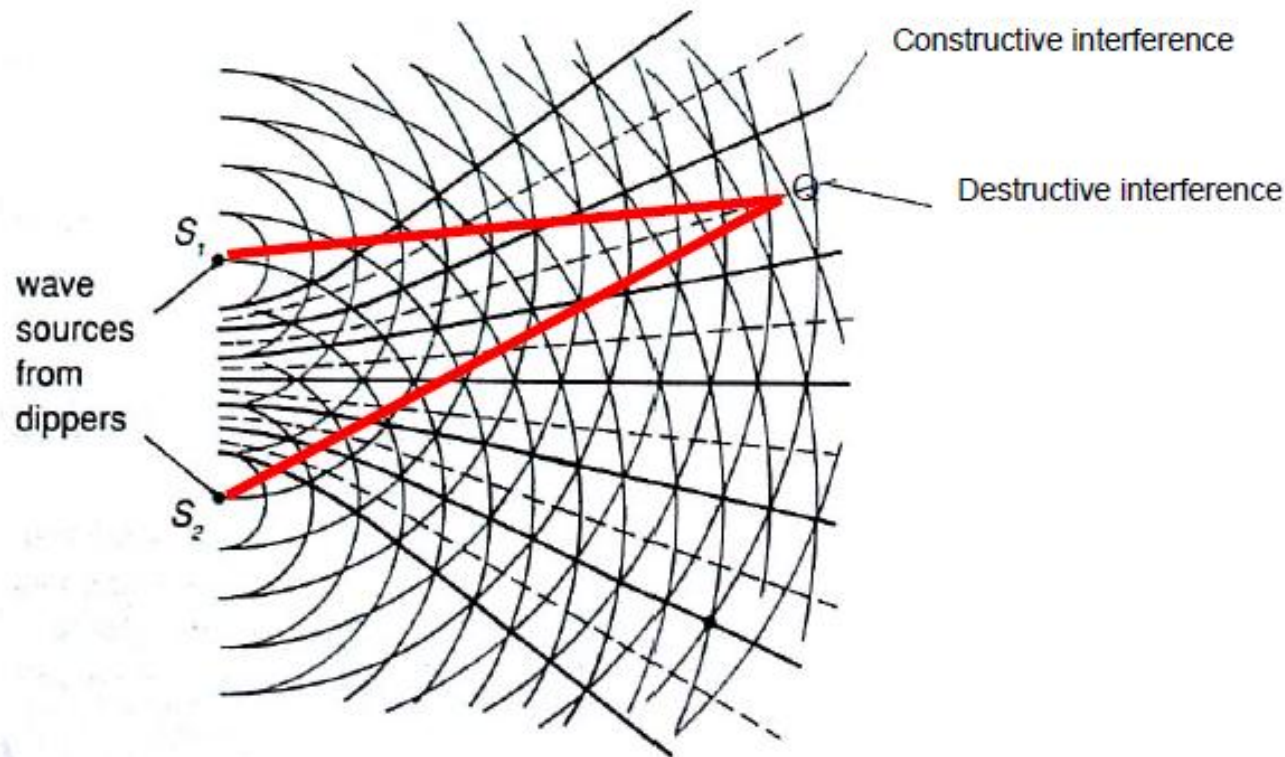


The 2 thick lines intersecting at P represent the paths of water waves from the 2 sources to produce a constructive interference at P .

Length $S_2P = 14\lambda$, length $S_1P = 13\lambda \rightarrow$ their path difference $= 14\lambda - 13\lambda = \lambda$

For other points with constructive interference, the **path difference must be $n\lambda$** , where n is an integer. The assumption here is that the sources are in phase.

Conditions for destructive interference



The 2 thick lines intersecting at Q represent the paths of water waves from the 2 sources to produce a destructive interference at Q.

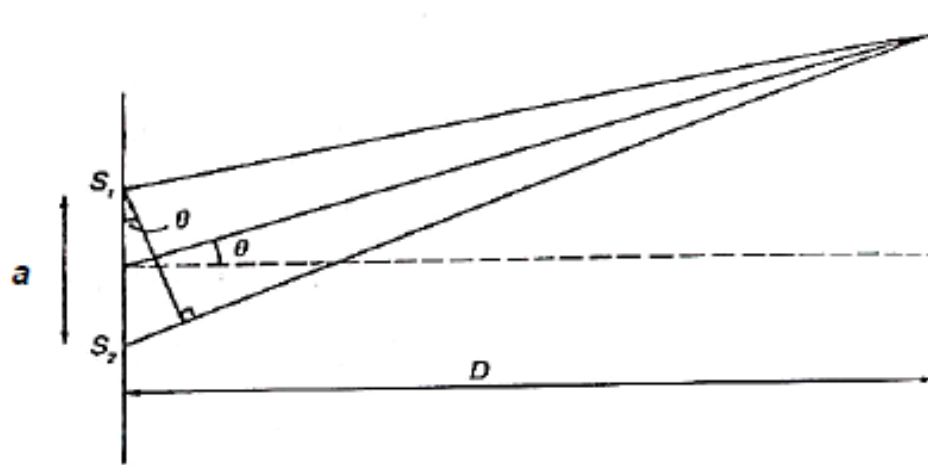
Length $S_2Q = 14\lambda$, length $S_1Q = 12\frac{1}{2}\lambda \rightarrow$ their path difference $= 14\lambda - 12\frac{1}{2}\lambda = 1\frac{1}{2}\lambda$

For other points with destructive interference, the **path difference must be** $(n + \frac{1}{2})\lambda$, where n is an integer. The assumption here is that the source are in phase.

Using the equation $\lambda = \frac{ax}{D}$
for double-slit
interference using light

Note : As per syllabus, you need to have an understanding of young's double slit experiment, but you don't have to prove/derive the equation.

Young's Double Slit



In the double slit experiment, bright & dark fringes alternate at **equal separation**.

The double slit interference is given by the equation

$$\frac{\lambda}{a} = \frac{x}{D}$$

λ is the **wavelength** of the light

a is the **separation** of the **slits**

x is the **separation** of the **fringes** on the screen (**fringe spacing**, separation between centres of bright fringes, or centres of dark fringes)

D is the **separation** between the **screen** and the **double-slit**

Check your understanding!

- **Do headlights from a car form interference patterns? Why?**

- The interference would be 'visible' if the two sources are oscillating in phase or have a constant phase difference. This is why a single light source (as in young's double slit experiment) is split to produce two which are then coherent.
- In addition the separation of the headlamps is so large that any interference fringes would be too close together to be easily measurable, and the path difference between waves would render them no longer coherent.
- Note that, because the wavelength of light is so small (of the order of 10^{-7} m) to produce observable fringes 'D' needs to be large and 'a' as small as possible. *(This is one of the application of equation of young's double slit experiment)*

Sample problem 7

Which of the following statements must be true about two wave-trains of monochromatic light arriving at a point on a screen if the wave-trains are coherent?

- A They are in phase.
- B They have a constant phase difference.
- C They have both travelled paths of equal length.
- D They have approximately equal amplitudes.
- E They interfere constructively.

Solution (Ans: B)

Only **[B]** is true as this is the meaning of coherence.

Sample problem 8

When a two-slit arrangement was set up to produce interference fringes on a screen using a monochromatic source of green light, the fringes were found to be too close together for convenient observation.

In which of the following ways would it be possible to increase the separation of the fringes?

- A Decrease the distance between the screen and the slits.
- B Increase the distance between the source and the slits.
- C Have a larger distance between the two slits.
- D Increase the width of each slit.
- E Replace the light source with a monochromatic source of red light

Solution: (Ans: E)

Since it is a double slit setup, using the equation $\frac{\lambda}{a} = \frac{x}{D} \rightarrow x = \frac{\lambda D}{a}$

- A decrease $D \rightarrow x$ will decrease
- B increase distance between source & slits does not affect the fringe separation
- C increase $a \rightarrow x$ will decrease
- D increasing the width of the slit will not affect x , but will allow more light through, hence a brighter pattern
- E replace green light with red light. $\lambda_{\text{red}} > \lambda_{\text{green}} \rightarrow$ if λ increases, x will increase.

Sample problem 9

- Calculate the observed fringe width for a young's double slit experiment using light of wavelength 600nm and slits 0.50nm apart. The distance from the slits to the screen is 0.80m.

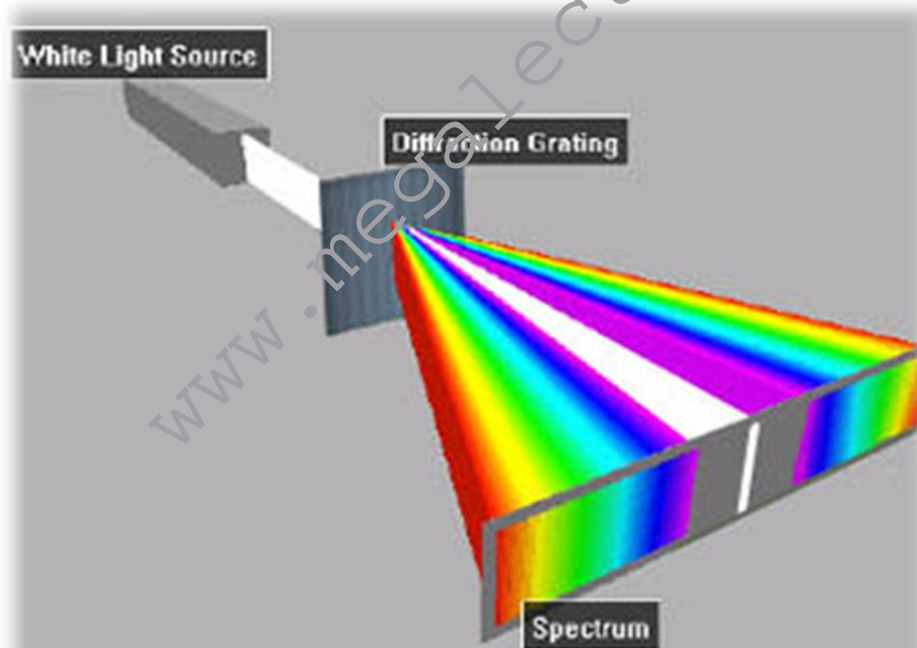
Solution :

$$\text{Using } \lambda = \frac{ax}{D}$$

$$\begin{aligned} x &= 600 \times 10^{-9} \times 0.80 / 0.50 \times 10^{-9} \\ &= 960 \text{ m} \end{aligned}$$

Use of a diffraction grating to determine the wavelength of light

- A **diffraction grating** is a plate on which there is a very large number of identical, parallel, very closely spaced slits.
- If a monochromatic light is incident on this plate, a pattern of narrow bright fringes is produced.

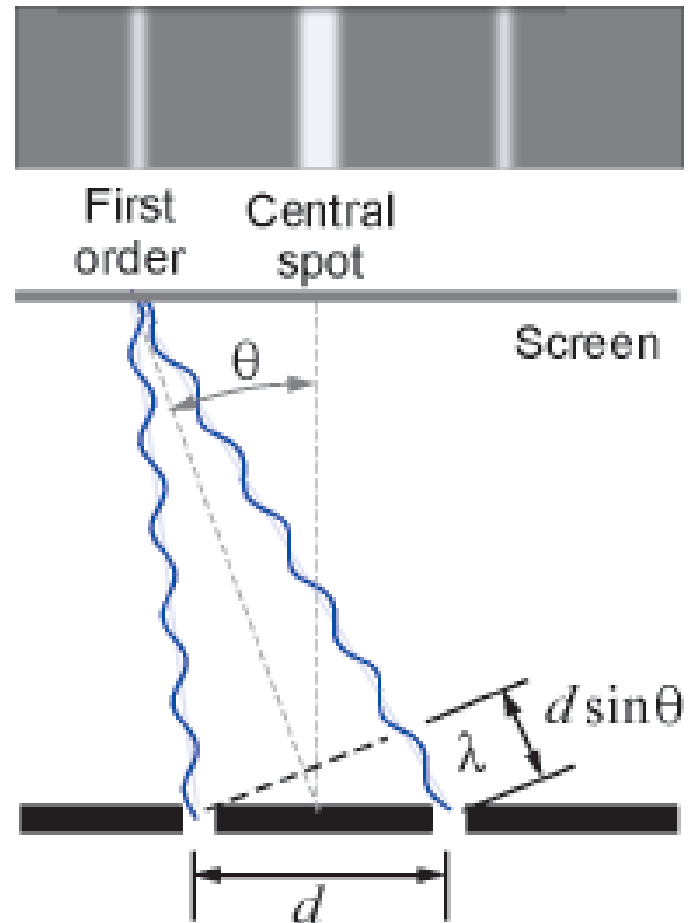


How a Diffraction Grating Works

When you look at a diffracted light you see:

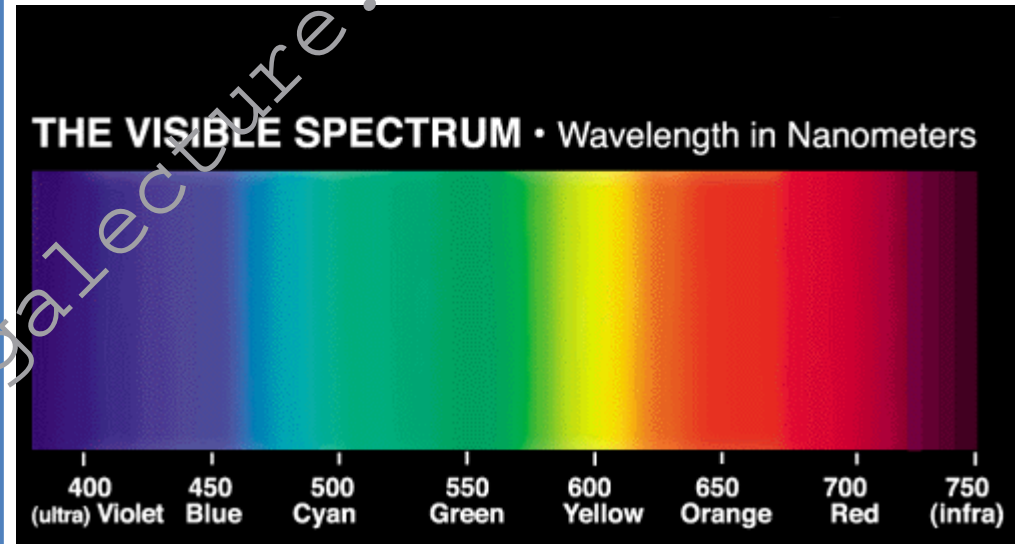
- the light straight ahead as if the grating were transparent.
- a "central bright spot".
- the interference of all other light waves from many different grooves produces a scattered pattern called a spectrum.

Interference pattern



Application of Diffraction Grating

- A diffraction grating can be used to make a spectrometer and a spectrometer is a device that measures the wavelength of light.

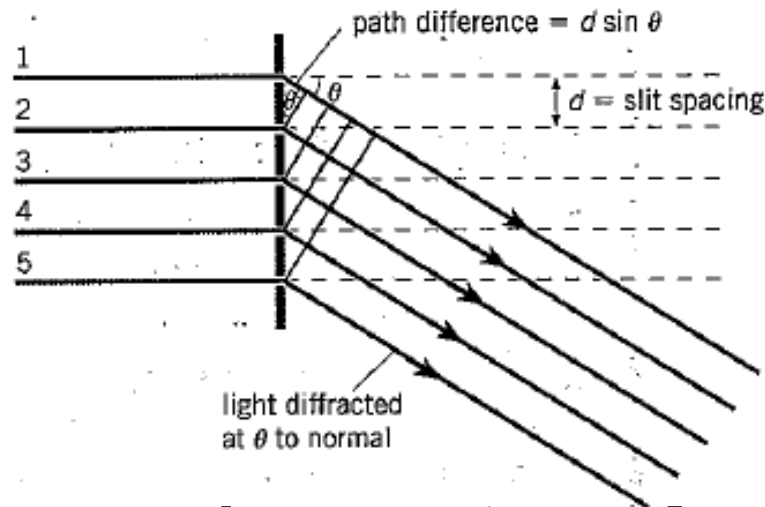


The equation : $d \sin \theta = n\lambda$

Figure 8.54 shows a parallel beam of light incident normally on a diffraction grating in which the spacing between adjacent slits is d . Consider first rays 1 and 2 which are incident on adjacent slits. The path difference between these rays when they emerge at an angle θ is $d \sin \theta$. To obtain constructive interference in this direction from these two rays, the condition is that the path difference should be an integral number of wavelengths. The path difference between rays 2 and 3, 3 and 4, and so on, will also be $d \sin \theta$. The condition for constructive interference is the same. Thus, the condition for a maximum of intensity at angle θ is

$$d \sin \theta = n\lambda$$

where λ is the wavelength of the monochromatic light used, and n is a whole number.



The equation : $d \sin \theta = n\lambda$

(continued from previous slide)

When $n = 0$, $\sin \theta = 0$ and θ is also zero; this gives the straight-on direction, or what is called the zero-order maximum. When $n = 1$, we have the first-order diffraction maximum, and so on (Figure 8.55).

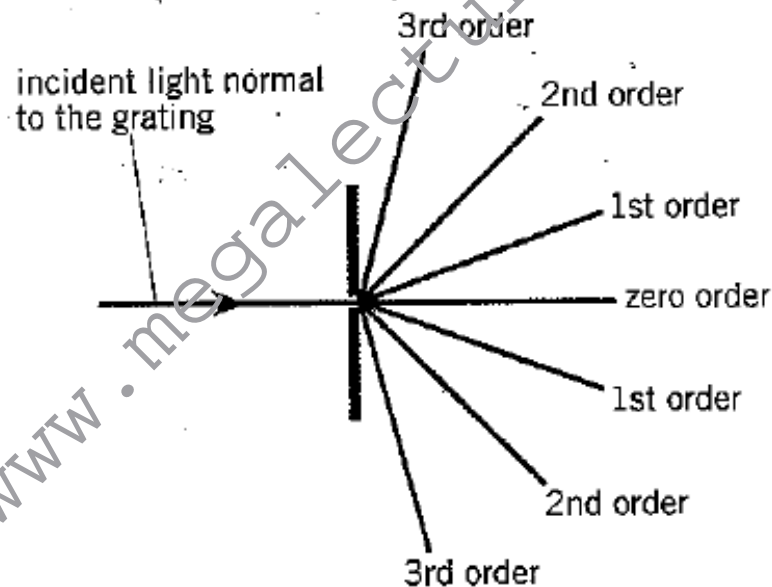


Figure 8.55 Maxima in the diffraction pattern of a diffraction grating

Sample problem 10

Monochromatic light is incident normally on a grating with 7.00×10^5 lines per metre. A second-order maximum is observed at an angle of diffraction of 40.0° . Calculate the wavelength of the incident light.

The slits on a diffraction grating are created by drawing parallel lines on the surface of the plate. The relationship between the slit spacing d and the number N of lines per metre is $d = 1/N$. For this grating, $d = 1/7.00 \times 10^5 = 1.43 \times 10^{-6}$ m. Using $n\lambda = d \sin \theta$,
 $\lambda = (d/n) \sin \theta = (1.43 \times 10^{-6}/2) \sin 40.0^\circ = \mathbf{460 \text{ nm}}$.

